DYNAMIC METEOROLOGY AND HYDROGRAPHY

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I

WASHINGTON, D. C.

PUBLISHED BY THE CARNEGIE INSTITUTION OF WASHINGTON

1910

CARNEGIE INSTITUTION OF WASHINGTON

PUBLICATION NO. 88

PRESS OF
THE NEW ERA PRINTING COMPANY
LANCASTER, PA.

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Insert in Volume One, Publication 88.

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DYNAMIC METEOROLOGY AND HYDROGRAPHY

PART I. STATICS

BY

V. BJERKNES AND J. W. SANDSTRÖM

CHAPTER I.

SYSTEM OF UNITS.

1. Meter-Ton-Second System.— In quantitative physical investigations the absolute units of the centimeter-gram-second system are now in general use. Sometimes these units are used directly. But, as one set of units can not have the proper magnitude for all sorts of measurements, special practical units are in many cases introduced which are derived from the corresponding fundamental units by the multiplication by suitable powers of 10. The choice of practical units is a question of great importance. It is of great advantage if they themselves form a connected system, or if they are at least in some simple relation to a connected system which can be used as a system of reference, the incessant troublesome return to the c.g.s. system being thus avoided.

For the purpose of dynamic meteorology and hydrography, the centimeter and gram are too small as units of length and mass. If for unit-length we choose the meter, and for unit-mass the metric ton, *i. e.*, the mass of a cubic meter of water at maximum density, great advantages are gained. The choice of a convenient unit of time unfortunately implies difficulties. Evidently the second is far too small a length of time for the measurement of changes in the state of the atmosphere and still more so in that of the sea. But the circumstance that the division of time is not decimal makes every change in the fundamental unit of time inadvisable. As fundamental units of reference we shall therefore use consistently

Meter $= 10^2$ centimeters. Metric ton $= 10^6$ grams. Second.

We shall refer to this system as the meter-ton-second system, or the m.t.s. system.

To the fundamental mechanical units we have to add, finally, the fundamental thermal unit. For this we shall choose the degree of the centigrade thermometer.

Unfortunately there is great confusion as to the units to which meteorological and hydrographical observations are referred. As we proceed we shall, therefore, give the tables required to derive from the observations recorded in the principal publications the results we wish to express in the units used in this treatise. These auxiliary tables, which would be superfluous if all observations were recorded in absolute units, are collected in the annexed "Appendix to meteorologic and hydrographic tables."

2. Simplest Derived Units. — From the values of the fundamental units those of the derived units are easily deduced. For completeness we shall add the dimensions of each derived quantity, expressed in the usual way in terms of length L, mass M, and time T.

The m.t.s. unit-velocity $[LT^{-1}]$ is the velocity of 1 meter per second, or 100 c.g.s. units of velocity.

The m.t.s. unit of momentum $[MLT^{-1}]$ is the momentum of the mass of a ton moving with the defined unit velocity. It is, therefore, equal to 100,000,000 c.g.s. units of momentum.

The m.t.s. unit of acceleration $[LT^{-2}]$ is the acceleration of 1 meter per second, or 100 c.g.s. units of acceleration. The acceleration of gravity is, therefore, in the m.t.s. system, equal to 9.8 approximately, or in rougher approximation equal to 10.

The m.t.s. unit of force $[MLT^{-2}]$ is the force which gives the mass of a ton the defined unit-acceleration. This unit of force is equal 100,000,000 c.g.s. units or dynes, *i. e.*, equal to 100 megadynes. Taking 10 for the acceleration of gravity, it will represent the weight of a tenth of a ton or of 100 kilograms.

The m.t.s. unit-impulse $[MLT^{-1}]$ is the change of momentum given by the defined unit-force during the time of 1 second. This unit-impulse is equal to 100,000,000 c.g.s. units of the same quantity. With respect to numerical value and dimensions this unit is identical with that of momentum.

The m.t.s. unit of force per unit-mass, sometimes called accelerating force $[LT^{-2}]$, is equal to the defined unit-force per ton of mass which is subject to the action of the force. It is equal, therefore, to 100 c.g.s. units of the same quantity. With respect to numerical value and dimensions the unit of force per unit-mass is identical with that of acceleration. The weight per unit-mass of a resting body is thus numerically equal to the acceleration which the body would take if it was free to fall. On account of this numerical accordance the expression "acceleration of gravity" is used to designate the intensity of gravity measured statically, *i. e.*, the weight per unit-mass of a heavy body.

The m.t.s. unit-work $[ML^2T^{-2}]$ is the work performed by the defined unit-force over the length of 1 meter. This unit-work is 10,000,000,000 c.g.s. units or ergs, *i. e.*, 10,000 meg-ergs. A unit of work in common use is the *joule*, which is equal to 10 meg-ergs. The m.t.s. unit of work, therefore, is a *kilojoule*. It represents approximately the work performed by lifting 1 ton 1 decimeter, or 100 kilogram-meters.

The m.t.s. unit of kinetic energy $[ML^2T^{-2}]$ is the kinetic energy of the mass of 1 ton moving with the unit-velocity defined above. The unit-increase of kinetic energy and also the unit-increase of potential energy $[ML^2T^{-2}]$ are obtained as equivalents for a unit of work performed. For this reason we can use numerically the kilojoule as a unit of kinetic and of potential energy as well as of work. When gravity is the acting force, unit-increase of potential energy is obtained by lifting 1 ton the approximate height of 1 decimeter.

The m.t.s. unit of activity $[ML^2T^{-3}]$ is the activity of 1 kilojoule per second. This is the kilowatt, an extensively used unit, introduced to replace the old unit of activity, the horsepower. The kilowatt is equal to 10,000,000,000 c.g.s. units of activity, and equal to 1.36 horsepower.

3. Units Used in Dynamics of Continuous Media. — In elementary dynamics definite masses are considered, to which the above-mentioned quantities are referred. In the dynamics of continuous media we have to deal with continuous distributions in space of mass, as well as of the quantities serving to define the static or the dynamic state of this distribution of mass. We then meet with the idea of *fields* of scalar as well as of vectorial quantities.

The purely kinematic quantities velocity and acceleration can be used at once for the description of fields in continuous media. But the quantities involving the idea of mass are not immediately serviceable. They must be referred either to ***unit-mass*** or to ***unit-volume*** of the medium.

The distribution of mass itself is described either by the volume per unit-mass or by the mass per unit-volume of the medium. The first of these quantities is the *specific volume* $[M^{-1}L^3]$, the second is the *density* $[ML^{-3}]$. They are reciprocal to each other, and the units in the m.t.s. system are the same as in the c.g.s. system.

Referring a mechanical quantity once to unit-mass and once to unit-volume of the medium, we arrive at two corresponding quantities. The passage from a quantity referred to unit-mass to the corresponding quantity referred to unit-volume involves the multiplication by a density, while the return involves the multiplication by a specific volume.

Most investigations in the dynamics of continuous media have been restricted to the case where the media are homogeneous. Then the fields of the corresponding quantities do not differ essentially from each other in their geometrical feature. This is the reason why the correspondence mentioned has attracted no greater attention hitherto. But in the problem now before us we shall have to treat the dynamics of essentially heterogeneous media. In this case the fields of corresponding quantities may differ widely from each other, and it is important to notice the analogies as well as the contrasts in these fields.

Momentum when referred to unit-mass leads back to the velocity, while momentum per unit-volume or specific momentum $[ML^{-2}T^{-1}]$ is the product of a velocity by a density. The m.t.s. unit of specific momentum is equal to 100 c.g.s. units of the same quantity, just as in the case of velocity. Velocity and specific momentum are the two corresponding quantities serving to describe the fields of motion in a continuous material medium.

Force when referred to unit-mass leads back to accelerating force, or acceleration, while force per unit-volume $[ML^{-2}T^{-2}]$ is equal to the product of an acceleration by a density. The m.t.s. unit of force per unit-volume is equal to 100 c.g.s. units of the same quantity, just as in the case of force per unit-mass. For the description of fields of force, the two defined kinds of force are theoretically equivalent to each other. The acceleration of gravity, used generally to describe the gravitational field of force, is a force per unit-mass. The gradient serving to describe the field of force due to a distribution of pressure in a fluid is a force per unit-volume. But for special reasons it may also be useful occasionally to describe the gravitational field by the force per unit-volume, and the field due to the pressure by the force per unit-mass of the medium.

The kinetic energy per unit-mass has the dimensions of the square of a velocity $[L^2T^{-2}]$. The kinetic energy per unit-volume is the square of the velocity multiplied by the density of the moving medium $[ML^{-1}T^{-2}]$. The units of each of these quantities in the m.t.s. system are equal to 10,000 of their c.g.s. units. They are perfectly equivalent to each other for the description of the field of kinetic energy in a moving medium. The work per unit-mass and per unit-volume have the same dimensions, respectively, as the kinetic energy per unit-mass and per unit-volume, and can be measured by the same units.

Activities referred either to unit-mass or to unit-volume come into consideration when processes of continuous transformations of energy are going on in the medium. The units of these quantities in the m.t.s. system are also equal to 10,000 of the corresponding c.g.s. units.

The gravity potential is a quantity which has the character of a work per unit-mass $[L^2T^{-2}]$, while a pressure is a quantity which has the character of a work per unit-volume $[ML^{-1}T^{-2}]$. The pressure is defined in a more elementary manner as a force per unit-area. But, however the definition be chosen, potential and pressure are closely related to each other from a theoretical point of view, and in a broader sense of the word they may be considered as corresponding quantities. Their dimensions differ by a quantity of the dimensions of a density, and their units in the m.t.s. system are equal to 10,000 of their c.g.s. units. As the units of these two quantities are of special importance to us, they will be discussed separately.

4. Units of Gravity Potential. — To every point in space we attribute a certain value of the gravity potential, defined numerically by this rule: It is equal to the potential energy relatively to sea-level possessed by a unit-mass situated in the point. The gravity potential of a point is therefore equal to the amount of work required to lift unit-mass from sea-level to the point against the action of gravity.

To unit-increase of gravity potential will therefore correspond, in any given locality, a definite increase of height, numerically equal to the reciprocal value of the acceleration of gravity. This increase of height will be slightly different in different localities, depending on the variations from place to place of the acceleration of gravity. But setting smaller variations aside, and taking 10 for the acceleration of gravity in the m.t.s. system, the height giving unit rise of potential will be equal to a decimeter.

To fix in our minds the approximate value of this height, we shall call the m.t.s. unit of gravity potential a *dynamic decimeter*. A ten times greater unit is the dynamic meter. Expressing gravity potentials in this latter unit, we gain the practical advantage that the number giving the gravity potential of a point will be very nearly equal to the number giving its height above or its depth below sea-level, expressed in common meters. This fortunate accordance makes it very convenient to use the dynamic meter as a technical unit of gravity potential. Values of the gravity potential expressed by an integer number of dynamic meters will be called *standard values*, and will be used very much as representatives for heights or depths.

But it should be emphasized that the dynamic meter and its subdivisions are units of gravity potential, not of length. In every given locality, however, they represent definite lengths measured along the plumb-line, and for this reason they can be used as full equivalents for the common length-measure when distances measured along the plumb-line are concerned.

5. **Units of Pressure.** — The unit-pressure of the m.t.s. system is the pressure of the unit-force defined above when it is exerted over the area of a square meter, and, as mentioned already, is equal therefore to 10,000 c.g.s. units of pressure, or 10,000 dynes per square centimeter. To avoid circumlocution, it will be necessary for us to have names for the employed units of pressure. The megadyne per square centimeter is approximately equal to the present practical unit, the atmosphere. It has often been proposed to introduce the megadyne per square centimeter as a practical unit of pressure, and to designate it by some name derived from the word "barometer." We shall choose the name *bar* as being the shortest, and designate the decimal parts of it as the decibar, centibar, and millibar. The m.t.s. unit of pressure will then be the *centibar*, while the c.g.s. unit will be the *microbar*.

Very simple rules are obtained for the columns of water exerting these pressures if we agree to have the heights of the water-columns represented by their values in dynamic meters, their multiples or subdivisions. Taking pure water at maximum of density, and neglecting its compressibility, we get these relations:

1 bar = pressure of 1 dynamic decameter of water.
1 decibar = pressure of 1 dynamic meter of water.
1 centibar = pressure of 1 dynamic decimeter of water.
1 millibar = pressure of 1 dynamic centimeter of water.

Finally, the c.g.s. unit, the microbar, is equal to the pressure of 10 dynamic microns of water.

Among these units we shall use often the decibar as a technical unit, on account of its correspondence to the dynamic meter as unit of gravity potential. Completing our terminology, we shall denote pressures represented by an integer number of decibars as *standard* pressures. In cases where we have to do with the relations between pressures and gravity potentials we shall often refer to these standard values of both quantities, using thus dynamic meter and decibar as connected units. But when the relation to other quantities comes in, we shall have to return to the m.t.s. units, the centibar, and the dynamic decimeter.

We shall also make frequent use of the millibar as that technical unit which is most convenient in reading the barometer. It will replace the present practical units, the millimeter or the inch of mercury. Using 13.59545 for the density of mercury at 0° C.,* and 9.80617 for the standard value of gravity (compare section 8 below), we find that 1 meter of mercury of 0° C. at a place where gravity has this standard value exerts the pressure of 1.333193 bars. Thus, a mercury

* THIESEN UND SCHEEL: Tätigkeitsbericht der Phys. Techn. Reichsanstalt, 1 Feb., 1897–31 Jan., 1898. Berlin, 1898.

barometer which gives, for standard value of gravity, direct readings in millibars is a barometer whose scale has its divisions at the mutual distance of 0.750079 mm. or 0.75 mm., practically, instead of at the distance of integer millimeters.

In meteorology it is common to give the barometric pressure either in millimeters or in inches of mercury. The millimetric division is not in the least more rational than the division into inches. Neither of them has anything to do with the system of absolute units. The consequences of this irrationality have not yet been seriously felt, because the barometric records have until now served for qualitative purposes mainly. But the further development of dynamic meteorology will compel us to introduce rational units sooner or later. Meanwhile we shall be obliged to change from the one system of units to the other by auxiliary tables.

The tables required for the direct passage from millimeters or inches of mercury to millibars are given in the Appendix. In many cases, however, it will be a saving of time and labor for a while to retain the units to which the original observations are referred, in order to carry out the transition to the rational units at a later stage of the work of computation, as will be developed in the proper places below.

CHAPTER II.

GRAVITY AND GRAVITY POTENTIAL.

6. Gravity. — The exterior force upon which the conditions of equilibrium and motion in the atmosphere and in the sea depend is gravity. By gravity without further specification we mean the force the intensity of which is found by the pendulum experiments. It is the resultant of two different actions — the attraction of the earth and the centrifugal force due to the earth's rotation. But in practical application we shall never make use of this decomposition of the force into the two components of different origin.

A first condition for the solution of concrete problems relating to the equilibrium and the motion of the air and the sea is therefore a knowledge of the intensity of gravity at every point of the space filled by these two media. This knowledge is founded on the actual measurements of the intensity of gravity at the earth's surface. But it is not necessary for us to take into consideration all the small irregularities in the variation of this force as they present themselves in geodetic investigations. Where no measured values of the intensity of gravity are at hand it will suffice to work with the "normal" values, as they can be calculated by the general formulæ of geodesy. They will give an approximation far closer than that by which we can find the values of any other force upon which the atmospheric or oceanic equilibrium or motion depends.

We shall therefore write down the formulæ necessary for the calculation of this normal intensity of gravity, and give a complete tabulation of these formulæ. According to the common terminology, we shall call the tabulated quantity the acceleration of gravity. But it should be remembered that it represents, as already mentioned (section 2), at the same time the intensity of gravity measured statically by the weight per unit-mass of the heavy body.

7. Normal Decrease of Gravity in the Atmosphere. — Let the numerical value g_1 of the acceleration of gravity be known at a point of the earth's surface. Its value g can then be calculated at any height z above this point from the decrease of the attraction with the increase of the distance from the attracting masses, and from the increasing influence of the centrifugal force with the increasing distance from the earth's axis. Setting aside quantities of the order of magnitude of the square of the ratio z/r, z being the height and r the radius of the earth, we find, according to Helmert,* as the best expression for the decrease of the gravity with the height,

$$(a) \qquad g = -(g_1 - 0.000003086z)$$

* Helmert: Ueber die Reduction der auf der physischen Erdoberfläche beobachteten Schweerebeschleunigungen auf ein gemeinsames Niveau, zweite Mitteilung. Sitzungsberichte der Akademie der Wissenschaften, Berlin, 1903, p. 650.

The sign $-$ is used because the acceleration of gravity is directed downwards, while we take the direction upwards as positive. The value of the correction term $-0.000003086z$ is given in table 1 M of the Meteorological Tables.

8. Reduction to Sea-Level and Normal Value of Gravity at Sea-Level. — By sea-level we mean on the one hand the surface of the sea in the case of perfect equilibrium and on the other an ideal continuation of this surface below the continents, determined by the condition of being always at right angles to the plumb-line.

Values of the acceleration of gravity, which are found by pendulum experiments at the surface of the earth, are reduced to sea-level to make them intercomparable. The purpose of the reduction is to arrive as closely as possible to the theoretical value of the acceleration of gravity, which is a function only of the latitude and which depends upon the figure and the rotation of the earth, all irregularities of topography and of local mass distribution being neglected. There has been much discussion as to how this reduction should be performed properly. Two different views have been advanced, based upon physically different conceptions of the nature of the equilibrium of the earth's crust. According to the first view the equilibrium is that of a solid elastic body. The masses of the continents present above sea-level are considered as additional masses whose weight is carried by the stress produced in the solid crust of the earth. According to the second view, the earth's crust has sufficient stiffness only to carry the weight of local elevations above the main level of the land, while on a larger scale the equilibrium is of a hydrostatical nature. The elevation of the continents above sea-level are, then, due to their buoyancy, their density being smaller than the average density of the earth's crust. The average density would be attained if the masses of the continents present above sea-level were absorbed by the underlying masses.

These two views of the nature of the equilibrium of the earth's crust lead of course to two different principles for the reduction to sea-level. According to the first view, the continental masses present above sea-level represent a surplus of mass, the attraction of which must be subtracted if the reduction should lead to the required normal value. This leads to the reduction according to the formula of Bouguer, which until lately has been used almost universally. According to the second view the reduction is made as if the continental masses were absorbed by the earth's crust below the continents, no mass being present between the physical surface of the earth and sea-level. The reduction is, then, simply the same as in the free air.

According to the result of recent geodetic investigations * this simple reduction leads with much closer approximation to the normal value of gravity at sea-level than the reduction according to the formula of Bouguer. Thus the theory of the

* G. R. Putnam: Results of a transcontinental series of gravity measurements. Phil. Soc. of Washington, February 2, 1895. Bulletin of the Society, vol. 13. Washington, D. C., 1900, p. 31.

G. K. Gilbert: Notes on the gravity determinations reported by Mr. G. R. Putnam. Phil. Soc. of Washington, March 16, 1895. Bulletin of the Society, vol. 13. Washington, D. C., 1900, p. 61.

R. v. Sterneck: Relative Schweerebestimmungen. Mitteilungen der Militär-Geographischen Institut. Wien, 1898, p. 100.

F. R. Helmert: Ueber die Reduction der auf der physischen Erdoberfläche beobachteten Schweerebeschleunigungen auf ein gemeinsames Niveau. Sitzungsberichte der Akademie der Wissenschaften. Berlin, 1902, p. 843; 1903, p. 650.

approximate hydrostatic equilibrium of the masses in the earth's crust is verified. More recently this verification has also been extended to the open sea by the measurements of the Nansen Expedition in the Polar Sea and those of Hecker on the Atlantic.* These results are very important for dynamic meteorology and hydrography, as they show that the gravitational field of force in atmosphere and sea is much more regular than originally supposed. The continental masses present above sea-level do not cause perturbations of the field. On the contrary, they make it more regular, because they compensate for subterranean mass defects. Neither does the sea, with its smaller density, complicate the field, because there are compensating excesses of mass below the sea-bottom. The only perturbations of the field are due to irregularities of local topography or of local mass distribution sufficiently small to be balanced by the elastic stresses which they produce in the earth's crust. We shall make no corrections for these local irregularities. The reduction to sea-level of the numerical value g_1 of the acceleration of gravity found by pendulum experiments at the earth's surface at the height z above sea-level will be given by the formula

$$(a) \qquad g_0 = g_1 + 0.000003086z$$

the correction term being the same as that of formula section 7 (*a*), or of table 1 M of Meteorological Tables, but with the sign reversed. We shall use this reduction consistently in cases where we start with really measured values of the acceleration of gravity at the earth's surface. It will be convenient, as all heights are measured from sea-level, and the reduction will bring in no errors in the values of gravity calculated for the free air, as errors possibly introduced by the use of formula (*a*) for reductions downward will drop out again by the reduction upward, according to formula section 7 (*a*).

If no measurements of the acceleration of gravity are at hand, we shall start with the "normal" value of gravity at sea-level, and derive from it by formula section 7 (*a*) or table 1 M the value at the earth's surface or at any height above sea-level. The normal value g_0 of the acceleration of gravity at sea-level we shall consider as given by the formula of Helmert: †

$$(b) \qquad g_0 = 9.80617\,(1 - 0.002644 \cos 2\phi + 0.000007 \cos^2 2\phi)$$

The values of g_0 are tabulated according to this formula in table 2 M of our Meteorological Tables.

9. Normal Increase of Gravity in the Sea. — Calculating the decrease of gravity in the atmosphere, we could simplify the problem by neglecting the mass of the air. But in view of the greater density of the water, the corresponding simplification will not be allowable for the case of the sea.

* O. E. SCHIÖTZ: Results of the pendulum observations. The Norwegian North Pole Expedition 1893–96, vol. II. Christiania, 1901.
O. HECKER: Bestimmung der Schwerkraft auf dem Atlantischen Ocean. Veröffentlichungen des preussischen geodätischen Instituts. Berlin, 1903.
† R. F. HELMERT: Der normale Teil der Schwerkraft im Meeresniveau. Sitzungsberichte der Akademie der Wissenschaften. Berlin, 1901, p. 328.

In order to calculate the correction in this case, we shall consider the earth as a sphere of radius r, and make use of the well-known theorem in the theory of attraction that a spherical shell of constant density does not exert any influence on a point inside it. In the depth z below sea-level we have therefore only to take into account the attraction of the mass contained within a sphere of radius $r - z$. M being the whole mass of the earth and m that of the shell, we have for the acceleration of gravity at sea-level

$$g_0 = k\frac{M}{r^2}$$

and at the depth z below sea-level

$$g = k\frac{M - m}{(r - z)^2}$$

Neglecting squares or products of the small ratios m/M and z/r, we conclude from these equations

$$g = g_0 + 2g_0\frac{z}{r} - g_0\frac{m}{M}$$

Denoting by ρ_m the mean density of the earth, and by ρ that of the spherical shell, we have

$$M = \tfrac{4}{3}\pi r^3\rho_m \qquad m = 4\pi r^2 z\rho$$

and thus

(*a*) $$g = g_0 + 2\frac{g_0}{r}\left(1 - \frac{3}{2}\frac{\rho}{\rho_m}\right)z$$

For the factor $2g_0/r$ we have to use, according to Helmert, the value 0.000003086. For the density of the spherical shell we shall use as an average value $\rho = 1.05$, corresponding to the density of the sea-water at the depth of nearly 5000 meters (compare table 14 H). Choosing finally $\rho_m = 5.5$ as the probable value of the average density of the earth, we get the formula

(*b*) $$g = g_0 + 0.0000022202z$$

by which we shall calculate the normal values of the acceleration of gravity in the sea.

The values of the correction term $0.0000022202z$ are given in table 2 H of the Hydrographic Tables.

Of course the normal values of the acceleration of gravity, which we are thus able to calculate, will generally slightly differ from the real local values, as a consequence of the local distribution of mass. It must also be remembered that the spherical shell does not consist exclusively of water, but also contains the land-masses below the continents. For this reason we might have chosen a still greater value for the mean density of the shell. But this heterogeneity of the shell will have different effects near the coasts and in the middle of the open sea, and we therefore leave it out entirely, the more so as the "normal" value of gravity gives a precision amply sufficient for the discussion of the dynamics of the sea in the present state of development of this science.

10. Level Surfaces and Dynamic Height or Depth. — A surface everywhere perpendicular to the plumb-line is a level surface. The free surfaces of liquids in equilibrium always form level surfaces, and the surface of the sea, together with its continuation below the continents as referred to above, is the fundamental level surface, to which all differences of level are referred.

If gravity is the only acting force, no work is required to move a weight along a level surface. But in order to lift it from one level surface to another, a certain amount of work is required, and always the same amount, irrespective of where on the two surfaces the two extreme points of the path are situated. Otherwise perpetual motion could be realized by lifting the weight at the place where less work is required and letting it down at the place where more work is required. Any level surface is therefore specified without ambiguity by the amount of work required to lift a certain mass, say unit-mass, from sea-level to any point of the surface. Or, in other words, a level surface is a surface of equal gravity potential (section 4) and is perfectly specified by the gravity potential of any of its points.

The level surfaces must be carefully distinguished from the surfaces of equal height above or equal depth below sea-level. The intensity of gravity decreases from the pole to the equator. Consequently the unit-mass must be lifted higher at the equator than at the pole, if the same amount of work is to be performed, and thus the same level surface be attained. A surface of equal height above or of equal depth below sea-level must therefore cut through the system of level surfaces. The surface of equal height or depth is a slanting surface, which is not normal to the plumb-line, and on which equilibrium is not possible under the sole action of gravity. If the surfaces were hard and smooth a ball would remain in equilibrium on a level surface. But on a surface of equal height above sea-level it would roll in the direction from the pole to the equator; and on a surface of equal depth below sea-level it would roll in the direction from the equator to the pole.

This property at once shows that the surfaces of equal height or depth are not suitable as coördinate surfaces in problems relating to the statics or the dynamics of the atmosphere or the sea. For this purpose only level surfaces are found suitable.

The introduction of the level surfaces as coördinate surfaces involves the use of gravity potentials for the specification of heights and depths. With this application of gravity potentials in view, we have introduced the names dynamic meter, dynamic decimeter, etc., for units of this quantity. To standard values of the gravity potential in the sense defined (section 4) will correspond *standard equipotential surfaces*. These will serve us as coördinate surfaces.

We shall also use the expressions dynamic height and dynamic depth as synonymous with gravity potential, with the difference only that we take the dynamic depth in the sea as a positive quantity, while the corresponding values of the gravity potential are negative. By this mode of expression *the level surfaces are surfaces of equal dynamic height above or of equal dynamic depth below sea-level*, the height or depth of the standard surfaces being an integer number of dynamic meters. We shall as a rule prefer the expressions dynamic height or depth when we refer to the dynamic meter as unit, and the expression gravity potential when we use the m.t.s. unit, the dynamic decimeter.

Denoting the gravity potential by ϕ, and the dynamic heights and depths respectively by H and D, we have the relations

$$(a) \qquad \phi = 10H \qquad \phi = -10D$$

by which we return from the technical unit, the dynamic meter, to the m.t.s. unit, the dynamic decimeter.

11. Fundamental Formulæ for the Gravity Potential. — The difference of potential between any two points can be found if we know the value of the acceleration of gravity everywhere along a curve s leading from the one point to the other. Let g_s be the component of the acceleration of gravity in the direction tangential to the curve s. The work per unit-mass performed against the action of gravity, when a mass is displaced the length ds along the curve is then $-g_s ds$. That is, the elementary difference of potential between the end-points of the line element ds is $-g_s ds$, and the finite difference of potential $\phi_2 - \phi_1$ between any two points joined by the curve s is found by the integration

$$(a) \qquad \phi_2 - \phi_1 = -\int_1^2 g_s ds$$

If the curve s coincides with the plumb-line, the acceleration of gravity will always come in with its full value g. If the lengths measured along the plumb-line be denoted by z, and the heights of the points 1 and 2 above sea-level by z_1 and z_2, the expression (a) takes the form

$$(b) \qquad \phi_2 - \phi_1 = -\int_{z_1}^{z_2} g dz$$

If from (b) we pass to dynamic heights in the atmosphere, expressed in dynamic meters, we have

$$(c) \qquad H_2 - H_1 = -\tfrac{1}{10}\int_{z_1}^{z_2} g dz$$

Correspondingly for the difference of dynamic depths in the sea we have

$$(d) \qquad D_2 - D_1 = \tfrac{1}{10}\int_{z_1}^{z_2} g_s dz$$

These formulæ serve to calculate the dynamic value of given geometric differences of height or depth.

12. Normal Relation between Geometric and Dynamic Heights.—Introducing the value (a), section 7, of the acceleration of gravity g in the integral 11 (c), and integrating from the initial height z_1 to any height z, we get for the corresponding difference of dynamic height

$$(a) \qquad H - H_1 = \frac{g_1}{10}(z - z_1) - 0.0000001543(z^2 - z_1^2)$$

By this formula we find the dynamic difference of height corresponding to any given geometric difference of height. It is to be noted that in the first approxima-

tion we can neglect the term containing $(z^2 - z_1^2)$, and instead of that use the approximate value 9.80 for the acceleration of gravity at sea-level. This gives the approximate relations

$$(a') \qquad H - H_1 = 0.98(z - z_1), \text{ or counted from sea-level, } H = 0.98z$$

$$(b') \qquad z - z_1 = 1.02(H - H_1), \text{ or counting from sea-level, } z = 1.02H$$

That is, the number expressing a height in dynamic meters is approximately 2 per cent smaller than the number expressing it in meters.

Supposing that the dynamic height be given, while the corresponding value of the geometric height should be found, we have to solve equation (a) with respect to $z - z_1$. To do this conveniently we first substitute from (b') the approximate values of z and z_1 in the correction term of equation (a), which is thus made linear in $z - z_1$. Solving and simplifying the correction term by the introduction of the approximate value 9.80 for the acceleration of gravity g_1, we get the equation

$$(b) \qquad z - z_1 = \frac{10}{g_1}(H - H_1) + 0.0000001637(H^2 - H_1^2)$$

by which the geometrical value of a given dynamic height can be calculated.

In practical application it will generally be most convenient to have all heights measured from sea-level. We then have $z_1 = 0$, $H_1 = 0$, $g_1 = g_0$, and formulæ (a) and (b) take the form

$$(a'') \qquad H = \frac{g_0}{10}z - 0.0000001543z^2$$

$$(b'') \qquad z = \frac{10}{g_0}H + 0.0000001637H^2$$

In order to tabulate conveniently these formulæ, we shall write them in a slightly modified form. In both the main term depends upon two variables, namely, g_0 and z or g_0 and H, respectively. But, thanks to the small variations of g_0, we can account for the influence of the variations of this quantity in a correction term, while the main term is made to depend upon one variable only. To attain this we shall write

$$(c) \qquad g_0 = 9.80\left(1 + \frac{g_0 - 9.80}{9.80}\right)$$

The fraction contained within the parentheses will have a value never exceeding 0.004. Neglecting squares of this quantity as well as products of it by quantities of its own order of magnitude, we bring the formulæ (a'') and (b'') to the forms

$$(a''') \qquad H = \{0.98z - 0.0000001543z^2\} + 0.1(g_0 - 9.80)z$$

$$(b''') \qquad z = \{1.020408H + 0.0000001637H^2\} - \frac{g_0 - 9.80}{9.60}H$$

The expressions inclosed within parentheses depend upon one variable only. Their values are given in tables 3 M and 5 M of Meteorological Tables. They give the relation between geometric and dynamic height for places where the acceleration

of gravity at the sea-level height z_1 has the special value 9.80. The last term in each equation gives the correction for other values of g_0. The value of this correction is given in tables 4 M and 6 M of Meteorological Tables. These tables can thus be used to pass from geometric to dynamic heights and *vice versa*, the only supposition being that we know the value g_0 of the acceleration of gravity at sea-level, which is found either by table 2 M, or by reduction to sea-level of the value of the acceleration of gravity found by direct determinations at the earth's surface. Proceeding in this way, we find the dynamic heights above sea-level both of the ground and of points in the free atmosphere. The height of the ground will contain an uncertainty due to that of the reduction of g to sea-level. But the heights of the points in the free atmosphere above the ground will contain no error due to this reduction.

13. Normal Relation between Geometric and Dynamic Depths.— Introducing the value (b), section 9, of the acceleration of gravity below the integral sign of (a), section 11, and integrating from sea-level, where $D = z = 0$ to any depth z, we find the corresponding value of the dynamic depth D

$$D = \frac{g_0}{10} z + 0.0000001101 z^2 \tag{a}$$

This formula serves to calculate the dynamic depth D corresponding to any given geometric depth z.

From this formula we draw as a first approximation

$$D = 0.98 z \tag{a'}$$

or, solving with respect to z,

$$z = 1.02 D \tag{b'}$$

That is, in the case of the sea we have the same approximate difference as in the atmosphere between the figures representing the two kinds of depth amounting to about 2 per cent.

Solving (a) by the method employed for (a), section 12, we find the equation

$$z = \frac{10}{g_0} D - 0.0000001168 D^2 \tag{b}$$

by which the geometric value of a given dynamic depth is calculated.

To make the formulæ (a) and (b) suitable for tabulation, we use the same artifice as above. Introducing (c), section 12, and neglecting small quantities of the second order, we can write the formulæ

$$D = \{0.98 z + 0.0000001101 z^2\} + 0.1(g_0 - 9.80) z \tag{a''}$$

$$z = \{1.020408 D - 0.0000001168 D^2\} - \frac{g_0 - 9.80}{9.60} D \tag{b''}$$

The expressions within the brackets depend on one variable only, and their values are given in tables 3 H and 5 H respectively of the Hydrographic Tables. They give the relation between geometric and dynamic depth in the special case that acceleration of gravity in sea-level has the value 9.80. The last term in each equation

gives the correction for other values of g, and the numerical values of these corrections are given in tables 4 H and 6 H respectively of the Hydrographic Tables.

14. Gravity Potential of Points at the Earth's Surface. — According to the modern principles of geodesy, levelings of high precision should always be combined with determinations of the acceleration of gravity. This combination of leveling with gravity measurements gives all the data required for the determination of gravity potentials of points at the earth's surface.

Leveling consists in sighting along level surfaces and in measurements of heights normal to them. A curve consisting of successive horizontal and vertical parts is thus traced out. Forming for this curve the integral (*a*), section 11, we have to take into account the vertical parts only. Let their lengths be z, z', z'', ... , and let g, g', g'', ... be the mean values of the acceleration of gravity along each of them. The integral then takes the form

$$\phi_2 - \phi_1 = gz + g'z' + g''z'' + \cdot \cdot \cdot \cdot \cdot \tag{a}$$

The sum on the right side thus gives the difference of gravity potential between the end-points of the curve.

All the measurements required for the determination of gravity potentials are thus performed by modern geodetic work. But unfortunately the results are not worked out and published in this form. Attention is directed mainly to the sum

$$Z = z + z' + z'' + \cdot \cdot \cdot \cdot \cdot \tag{b}$$

which is supposed to represent the difference of height between the two end-points. This Z is, however, no well-defined quantity, because the level surfaces are not parallel to each other. If the leveling be performed along another route, a slightly different sum Z will generally be found. The discrepancies caused by the lack of parallelism between the level surfaces may be diminished by suitable corrections, but no general method can be conceived which would make them disappear, and the real relation of the determined Z to the vertical distance of the one point from the level surface passing through the other will remain obscure.

The only quantity which can be determined without ambiguity is the gravity potential ϕ. The same will be the case if we pass to the other fundamental method for the determination of heights, the barometric method. As we shall have occasion to show later, this method also gives gravity potentials as its direct natural result, while the passage to heights brings uncertainties.

That under these circumstances gravity potentials, when wanted, must be found by recalculation from the published heights, is very unsatisfactory, so much the more so as it will probably presently become apparent that gravity potentials are what are really needed for scientific purposes, heights being only of secondary importance. Such at least is the case in meteorology, and will also be that of geology as soon as the question of the statics and dynamics of the earth's crust is taken up seriously. It would therefore be a great advance if gravity potentials were published as the main scientific result of geodetic work, and heights only as results computed from gravity potentials.

Provisionally we have to do the reverse. The problem to compute the most probable values of the gravity potential from the published heights is therefore of some importance. The method will mainly consist in removing the corrections originally introduced to pass from gravity potentials to heights, and will therefore turn out somewhat differently according as the barometric, the leveling, or trigonometric methods have been used. Further, it will differ with the different rules for the reduction used in each of these methods. Thus different methods would have to be used on different occasions, and the data determining the choice of method would not always be at hand. In this state of confusion the normal reduction, which we have developed in the case of points in the free atmosphere (section 12), seems to be the most worthy of recommendation, also for the determination of gravity potentials at points on the earth's surface.

15. Maps of Dynamic Topography. — When the gravity potential or the dynamic height is known for a sufficient number of points of the earth's surface we shall be enabled to draw a new kind of topographic maps, representing not the geometric but the dynamic heights of the country. The curves of these maps would be real level curves, which would represent the coast-lines if the country were partially submerged under the sea. The number of curves between two points would represent the amount of work per unit-mass which had to be performed against the action of gravity, if a body should be moved from the one point to the other. The maps would thus represent the height of a mountain, not by the vertical distance of its summit from sea-level, but by the work required to reach the summit. They would further directly give the amounts of potential energy possessed by the masses of water stored in the lakes and would show how this potential is given up during the flow of the water down the rivers.

The motion of the air is restricted by the condition of tangential contact with the earth's surface. The knowledge of the topography of the land is therefore indispensable for the study of this motion. Both the geometric and the dynamic topography must be known, but for evident reasons the dynamic topography is of first importance.

For the construction of these maps the close accordance of the common and the dynamic meter is of great practical value. Especially if the maps should represent large parts of the world on a moderate scale, there will be no visible difference between the course of two curves, one of which represents the height of a certain number of common meters, while the other represents the height of the same number of dynamic meters. To make such maps practically useful in meteorology it will be necessary to simplify the topography, smoothing out all the small irregularities. These maps of idealized topography, drawn on a moderate scale, can therefore, according to circumstances, be considered as representing both the geometric and the dynamic topography.

If the topography of the earth's surface is of importance for the motion of the air, that of the bottom of the sea is of still higher importance for the motion of the sea. As in the case of the air, the dynamic topography is of the greatest importance,

the geometric being only of secondary interest. But for maps on moderate scales we can identify both kinds of topography. Near the coasts it will generally be necessary to simplify the course of the curves. But for greater distances from the coasts the bottom configuration is generally so regular, or our knowledge of it so incomplete, that artificial simplifications may be more or less dispensed with.

The topographical maps accompanying this work can be considered as representing both geometric and dynamic topography. On the map of the world, giving the topography of the earth's surface both above and below sea-level, the main curves are drawn for the interval of 1000 meters, which may be interpreted as geometric or dynamic meters according to circumstances. For the displacement from curve to curve of a unit-mass, we have a gain or loss of potential energy of 10,000 m.t.s. units.

16. Scalar Field. — It will be useful to refer here to some fundamental notions relating to scalar fields, and their variations from place to place in space. Let α be a scalar quantity which has a uniquely determined value in every point of space. To represent distinctly the distribution in space of these values, or, in other words, to represent the field of the scalar α, we can draw a set of *equiscalar surfaces*

$$\alpha = \alpha_0, \quad \alpha = \alpha_1, \quad \alpha = \alpha_2, \quad \cdots\cdots$$

Each of these contains the points in space where the scalar has a certain constant value, $\alpha_0, \alpha_1, \alpha_2 \cdots\cdots$, respectively. This is the well-known method of representing the distribution of potential by equipotential surfaces, that of pressure by isobaric surfaces, that of temperature by isothermic surfaces, and so on.

The sheet between two equiscalar surfaces α_0 and α_1 will be called an *equiscalar sheet.* The use of the word "equiscalar" in connection with a sheet must not be misunderstood. The scalar is not constant in the sheet, but it has limited variations, the limits being given by its values α_0 and α_1 on the boundary. The word "equiscalar" used for a sheet should remind us of this limitation of the variations, as well as of the possibility of defining an average value of the scalar, which is constant all along the sheet.

In most cases it will be found convenient to draw the equiscalar surfaces for unit differences of the scalar. These surfaces will then divide the space into a set of equiscalar *unit-sheets.* Choosing a unit of suitable magnitude, we can always be certain that the unit-sheets get a suitable thickness for a perspicuous distinct representation of the field. If sufficiently thin sheets are obtained we can always say that the difference between the values α_1 and α_0 of the scalar in two points of space 1 and 0 is equal to the number of unit-sheets contained between them.

This difference, $\alpha_1 - \alpha_0$, divided by the length s of any curve joining the points 0 and 1,

$$\frac{\alpha_1 - \alpha_0}{s} \tag{a}$$

gives the *average* rate of variation of the scalar along the curve s.

Now let the curve s be a straight segment of line, and let its length diminish indefinitely. In this limiting case (a) gives the *local* rate of variation of the scalar α in the direction determined by the elementary segment of line s. This rate will vary with the direction of s. To examine this variation let us choose the unit of the scalar quantity so small that the thickness of the unit-sheets is small in comparison to the elementary length s. Further, let s have one end-point fixed and let it have a constant length, while it can have any direction. Within the spherical space of radius s the equiscalar surfaces separating the unit-sheets can be considered as parallel and equidistant. Then the number of unit-sheets cut by the segment s will evidently be proportional to the cosine of the angle which this segment forms with the normal n to the equiscalar surfaces. The rate of variation of the scalar being in direct proportion to the number of unit-sheets cutting the segment of line s, we get this result:

The rate of variation of a scalar quantity in any direction s is equal to its rate of variation along the normal to the equiscalar surfaces, multiplied by the cosine of the angle contained between this direction s and the normal n to the equiscalar surfaces.

17. Gradient and Ascendant. — In accordance with this result we can represent the main rate of variation of the scalar field by a vector directed along the normal to the equiscalar surfaces. The rate of variation along any direction is, then, represented by the component of the vector along this direction. The vector may be defined with the positive or with the negative sign, according as the rate of variation be interpreted as the rate of increase, or as the rate of decrease of the scalar quantity. The vector representing the rate of decrease is generally called the *gradient* and, more specially, potential gradient, pressure gradient, temperature gradient, etc., in accordance with the nature of the scalar quantity. To have a name for the vector representing the rate of increase of the scalar, we shall call it the *ascendant*. Generally the gradient has the most perspicuous physical sense. But still in some cases the use of the ascendant is to be preferred for practical reasons.

From what precedes it will be seen that the gradient G and the ascendant A of the scalar α may be defined by the equations

$$G = -\frac{d\alpha}{dn} \qquad (a)$$

$$A = \frac{d\alpha}{dn} \qquad (b)$$

n being the normal to the equiscalar surfaces, counted positive in the direction of increasing values of α. In the same way the components G_s and A_s of these vectors along any direction s are given by the rates of decrease or of increase respectively along the direction s

$$G_s = -\frac{\partial\alpha}{\partial s} \qquad (c)$$

$$A_s = \frac{\partial\alpha}{\partial s} \qquad (d)$$

The equiscalar surfaces or the unit-sheets ? representing the field of the scalar quantity a give at the same time a complete representation of the field of the vector G or A. From what is stated above we can immediately draw these conclusions:

(1) The direction of the vectors is that of the normal to the equiscalar surfaces.

(2) If a sufficiently small unit be used, the magnitude of the vector will be represented numerically by the number of unit-sheets per unit-length of the normal; or, what comes to the same thing, by the reciprocal thickness of the unit-sheet.

(3) The component of any of the vectors in any direction s is numerically equal to the number of unit-sheets per unit-length in this direction; or, in other words, it is equal to the reciprocal length of that segment of the line s which is contained in a unit-sheet.

If, finally, we add that the gradient points in the direction of decreasing and the ascendant in the direction of increasing values of the scalar, we see that the equiscalar surfaces and the unit-sheets give a full representation of the field of the gradient or of the ascendant. If greater units be used, so that the unit-sheets have greater thickness than supposed above, perfectly corresponding theorems may be formed for the average values of the vectors or their components referred to definite lengths of the segment s.

As we can pass by a process of differentiation from the field of a scalar to the field of its gradient or its ascendant, we can, *vice versa*, return by a process of integration from one of the latter fields to the first. To show this, say for the gradient, we can multiply equation (c) by the line element ds and integrate along the curve s from a point 0 to a point 1. This gives

$$(e) \qquad \int_0^1 G_s ds = -\int_0^1 \frac{\partial a}{\partial s} ds = -\int_0^1 da = a_0 - a_1$$

a_0 and a_1 being the values of a at the points 0 and 1, respectively. The first member of this equation is the line-integral of the component of the vector G tangential to the curve s. As we shall usually have to take line-integrals only of the tangential vector-components, we may denote an integral of this nature simply as ***the line-integral of the vector***. This line-integral of the gradient gives us the means of reconstructing the field of the scalar. For, knowing the field of the gradient and the value of the scalar quantity in one point of space, we can find the value of the scalar in any point by integrating the gradient along any curve leading from the first point to the second.

It will be useful, finally, to express in terms of the gradient the ratio (a), section 16, from which we derived originally the definition of this vector. Taking in the integral (e) the mean value $G_{s,m}$ of the tangential component of the gradient outside the integral sign, the integration can be performed, and gives the length s of the curve. Dividing by this s, we get

$$(f) \qquad G_{s,m} = \frac{a_0 - a_1}{s}$$

Thus, the mean value of the component of the gradient tangential to any curve s is

equal to the difference of the values which the corresponding scalar quantity has in the end-points of s, divided by the length of s.

18. The Gravitational Field of Force. — The relation of gravity potential to the acceleration of gravity is that of a scalar quantity to its gradient. The gravity potential will therefore serve to give us not only a rational system of coördinates; it will also give us a full representation of the gravitational field of force.

To sum up the facts relating to this representation, we see that formula (*a*), section 11, which defined the gravity potential in terms of the acceleration of gravity, has exactly the form of the formula (*e*), section 17, which defines a scalar quantity in terms of the gradient. *Vice versa*, the acceleration of gravity can be represented by the rate of decrease of the gravity potential along the normal to the equipotential surfaces, *i. e.*, along the plumb-line z,

$$(a) \qquad g = -\frac{d\phi}{dz}$$

In the same way the component of the acceleration of gravity along any direction s is given by the rate of decrease of the gravity potential along this direction

$$(b) \qquad g_s = -\frac{\partial\phi}{\partial s}$$

Another form of expressing the facts contained in the formulæ (*a*) and (*b*) is the statement that the equipotential surfaces and the unit-sheets give a full representation of the gravitational field of force. First, the acceleration of gravity is directed along the normal to these surfaces, *i. e.*, along the plumb-line. Second, it is numerically equal to the reciprocal thickness of the unit-sheets. Thirdly, its component along any line s is numerically equal to the reciprocal length of the segment of this line which is contained in the unit-sheet. As we see, these statements are simply the reversal of the statements by which we defined originally our unit of gravity potential, the dynamic decimeter, in terms of the acceleration of gravity. Corresponding to (*f*), section 17, we get finally

$$(c) \qquad g_{s,m} = \frac{\phi_0 - \phi_1}{s}$$

where $g_{s,m}$ is the average value of the component which the acceleration of gravity has tangentially to the curves, while ϕ_0 and ϕ_1 are the values of the gravity potential in the end-points of the curve.

The gravitational field of force is a field in space and thus a 3-dimensional field. The components of its field intensity tangentially to a surface will represent a 2-dimensional field of force. These 2-dimensional fields, which will be of great importance for us, are represented fully by a map giving the dynamical topography of the surface.

We can exemplify this by reference to our maps of dynamic topography. Formula (*c*) can be used to find the average value of the acceleration of gravity along any part of a curve contained in the surface represented by the map. Any such curve will be divided into segments s by the level curves of the maps, and to each such segment the formula (*c*) can be brought into application.

In this case it will, however, be inconvenient to measure the length of the curve in meters ; but the m.t.s. value of $g_{s,m}$ will come out correctly also when the length of the curve s is measured in kilometers and the difference of potential $\phi_0 - \phi_1$ for each 100 dynamic meters of height is taken for unity. The average value of the component of the acceleration of gravity along a segment limited by two successive level curves on our map of the world will therefore be

$$(d) \qquad g_{s,m} = \frac{10}{s}$$

If the map represented the topography of a perfectly hard and smooth surface, and if the curve s be the path of a particle forced to slide on it with any initial velocity under the sole action of gravity, this formula would serve to find the average acceleration of the particle in any part of its path.

CHAPTER III.

SPECIFIC VOLUME AND DENSITY OF ATMOSPHERIC AIR AND SEA-WATER.

19. **Distribution of Mass.** — Every motion consists in the displacement of masses. Only in certain definite distributions of mass will the causes of motion cease to act. As introductory to the investigation of the conditions of equilibrium and motion of the atmosphere and the hydrosphere, we will therefore have to consider the distribution of mass in general, and the methods of finding and representing it.

For the numerical representation of the distribution of mass in a continuous medium, such as air or water, we have, as mentioned already (sec. 3), two methods: We can specify the volume occupied by the different unit-masses, or we can specify the masses present in the different units of volume. In the first case we register the *specific volume*, in the second the *density* of the medium. The number representing one of these quantities is the reciprocal of that representing the other. These quantities are completely equivalent in representing the distribution of mass. But which to choose is a question of importance, as it leads to one or the other of two different methods already referred to (section 3) of formulating the conditions of equilibrium and motion of the medium. Theoretically neither of these methods has any advantage over the other, but they supplement each other in a convenient manner. We shall therefore develop both side by side.

Specific volume or density of atmospheric air or of sea-water are as a rule not observed directly. Generally they will have to be calculated from other quantities, more easily observed with sufficient precision. These quantities are pressure, temperature, and humidity in the case of the atmosphere; depth, temperature, and salinity in the case of the sea.

20. **Equation of State of the Atmospheric Air.** — To calculate the specific volume of dry atmospheric air, we use the equation of Boyle-Gay-Lussac. As the letter t will be reserved for the most fundamental of all independent variables, time, and the letter v for the most important vector quantity related to the motion of the atmosphere or the sea, velocity, we shall represent the temperature according to the common centigrade scale by τ, and the corresponding temperature referred to the absolute zero by ϑ, thus

$$(a) \qquad \vartheta = \tau + 273$$

while we shall denote the specific volume by α. The equation connecting pressure, specific volume, and temperature of a perfect gas is then

$$(b) \qquad p\alpha = R\vartheta$$

The gas constant R of dry atmospheric air is 2153 when the pressure is expressed in millimeters of mercury, and 2870 when it is measured in m-bars.

If the air be more or less moist, an equation of the form (b) can still be used, only with a new value R' of the gas constant

$$pa = R'\vartheta$$

If the unit-mass of moist air contains m parts of water-vapor, and consequently $1 - m$ parts of dry air, the laws for the mixtures of gases give for the constant R' the expression

$$R' = (1 - m)R + mR''$$

R being the gas constant of dry atmospheric air, and R'' that of water-vapor. Now the constants of two gases are in proportion to their specific volumes. For the case of water-vapor and dry atmospheric air this proportion has the well-known approximate value 8/5, which will give sufficient accuracy for our purposes. Consequently $R' = R(1 + 0.6m)$.

21. Virtual Temperature. — The gas constant R' of moist air is thus a variable quantity, changing with the variable mass m of water present. The second member of the equation for moist air will therefore contain two variable quantities, R' and ϑ. The first of these will be, however, variable only between narrow limits. We can therefore advantageously use a well-known artifice, considering the slightly variable quantity R' constant and equal to R, while we for compensation add a small correction to the widely variable quantity ϑ. Thus, introducing

$$\vartheta_r = \vartheta(1 + 0.6m) \tag{a}$$

we can write the equation for moist air in the form

$$pa = R\vartheta_r \tag{b}$$

R being the gas constant for dry air, and ϑ_r a somewhat increased temperature, namely, the temperature which dry air ought to have in order to get the same specific volume as the assumed mass of moist air of temperature ϑ. With Guldberg and Mohn, who first introduced this useful auxiliary quantity, we shall call ϑ_r the virtual temperature.

As may prove most convenient, we shall count this virtual temperature either from the freezing-point of water or from the absolute zero, and denote it by τ_r and ϑ_r respectively, thus

$$\tau_r = \tau + \epsilon_r \qquad \vartheta_r = \vartheta + \epsilon_r \tag{c}$$

where, according to (a), the correction ϵ_r has the value

$$\epsilon_r = 0.6m\vartheta \tag{d}$$

m being the mass of water-vapor per unit-mass of atmospheric air.

22. Tables for the Virtual-Temperature Correction. — The formula (d) above can be used to calculate the virtual temperature when the mass m of water-vapor

per unit-mass of the air is known. But this quantity m is never observed directly. What is generally determined is the relative humidity, *i. e.*, the proportion of the quantity of water-vapor really present to that which would be present if, at the same pressure and temperature, the air were saturated with moisture.

If f be the pressure of the saturated vapor at the temperature considered, p the total pressure, and r the relative humidity, rf will represent the pressure of the vapor and $p - rf$ that of the dry air in the mixture. The partial pressures of each constituent in a mixture are in the same ratio as their volumes were before mixing. The specific volumes of air and water-vapor being in the ratio $5:8$ the volumes of $1 - m$ parts of air and m parts of water-vapor are in the ratio $5(1 - m):8m$ and consequently

$$(p - rf) : rf = 5(1 - m) : 8m$$

If the value of m found from this equation be substituted in the equation (d), section 21, we find the temperature correction

$$\epsilon_r = \frac{3rf}{8p - 3rf}\vartheta \tag{a}$$

It appears as a function of four variable quantities. But only three of these — pressure p, temperature ϑ, and humidity r—are independent, while the vapor pressure f is a known function of temperature.

To conveniently calculate ϵ_r we can first put $r = 1$, and calculate the temperature correction

$$\epsilon_{100} = \frac{3f}{8p - 3f}\vartheta \tag{b}$$

corresponding to 100 per cent humidity. This ϵ_{100} being calculated, the value of ϵ corresponding to any relative humidity is easily found. Division of (a) by (b) gives

$$\frac{\epsilon_r}{\epsilon_{100}} = \frac{8 - 3\dfrac{f}{p}}{8 - 3\dfrac{rf}{p}} \cdot r$$

where the temperature ϑ has dropped out. Numerical calculation easily shows that, even under unfavorable circumstances, the coefficient of r can be set equal to unity without producing any error in the tenths of the centigrade degree. Thus the equation is reduced to

$$\epsilon_r = r\epsilon_{100} \tag{c}$$

As an immediate result of equations (b) and (c) we get the following rule for the calculation of the virtual-temperature correction ϵ_r for air of r per cent relative humidity: First calculate the correction ϵ_{100} for saturated air of the given temperature and pressure; then r per cent. of ϵ_{100} gives the required correction ϵ_r.

The virtual-temperature correction ϵ_{100} for saturated air is given in table 7 M of the Meteorological Tables as function of the pressure in m-bars and degrees of the

centigrade thermometer.* The tabulated numbers are, as seen, all very small, and r per cent of any of them is easily found with sufficient accuracy.

When the pressure is given in millimeters of mercury, table 11 A of the Appendix gives the virtual-temperature correction for saturated air, expressed as above in degrees of the centigrade thermometer.

When the observations are made in inches of mercury and degrees of the Fahrenheit thermometer, it will be usually found most convenient, also, to calculate the virtual-temperature correction in Fahrenheit degrees and to perform at a later stage the transition to the other system of units. The virtual-temperature correction for saturated air, expressed in Fahrenheit degrees, and with the pressure in inches of mercury as argument, is given in table 12 A of the Appendix.

23. **Virtual-Temperature Diagrams.** — The calculation of the virtual-temperature correction for every special observation of a long series will be found to be a great waste of time. In such cases it is easy to find a curve, *the virtual-temperature diagram*, giving the relation between virtual temperature and pressure. This diagram is obtained by the following procedure: On coordinate paper the pressures are measured out along the axis of the ordinates and the temperatures along the axis of the abscissæ. In this plane the observed values of temperature and pressure give a number of points by use of which a curve representing the relation of pressure and temperature is drawn. This curve immediately gives the temperatures corresponding to the pressures 1100, 1050, 1000, 950, ... m-bars, serving as argument in table 7 M. The corresponding virtual-temperature corrections for saturated air are taken from this table with great ease, no interpolation respecting the pressure now being required. By means of these corrections a second curve is drawn, the curve of virtual temperature for saturated air. The curve for the virtual temperature corresponding to the observed relative humidities will run between these curves at a horizontal distance from the curve of real temperature, which is r per cent of the horizontal distance between the two curves. This final curve is then easily drawn by estimation, in accordance with the observed relative humidities.

The method is easily understood by inspection of the diagrams accompanying the examples worked out in Chapter VI. In each of them the curve to the left is that of real temperatures, as immediately given by the observed temperatures and pressures. The curve to the right is that of virtual temperature for saturated air, as obtained by the use of table 7 M as described above. The line between the two others is the required curve of virtual temperatures, as drawn by means of the observed relative humidities.

The observations being made in millimeters of mercury and centigrade degrees, or in inches of mercury and Fahrenheit degrees, the virtual-temperature curve will be obtained in exactly the same way, table 11 A or 12 A of the Appendix being used to obtain the curve, as shown in the examples of Chapter VI.

* The values of the vapor-tension f used for calculating table 7 M have been taken from Broch's well-known table (Travaux et Mémoires du Bureau International des Poids et des Mesures, T. 1, Paris, 1881) for temperatures above zero, and from Juhlin's table (Bihang till k. svenska Vetenskapsakademiens Handlingar, T. 17, Afdelning I, Stockholm, 1891), for temperatures below zero.

24. Virtual Temperature as a Function of the Height. — In some cases the height will appear instead of the pressure as one of the observed quantities. It will then be convenient to be able to calculate the virtual-temperature correction as a function not of pressure but of height. For this we must first know the average pressure for the heights to be used as argument in the table. Using the international kite and balloon ascents performed in Europe for the years 1901 to 1903, we have found the values of the average pressure in given dynamic heights *above the 1000 m-bar surface* (table A).

TABLE A.—*Average Pressures in Given Dynamic Heights above the 1000 m-bar Surface, Calculated from the International Kite and Balloon Ascents in Europe for the Period 1901–1903.*

Height (dynamic meters).	Pressure (m-bars).	Height (dynamic meters).	Pressure (m-bars).	Height (dynamic meters).	Pressure (m-bars).
3000	685	6500	428	10000	256
2500	731	6000	459	9500	276
2000	779	5500	492	9000	298
1500	830	5000	526	8500	321
1000	883	4500	563	8000	346
500	940	4000	600	7500	372
0	1000	3500	642	7000	399

Table A would give the average pressures in the corresponding heights above *sea-level*, if the pressure at sea-level was 1000 m-bars. This pressure being about 760 mm. of mercury, or 1013 m-bars, all pressures in table A would have to be increased by about 1.3 per cent in order to give the average pressures in the corresponding heights above sea-level. But this difference is quite insignificant for our present purpose.

By means of these values of the average pressure in the standard heights, table 8 M has been calculated from table 7 M. The virtual temperature varying very slowly with the pressure, even a great deviation of the actual pressure from the supposed average value will have no influence on the correctness of the results obtained from table 7 M.

The use of table 8 M is perfectly analogous to that of table 7 M. An example of a virtual-temperature diagram, drawn with the height as the independent variable is also given in Chapter VI. The heights are given originally in common meters. Before using table 8 M these are first to be transformed, by tables 3 M and 4 M, to dynamic meters.

The height being given originally in feet and the temperature in Fahrenheit degrees, tables 2 A and 3 A of the Appendix are first used to transform the heights from feet to dynamic meters, and afterwards table 13 A of the Appendix to draw the virtual-temperature diagram in Fahrenheit degrees.

25. Specific Volume and Density of the Air. — The value of the virtual temperature being found, it is easy to calculate the specific volume or the density by the equation of state, which gives

(*a*) $$\alpha = R\frac{\vartheta_r}{p}$$

(*b*) $$\rho = \frac{p}{R\vartheta_r}$$

Two variables ϑ_r and p appearing on the right side of each of these equations, and each of the variables having a wide range of variation, the complete tabulation of α and ρ would be very laborious and lead to very bulky tables. We shall therefore use the equations (*a*) or (*b*) for eliminating α or ρ of our equations. Afterwards we shall give an indirect way to obtain the geometric representation of the fields of specific volume or of density in the atmosphere. As an aid for this, table 14 M of Meteorological Tables, giving the value of the specific volume for the standard pressures, will be found useful.

26. Investigations of the Physical Properties of Sea-Water. — The physical properties of sea-water have been subject to elaborate investigations in connection with the international exploration of the northern European waters.*

The specific volume of sea-water and its reciprocal value, the density, depend upon three variables — pressure, temperature, and salinity. Generally the salinity is not determined directly, but deduced from the content of chlorine found by titration, s denoting the salinity and Cl the quantity of chlorine, both expressed in per milles ($^0/_{00}$) of weight. s and Cl are, according to Martin Knudsen, connected by the equation

(*a*) $$s = 0.030 + 1.8050\ \mathrm{Cl}$$

By this equation, which is tabulated in Martin Knudsen's tables, we can pass from the independent variable Cl used by Martin Knudsen to the independent variable s, which we shall use consistently.

To express the results obtained for the specific volume or the density of sea-water, we shall introduce the following notations: $\alpha_{s\tau p}$ means the specific volume and $\rho_{s\tau p}$ the density of sea-water of salinity $s\ ^0/_{00}$, temperature $\tau°$ C., and sea-pressure of p decibars. By sea-pressure we then mean the total pressure diminished by the pressure exerted by the atmosphere against the surface of the sea. The decibar is employed as a practical unit instead of the unit centibar belonging to the m.t.s. system, because the pressure increases approximately by 1 decibar for every meter increase of depth.

Instead of writing the whole number representing a value of the density ρ, say the number 1.02674, practical hydrographers usually write the four last figures 26.74. This quantity being denoted by σ, we have thus

$$\sigma_{s\tau p} = (\rho_{s\tau p} - 1) \cdot 1000$$

* Martin Knudsen: Berichte über die Konstantenbestimmungen zur Aufstellung der hydrographischen Tabellen von Carl Forch, Martin Knudsen, und S. P. L. Sörensen. Mémoires de l'Academie Royale des Sciences de Danemark. Copenhague, 1902.

Martin Knudsen: Berechnung der hydrographischen Tabellen und Diskussion der Ergebnisse. Wissenschaftliche Meeresuntersuchungen herausgegeben von der Komission zur Untersuchung der deutschen Meere in Kiel, Band 2, 1903.

Martin Knudsen: Hydrographical Tables according to the measurings of Carl Forch, J. P. Jacobsen, Martin Knudsen, and S. P. L. Sörensen. Copenhagen and London, 1901.

V. Walfrid Ekman: Die Zusammendrückbarkeit des Meerwassers, etc. Conseil permanent International pour L'Exploration de la Mer. Publication de Circonstance N° 43. Copenhague, 1908.

By measurements under atmospheric pressure on different samples of sea-water of different salinities at a series of different temperatures, Martin Knudsen has determined the quantity $\sigma_{s\tau 0}$. The result is contained in the following formulæ:

For the case of $\tau = 0$ the quantity σ_{s00} is determined as function of the quantity of chlorine by the equation

$$(b) \qquad \sigma_{s00} = -0.069 + 1.4708\,\mathrm{Cl} - 0.001570\,\mathrm{Cl}^2 + 0.0000398\,\mathrm{Cl}^3$$

Then the quantity $\sigma_{s\tau 0}$ is determined as a function of the temperature τ and the quality σ_{s00} found from (b) by the equation

$$(c) \qquad \sigma_{s\tau 0} = \Sigma_\tau + (\sigma_{s00} + 0.1324)[1 - A_\tau + B_\tau(\sigma_{s00} - 0.1324)]$$

the quantities Σ_τ, A_τ and B_τ being the following functions of temperature:

$$(c') \qquad \begin{aligned} \Sigma_\tau &= -\frac{(\tau - 3.98)^2}{503.570} \cdot \frac{\tau + 283}{\tau + 67.26} \\ A_\tau &= \tau(4.7867 - 0.098185\tau + 0.0010843\tau^2) \cdot 10^{-3} \\ B_\tau &= \tau(18.030 - 0.8164\tau + 0.01667\tau^2) \cdot 10^{-6} \end{aligned}$$

The quantities σ_{s00} and $\sigma_{s\tau 0}$ determined by these formulæ are tabulated in Martin Knudsen's tables. From the tabulated numbers we pass to the corresponding values $\rho_{s\tau 0}$ of the density by the formula

$$\rho_{s\tau 0} = 1 + \frac{\sigma_{s\tau 0}}{1000}$$

and from these we can pass by an inversion table (table 23 H) to the corresponding values of the specific volume. The equations (b) and (a) or the corresponding tables in Knudsen's collection allow us to bring in s as the independent variable.

V. Walfrid Ekman has determined the influence of the pressure upon the volume of sea-water of different salinities and at different temperatures. By these measurements the pressures were calculated from the compression of distilled water of 0° C., the measurements of Amagat* being, after an independent control of their reliability, used as a base for this calculation. Mr. Ekman has kindly furnished us with the following formula computed from his own measurements in connection with those of Amagat:

$$(d) \qquad \begin{aligned} \alpha_{s\tau p} = \alpha_{s\tau 0} - p\alpha_{s\tau 0} \cdot 10^{-9} \Bigg\{ & \frac{4886}{1 + 0.0000183p} - \Big[227 + 28.33\tau - 0.551\tau^2 + 0.004\tau^3\Big] \\ & + p \cdot 10^{-4}\Big[105.5 + 9.50\tau - 0.158\tau^2\Big] - 1.5p^2\tau \cdot 10^{-8} \\ & - \frac{\sigma_{s00} - 28}{10}\Big[147.3 - 2.72\tau + 0.04\tau^2 - p \cdot 10^{-4}(32.4 - 0.87\tau + 0.002\tau^2)\Big] \\ & + \left(\frac{\sigma_{s00} - 28}{10}\right)^2 \Big[4.5 + 0.1\tau - p \cdot 10^{-4}(1.8 - 0.06\tau)\Big] \Bigg\} \end{aligned}$$

* **Amagat**: Mémoires sur l'élasticité et le dilatation des fluides jusqu'aux très hautes pressions. Annales de Chimie et de Physique, t. 29, 1893, p. 544.

the quantities $\alpha_{s\tau 0}$ and σ_{s00} being calculated as explained above from the formulæ or the tables of Martin Knudsen. Mr. Ekman's discussion of his results shows that the specific volume calculated by the formula will not probably contain greater error than 0.00001 for the pressure of 1000 d-bars, and proportionally 0.0001 for 10,000 d-bars. Still more important for the oceanic dynamics is the following conclusion from Mr. Ekman's discussion: Differences in the specific volume of two samples of sea-water taken from the same depth, which have been calculated by this formula, will be perfectly reliable in the fifth decimal in all cases met with in the sea.

27. Tables for the Specific Volume of Sea-Water. — We shall never use the preceding formulæ directly in investigations in statics or dynamics of the sea. They will only serve for the construction of tables giving the specific volume or the density of sea-water for all necessary values of the independent variables. This tabulation, however, contains difficulties. The greatest depth hitherto sounded in the sea being 9636 meters, the sea-pressure can vary from 0 to about 10,000 d-bars. The temperature can vary from -2 to about 30° C. and the salinity from 0 to about 40 °/₀₀. Modern observations being taken with a precision of about 0.01° C. and of 0.01 °/₀₀ of salinity, the tables should not have greater intervals than 0.1° C. and 0.1 °/₀₀ salinity. To be able to interpolate conveniently to any depth, the pressure should not be taken with greater intervals than 10 d-bars. The direct tabulation would thus involve the calculation of $320 \times 400 \times 1000 = 128{,}000{,}000$ different values of the specific volume. By printing 500 numbers on each page the tables would cover 256,000 pages. The direct tabulation must thus be given up, and we shall have to use in a more developed form the principles exemplified for the case of two variables in tables 3 M to 6 M and 3 H to 6 H, viz, to break up the quantity to be tabulated in a sum of terms, each of which is more easily subject to tabulation.

To carry this through in the present case, we can use a development analogous to that of Taylor. We first write

$$(a) \qquad \alpha_{s\tau p} = \alpha_{35,0,p} + \delta$$

Here $\alpha_{35,0,p}$ denotes the specific volume of sea-water of the constant salinity 35 °/₀₀ and the constant temperature 0° C. under any pressure p. These special values of salinity and temperature are not very far from the average values in the deep oceans, and we shall therefore denote $\alpha_{35,0,p}$ as the *normal* specific volume of the sea-water under the pressure p. The value $\alpha_{s\tau p}$ representing the specific volume of any kind of sea-water under the same pressure p is then found from $\alpha_{35,0,p}$ by the addition of a correction δ, which we shall call the *anomaly* of the specific volume This correction will be a function of salinity, temperature, and pressure, and can be broken up in a series of terms

$$(b) \qquad \delta = \delta_s + \delta_\tau + \delta_{s\tau} + \delta_{sp} + \delta_{\tau p} + \delta_{s\tau p}$$

where the indices show the variables upon which the different terms depend.

Thus δ_s and δ_τ depend each on one variable, $\delta_{s\tau}$, δ_{sp}, $\delta_{\tau p}$ each on two, and only $\delta_{s\tau p}$ on all three variables. The main point is now to determine these terms so that the terms depending upon more than one variable become as small as possible. This is done if we give them the following values:

$$
(c)\qquad
\begin{aligned}
\delta_s &= \alpha_{s,0,0} - \alpha_{35,0,0}\\
\delta_\tau &= \alpha_{35,\tau,0} - \alpha_{35,0,}\\
\delta_{s\tau} &= (\alpha_{s,\tau,0} - \alpha_{35,\tau,0}) - (\alpha_{s,0,0} - \alpha_{35,0,0})\\
\delta_{sp} &= (\alpha_{s,0,p} - \alpha_{35,0,p}) - (\alpha_{s,0,0} - \alpha_{35,0,0})\\
\delta_{\tau p} &= (\alpha_{35,\tau,p} - \alpha_{35,0,p}) - (\alpha_{35,\tau,0} - \alpha_{35,0,0})\\
\delta_{s\tau p} &= [(\alpha_{s,\tau,p} - \alpha_{35,\tau,p}) - (\alpha_{s,\tau,0} - \alpha_{35,\tau,0})] - [(\alpha_{s,0,p} - \alpha_{35,0,p}) - (\alpha_{s,0,0} - \alpha_{35,0,0})]
\end{aligned}
$$

It is easily verified that the substitution of (*c*) and (*b*) in (*a*) makes this equation identical. Now, each of the quantities $\alpha_{35,0,p}$, δ_s, δ_τ, $\delta_{s\tau}$, δ_{sp}, $\delta_{\tau p}$, $\delta_{s\tau p}$ are easily tabulated separately. The values of $\alpha_{35,0,p}$ are found from Ekman's formula, putting $s = 35$, $\tau = 0$, and using the values $\alpha_{35,0,0} = 0.97364$ and $\sigma_{s00} = \sigma_{35,0,0} = 28.13$, calculated from Martin Knudsen's tables. The result for 1000 values of the pressure is given in table 8 H. The correction δ_s for salinity at temperature zero and the correction δ_τ for temperature at salinity $35\,^0/_{00}$ are both found from Martin Knudsen's formulæ or tables. The result is given in tables 9 H and 10 H for 400 values of the salinity and 320 values of the temperature, respectively.

Being differences of the second order, the quantities depending upon two variables $\delta_{s\tau}$, δ_{sp}, $\delta_{\tau p}$ are sufficiently small to be tabulated for ten times greater intervals of the independent variables, viz, for 40 values of the salinity, 32 values of the centigrade degrees, and 100 values of the pressure. $\delta_{s\tau}$ is found from Martin Knudsen's formulæ or tables. The expressions of δ_{sp} and $\delta_{\tau p}$, which are rather long in spite of the smallness of the numerical values, are formed from Ekman's formula. The results are given in tables 11 H, 12 H, and 13 H. The quantity $\delta_{s\tau p}$ finally, depending upon three variables, is given by a very long expression deduced from Ekman's formula. But being a difference of the third order, it is sufficiently small to be tabulated for still greater intervals of the independent variables. The result is given in table 14 H as a system of 17 small tables, each corresponding to a certain salinity, while within each table temperature and pressure figure as the independent variables, the intervals of pressure being 1000 decibars.

The tabulation of the specific volume of sea-water has thus been accomplished by seven small tables covering 10 pages. This system of small tables is equivalent to the one table of 256,000 pages on account of the possible permutations in the sum (*a*) and (*b*) of the values taken from the different small tables.

The three last tables, involving the pressure as one of the independent variables, are not calculated completely, those combinations of the variables being left out which are not found in the sea. The general distribution of temperature and salinity in the sea is readily seen from the charts worked out by Dr. G. Schott.* The greatest variation of temperature is found on the surface of the sea, extending from the evident lower limit, the freezing-point of sea-water, between −1 and

* Wissenschaftliche Ergebnisse der deutschen Tiefsee-Expedition auf dem Dampfer "Valdivia," 1898–99. T. I. V. Oceanographie und maritime Meteorologie von Dr. G. Schott. Jena, 1902.

$-2°$ C., to the maximum values in the tropics, hardly anywhere exceeding 30° C. in the open sea. As we proceed downwards, the freezing-point of sea-water is retained as the lower limit, while the upper gradually decreases, but at a very different rate, in the open ocean and in the more or less closed seas. The temperature in the open sea will hardly anywhere exceed 10° C. at a depth of 1000 meters, and in still greater depths it will be found between the limits $+2°$ and $-2°$. In more or less closed seas the temperature may be much higher in corresponding depths. Thus, in the Red Sea there is a temperature of 21.5° C. at the depth of 2100 meters, in the Sulu Sea (between Borneo and Philippines) 10.2° C. at the depth of 4300 meters, and in the Mediterranean 13.9° C. at the depth of 4400 meters. The salinities also have their greatest range of variation at the surface, namely, from zero at the mouths of great rivers to 39 ‰ in the Mediterranean and 40.4 ‰ in the Red Sea. As we proceed downwards the low salinities rapidly disappear, but at a different rate and converging towards different limits in the open ocean and in the closed seas. In the open ocean the salinity rapidly converges towards the almost constant salinity of about 35 ‰. The Mediterranean has the higher salinity of about 39 ‰ and the Red Sea of about 40 ‰ at the bottom, while the Baltic has the low salinity of about 10 ‰ in its greatest depths, somewhat exceeding 400 meters. Lower salinity than that of the Baltic and higher than that of the Mediterranean or the Red Sea will hardly be found anywhere in corresponding depths. These general data have determined the limits of the three last tables.

The method of tabulation which we have used, while leading to very small and convenient tables, has a defect which must be mentioned. The result is found as the sum of seven terms. Each term may have an error of five units in the sixth decimal of the specific volume, or, on account of the double interpolations in the tables containing more than one variable, even somewhat more. Thus in exceptional cases errors may occur exceeding 3.5 in the fifth decimal, corresponding to an error of about 0.03 ‰ in the salinity and thus exceeding somewhat the errors of careful observations, which may be carried to about 0.01 ‰. This error may perhaps be of importance for the investigation of the conditions of equilibrium or motion in very homogeneous parts of the sea. It might have been avoided if we had calculated all our tables with one decimal more. But this would have made the use of the tables in the most common cases much less convenient. When more accurate tables are required it will probably be the best plan to construct three different sets of tables for three different types of sea-water: the oceanic type, the Mediterranean type, and the Baltic type. For the oceanic type the tables should be extended to all pressures, but for all greater pressures the temperature and salinity could be contained between very narrow limits, approaching 0° C. for temperature and 35 ‰ for salinity. For the Mediterranean type the tables should extend to pressures somewhat exceeding 4000 d-bars, while the temperature in greater depths has about 10 for its lower and 20 for its higher limit. The salinity in greater depths should have 35 for its lower and a little more than 40 for its higher limit. These tables might be used in closed tropic or subtropic seas, as the Mediterranean, the Red Sea, and the Sulu Sea. For the Baltic type the tables could be

limited to the pressure of 500 d-bars, while the temperature must have all values from the freezing-point of water to that of the tropic sea, and the salinity all values from that of fresh water to that of oceanic water. These tables could be used for the Baltic, the St. Lawrence Bay, the mouths of great rivers, the shallow waters along the Arctic coasts, etc. The variations of the independent variables being limited in this way, the tables could be constructed for so small intervals of the independent variables that convenient differences would be obtained even if the specific volume was calculated with six decimals.

28. Control of the Accuracy of the Tables. — A question of the highest importance is that of the absolute reliability of the tables. The test is given in as direct form as possible by the annexed tables B and C. The first table contains the volumes of the samples of sea-water under atmospheric pressure examined by Martin Knudsen. These volumes are not given in absolute measure in Martin Knudsen's paper, but by the additional data they have been reduced from the relative measure employed in the course of the experiments to the values in absolute units given in table B. In the same manner table C contains the specific volumes of the samples of sea-water under different pressures examined by Ekman. These

TABLE B.—*Fundamental Values $a_{s,\tau,0}$ of Specific Volume of Sea-Water under Atmospheric Pressure.*

I. $s = 3.20$ ‰		II. $s = 8.35$ ‰		III. $s = 10.56$ ‰		IV. $s = 14.634$ ‰		V. $s = 18.818$ ‰	
τ	a	τ	a	τ	a	τ	a	τ	a
0.000	0.9974973	−0.296	0.9933732	−0.298	0.9916115	−0.164	0.9883820	−0.783	0.9851090
0.051	.9974948	0.000	.9933633	0.000	.9916041	0.000	.9883806	0.000	.9851091
5.085	.9974391	4.885	.9933790	5.000	.9916623	4.728	.9884897	4.986	.9852914
9.485	.9977004	9.936	.9937553	9.697	.9920317	9.981	.9889588	9.780	.9857721
14.895	.9983671	15.004	.9944515	14.907	.9927631	14.868	.9896916	14.793	.9865494
19.450	.9919899	20.212	.9954682	19.824	.9937177	20.118	.9907537	20.417	.9877318
25.319	1.0058375	24.754	.9965707	25.237	.9950581	24.661	.9918878	24.828	.9888584
30.088	1.0195311	29.899	.9980556	30.211	.9965190	30.607	.9936518	30.486	.9905582

VI. $s = 23.204$ ‰		VII. $s = 25.83$ ‰		VIII. $s = 28.956$ ‰		IX. $s = 35.37$ ‰		X. $s = 33.93$ ‰	
τ	a	τ	a	τ	a	τ	a	τ	a
−0.203	0.9816800	−0.262	0.9796612	−1.720	0.9772362	−2.668	0.9745978	−2.215	0.9734010
0.000	.9816837	0.000	.9796680	0.000	.9772745	−0.245	.9746578	0.000	.9734804
4.940	.9819294	5.310	.9799810	0.079	.9772773	0.000	.9746684	0.182	.9734897
9.737	.9824525	10.124	.9805596	5.016	.9776046	4.917	.9750309	4.777	.9738523
14.946	.9833107	14.569	.9813135	9.747	.9781858	10.036	.9757038	9.767	.9745102
19.879	.9843669	20.189	.9825299	14.518	.9790097	14.843	.9765761	14.784	.9754332
25.054	.9857070	25.068	.9838162	20.044	.9802288	19.941	.9777241	19.722	.9765432
30.834	.9874744	30.303	.9854087	24.988	.9815413	24.880	.9790510	24.635	.9778540
				30.683	.9832852	30.435	.9807592	29.810	.9794444

XI. $s = 34.93$ ‰		XII. $s = 35.05$ ‰		XIII. $s = 35.37$ ‰		XIV. $s = 39.35$ ‰	
τ	a	τ	a	τ	a	τ	a
−2.734	0.9725994	−2.488	0.9725204	−0.387	0.9723336	0.000	0.9693332
−0.024	.9726923	−0.028	.9726107	0.000	.9723540	0.266	.9693603
0.000	.9726936	0.000	.9726122	5.245	.9727937	5.066	.9699236
5.104	.9731106	5.096	.9730290	10.313	.9735072	9.670	.9706203
9.768	.9737433	10.225	.9737397	14.523	.9742959	14.868	.9716045
15.273	.9747742	15.287	.9746942	19.770	.9754929	20.366	.9728974
19.924	.9758557	19.917	.9757658	24.531	.9767758	24.798	.9741282
25.508	.9773670	25.645	.9773348	29.840	.9784143	30.327	.9758659
30.024	.9787782						

values do not appear explicitly in Mr. Ekman's paper either, but he has kindly calculated them for us as directly as possible from the compressibilities measured, using the necessary additional data from Amagat and Martin Knudsen.

TABLE C. — *Fundamental Values of $\alpha_{s,\tau,p}$ of the Specific Volume of Sea-Water under Different Pressures.*

Sea-pressure (d-bars).	$t=0$	$t=4.97°$	$t=9.97°$	$t=14.96°$	$t=19.96°$
I. Sea-water of salinity 31.130 ‰ (Cl = 17.230).					
0	0.975600	0.975953	0.976598	0.977493	0.978617
2000	.966722	.967312	.968147	.969189	.970425
4000	.958396	.959192	.960192	.961364	.962701
6000	.950561	.951535	.952679	.953966	.955394
II. Sea-water of salinity 38.525 (Cl. = 21.327).					
0	0.969960	0.970408	0.971130	0.972087	0.973257
2000	.961296	.961964	.962864	.963957	.965232
4000	.953166	.954028	.955079	.956293	.957659
6000	.945510	.946537	.947723	.949042	.950493

To control the accuracy of tables 8 H to 14 H, we can calculate from the tables the specific volume for those values of salinity, temperature, and pressure appearing in tables B and C. It will be seen, then, that all values are found with differences only in exceptional cases exceeding 1 or 2 units in the fifth decimal, these discrepancies being easily explained by the possible errors obtained as developed above by the addition of a sum of terms containing small errors. The test thus shows that no error of any importance can have been made, either by the calculation of the formulæ of interpolation from the observations or in the calculation of the tables from these formulæ.

29. Tables of the Density of Sea-Water. — The density being the reciprocal of the specific volume, tables for the density are easily deduced from those for the specific volume, provided that the same independent variables be retained. For reasons which will become evident later it will be convenient, however, to have the density registered as a function of the dynamical depth instead of as a function of the pressure. Of course there exists no intrinsic physical relation between depth and density. But some measures of precaution being taken, this method of tabulation can be used, thanks to the close relation between depth and pressure in the sea.

Using the index D to denote that the dynamic depth appears as an independent variable, we write, corresponding to section 27 (*a*) and section 27 (*b*):

(*a*) $$\rho_{s\tau D} = \rho_{35,0,D} + \varepsilon$$

(*b*) $$\varepsilon = \varepsilon_s + \varepsilon_\tau + \varepsilon_{s\tau} + \varepsilon_{sD} + \varepsilon_{\tau D} + \varepsilon_{s\tau D}$$

For the right-hand terms of the last equation we can write expressions corresponding to section 27 (*c*), ε being substituted for δ, ρ for α, and D for p.

The three terms corresponding to the case $p = D = 0$, viz, ϵ_s, ϵ_τ, $\epsilon_{s\tau}$, are calculated directly from Martin Knudsen's formulæ or tables. Thus the only difficulty concerns the terms $\rho_{35,0,D}$, ϵ_{sD}, $\epsilon_{\tau D}$, $\epsilon_{s\tau D}$ containing the depth as an independent variable. To perform the transition from the variable p to the variable D, we must know with a sufficient approximation the relation between pressure and depth in the sea. For the case of normal sea-water of 35 ‰ and 0° C. this relation is easily determined by a method explained in the next chapter. The result is contained in table 7 H, which gives the dynamic depth of any given pressure for intervals of 10 d-bars. By interpolation in this table we can determine the pressure in any depth expressed by any integer number of dynamic meters. The result of these interpolations is given in table 15 H, which contains the pressures in depths expressed by any integer number of dynamic meters, registered for intervals of 10 dynamic meters. We can now by table 8 determine the specific volume of the sea-water corresponding to the pressures registered in table 15. These will be the specific volumes of normal sea-water for the depths figuring as arguments in table 15 H. Passing to the reciprocal values by use of the inversion-table 23 H, we get table 16 H, giving the normal density $\rho_{35,0,D}$ of sea-water in 1000 different dynamic depths.

For the calculation of the small quantities ϵ from the corresponding quantities δ, we can make use of simple approximation rules. Differentiating the equation connecting the density ρ with the corresponding specific volume α, we get $d\rho = -d\alpha/\alpha^2$. Applying this for the transition from the correction δ of any value of the specific volume to the corresponding correction ϵ of the density, we get

$$(a) \qquad \epsilon = -\frac{\delta}{\alpha^2}$$

Using, as above, table 15 H to find the pressure corresponding to the given dynamic depth, and this pressure to find δ-values from tables 12 H or 13 H, the corresponding values of ϵ are calculated by equation (a). In this way the main tables 20 H and 21 H are calculated. These would give exactly the required corrections ϵ_{sD} and ϵ_{τ} if the pressure in the depth considered had exactly the normal value given by table 15 H. But if the water above the level considered has other than the normal salinity 35 ‰ or other than the normal temperature 0° C., the pressure will be slightly different, and this will have a slight influence upon the density of the sea-water in the level considered. The anomaly of pressure in question can easily be estimated with sufficient approximation from the average salinity and the average temperature of the water above the level considered, and thus the corresponding correction of the density as the consequence of the compression found. These corrections are given in the small tables placed at the foot of the main tables 20 H and 21 H, having the average instead of the local values of salinity and temperature as argument.

It is seen that as α^2 never differs very much from unity, corresponding values of δ_{sp} and ϵ_{sD}, as well as of $\delta_{\tau p}$ and $\epsilon_{\tau D}$, are very nearly like each other, but with opposite signs. Passing to the calculation of the term of the third order, $\epsilon_{s\tau p}$, we can simply put $\alpha^2 = 1$, and identify the numbers expressing depths in dynamic meters with those expressing pressures in decibars. Table 22 H, giving the values of $\epsilon_{s\tau D}$, is

therefore identical with table 14 H, giving $\delta_{\sigma\tau p}$, but with the difference that all terms appear with the sign reversed.*

30. Important Features of Specific Volume or Density of Sea-Water. — Tables 9 H and 17 H show the regular decrease of the specific volume, or the increase of density with increasing salinity. In the same way tables 8 H and 16 H show the regular decrease of the specific volume, or increase of density with increasing pressure. These tables do not show any marked peculiarities of the sea-water. But very marked peculiarities are shown by the volume-tables 10 H, 11 H, and 12 H and the corresponding density-tables 18 H, 19 H, and 20 H.

Table 10 H shows for sea-water of the salinity 35 ‰ a regular decrease of volume, and table 18 H the corresponding increase of density for decreasing temperature. No maximum of density is found. Table 10 H used together with 11 H, or 18 H used together with 19 H, shows that we have a minimum of specific volume or a maximum of density at 4° C. for fresh-water, at 2° for a salinity of about 9.5 ‰, at 0° for a salinity of about 19 ‰ and at —2° for a salinity of 28 to 29 ‰. But for the normal oceanic salinity of about 35 ‰, there exists no maximum of density, and we shall, in the case of equilibrium, always have warmer water above and colder below. This circumstance makes the equilibrium condition of the ocean quite different from that of fresh-water lakes.

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*The first determinations of the compressibility of the sea-water were performed by P. G. Tait (*Challenger* Report, Physics and Chemistry, vol. II, 1889). The first hydrographical tables taking into account the compressibility were calculated by Sandström and Helland-Hansen (Report on Norwegian Fishery and Marine Investigations, vol. II, No. 4, Bergen, 1903). When our tables were first calculated, the only measurements performed upon sea-water were still those of Tait, which had not by far the exactitude of Ekman's. Especially, no care had been taken to determine the salinity of the samples of water experimented upon at the time when their compressibility was measured. We therefore combined Tait's measurements for sea-water with those of Amagat for distilled water, using Amagat's as absolute determinations and those of Tait only as relative comparisons of the compressibility of sea-water and fresh-water of the same temperature. In the developments, section 27 (*c*), we therefore had to subdivide every term where the pressure and the salinity entered into a main term depending upon the measurements of Amagat upon fresh-water and a correction term depending upon the corresponding measurements of Tait upon sea-water. Thanks to this method of calculation we had obtained tables which were in unexpectedly good accordance with the new tables, which we have now calculated after the manuscript of the first tables had been sent to press. The degree of accordance between the two sets of tables will be seen from the following data.

The specific volume of normal sea-water (Amagat-Tait) was smaller than that registered in our new table 8 H for the pressures from zero to about 2200 d-bars, the maximal error being 0.00007 at 800 to 900 d-bars. Then the volume was found greater from 2200 to 4400, the error varying between 0.00001 and 0.00002. For greater pressures the specific volume was always found smaller, the error increasing gradually to 0.0001 at 5700 d-bars, to 0.001 at 8500 d-bars, and to 0.002 at 10,000 d-bars. Thus the error here runs up to 1/500 of the total volume. It should be remarked, however, that these greater discrepancies only occur for the values which have been extrapolated, Tait's experiments being extended to the pressure of 4629 and Ekman's to the pressure of 6000 d-bars. For depths in the sea not exceeding 5000 meters the accordance is remarkably good, and hydrographic surroundings very seldom go to a greater depth. The degree of accordance is very well illustrated by the following fact: The normal depth of the isobaric surfaces calculated according to Tait never differs by so much as one decimeter from the corresponding depths according to Ekman (table 7 H) for the first 5000 meters. For greater depths there are gradually increasing discrepancies, the depth being found according to Tait 1 m. too small for the pressure of 8000 d-bars, 2m. too small for the pressure of 9000 d-bars, and 3.6 m. too small for the pressure of 10,000 d-bars. But even these discrepancies are of a secondary importance, for, as will be seen later, an error in the estimation of the normal depth of the isobaric surfaces will have practically no influence upon the discussion of the state of equilibrium or motion. Of much greater importance are the much smaller corrections in tables 12 H and 13 H. The greater part of the numbers in these tables have remained unaltered. But still there is a marked difference, the numbers being found numerically too small in both tables, the discrepancies in the most extreme cases mounting to 0.00004 in the values of δ_{sp} and to 0.00007 in the values of $\delta_{\tau p}$. In spite of their smallness, these corrections are of real importance for the estimation of the conditions of equilibrium and motion in the sea.

The value of the quantity $\delta_{\sigma\tau p}$ was underestimated, so that no tabulation was found necessary. This may have been an error due to the difficulties caused by the complicated method of calculation which had to be employed in order to eliminate so much as possible the errors due to the inaccuracies of Tait's measurements. Thus table 14 H has been calculated only by Ekman's formula.

The examples in Chapter VIII have been corrected according to the new tables, but the charts in Chapter IX were already printed. However, the changes in these charts would in most cases be almost microscopical.

Tables 12 H and 20 H show an increasing resistance of the sea-water against compression for increasing salinity, and tables 13 H and 21 H show, for the interval of the temperatures in question, a similar increased resistance against compression with increasing temperature. This dependence of the compressibility upon the temperature and the salinity is of great importance for the internal conditions of equilibrium or of motion in the sea. To consider a definite example: At a pressure of 5000 decibars, *i. e.*, at a depth of 5000 meters, water of 35 ‰ salinity and at the temperature of $-1°$ C. will have the same specific weight as water of 35.48 ‰ salinity at a temperature of $+1°$ C. But under the diminished pressure of 2000 decibars, *i. e.*, at the depth of 2000 meters, the specific volume of the first water will be 0.00015 greater than that of the second, and at a depth of 9000 meters the reverse will be the case. The extreme importance of these differences of compressibility will thus be perfectly clear.

31. Isosteric and Isopycnic Surfaces. — The value of the specific volume being known in a sufficient number of points in the atmosphere or the sea, we can represent the distribution of mass in each of these media by drawing a set of equiscalar surfaces, joining all points where the specific volume has certain constant values. We shall call these surfaces *isosteric surfaces*. If, on the other hand, the value of the density be known in a sufficient number of points, we may represent the distribution of mass by drawing surfaces of constant value of the density, or *isopycnic surfaces*.

The two fields representing the distribution of mass are closely related to each other, every isosteric surface being also an isopycnic surface, and *vice versa*. But one important difference should be emphasized: if the isosteric surfaces be drawn for unit-differences of the specific volume, the corresponding isopycnic surfaces will not have unit-differences of the density, and *vice versa*. This will be well illustrated if we consider the conditions in the atmosphere. Here the density decreases upwards, converging toward a very small limit, or perhaps toward zero. The specific volume, on the contrary, which is the reciprocal of the density, increases upwards, converging toward a very great limit, or perhaps toward infinity.

Drawing the isopycnic surfaces for unit-differences of the density (a unit of convenient magnitude being chosen), the thickness of the unit strata will increase upward, approximately in geometric series. If, on the other hand, the isosteric surfaces are drawn for unit-differences of the specific volume, the thickness of the corresponding unit-sheets will decrease, approximately in geometric series. Even in the sea there is a corresponding difference between the two systems of surfaces, only much less pronounced.

The equiscalar surfaces of the specific volume or the density being very nearly level, the gradient or the ascendant of these quantities will be directed very nearly along the plumb-line. For reasons which will appear later it will be more convenient to use the ascendants than the gradients in this case. The ascendant of the specific volume points upward, that of the density downward, forming a very small angle with the plumb-line.

CHAPTER IV.

PRINCIPLES OF HYDROSTATICS.

32. Pressure, Isobaric Surfaces, and Gradient.— The theory of pressure as met with under general conditions in strained elastic bodies or in moving viscous fluids is of great complexity. But in the special case of the equilibrium of any fluid, as well as in the case of the motion of a frictionless fluid, it is reduced simply to a scalar quantity. The field of a hydrostatic pressure can therefore be described according to the common principles for the description of scalar fields (section 16). Thus for the geometric representation of this field we draw surfaces of equal value of pressure, or *isobaric* surfaces. As a rule we shall draw them for unit-differences, so that they divide the space into a set of isobaric unit-sheets. To get unit-sheets of the proper thickness we are free to choose a unit-pressure of suitable magnitude.

The *pressure gradient*, or simply the *gradient* G, is given by the rate of decrease of the pressure p along the normal n to the isobaric surfaces

(*a*) $$G = -\frac{dp}{dn}$$

and the component G_s of the gradient along any direction s is given by the rate of decrease of the pressure along this direction

(*b*) $$G_s = -\frac{\partial p}{\partial s}$$

The isobaric surfaces and the unit-sheets, drawn for a unit of suitable magnitude, give the full representation of the field of the gradient G (section 17). The vector itself is directed along the normal to the surfaces, its numerical value being equal to the reciprocal thickness of the sheet. Its component G_s along any line is equal to the reciprocal length of that segment s of this line which is contained in the unit-sheet. In accordance with formula (*f*), section 17, we have finally

(*c*) $$G_{s,m} = -\frac{p_1 - p_0}{s}$$

which gives the mean value along the curve s of the component of the gradient tangential to the curve. The mean tangential gradient is thus equal to the difference of pressure at the end-points of the curve, divided by the length of the curve.

33. Dynamic Significance of the Pressure Gradient. — Like every scalar quantity, pressure has a gradient. But the gradient of the pressure has at the same time a dynamical significance, making it the fundamental vector of hydrostatics and hydrodynamics.

Let us determine the elementary force component dF_s, which, as a consequence of the pressure, tends to move a volume element $d\tau$ of the fluid in the direction s. To consider the simplest case, let the volume element have the form of a straight cylinder with its axis in the direction s and with its bases normal to this direction. As the pressure in a perfect fluid acts normally to the surface, the pressure against the lateral surface can be disregarded, as giving no addition to the component of force along the axis s. We have thus only to consider the pressure against the two bases of the cylinder. Let the value of the pressure at the first base be p. At the other it will then be $p + \frac{\partial p}{\partial s}ds$, ds being the height of the cylinder. The area of each base being $d\sigma$, we find that the exterior fluid exerts the force $pd\sigma$ against the first and the oppositely directed force $-(p + \frac{\partial p}{\partial s} ds)\, d\sigma$ against the second base. From these two oppositely directed forces will therefore result the force $dF_s = -\frac{\partial p}{\partial s} dsd\sigma$. Now $dsd\sigma$ is the volume $d\tau$ of the element. Further, $-\partial p/\partial s$ is the component G_s of the gradient in the direction s (section 33, b). We therefore get

(a) $$dF_s = G_s d\tau$$

Thus the elementary force tending to move a volume element of the fluid in any direction is equal to the component of the gradient in this direction, multiplied by the volume of the element. Or, in other words: *The gradient represents the force per unit-volume due to the field of pressure in the fluid.*

By this we see that there is a close relation between potential gradient and pressure gradient. For both gradients represent moving forces. But there is this important difference, that the potential gradient represents the force of gravity *per unit-mass*, while the pressure gradient represents the force of pressure *per unit-volume* (section 3). To get the force of pressure per unit-mass we have to multiply the gradient by the specific volume, exactly as we get the force of gravity per unit-volume by multiplying the acceleration of gravity by the density of the body considered. Force of gravity and force of pressure, both referred to unit of mass, are therefore, respectively,

(b) $$g \qquad \text{and} \qquad \alpha G$$

while the same two forces, both referred to unit-volume, are, respectively,

(c) $$\rho g \qquad \text{and} \qquad G$$

The consistent use either of forces per unit-mass or of forces per unit-volume leads to mutually equivalent but formally different methods of formulating the principles and of treating the problems of hydrostatics. We shall develop both of them in parallel, as they are complements of each other from a practical point of view.

34. **Condition of Equilibrium in Terms of Forces per Unit-Mass.** — The condition of internal equilibrium of a fluid is fulfilled if the force of gravity and the

force of pressure are everywhere directed oppositely to each other, and if their amounts per unit-mass are equal,

(*a*) $$g = -\alpha G$$

Other forms for this condition are easily deduced. Remembering that the negative derivatives of ϕ and p along the direction s are equal to the components of g and G in this direction, we get

(*b*) $$\frac{\partial \phi}{\partial s} = -\alpha \frac{\partial p}{\partial s}$$

Writing equations of this form for each of the three rectangular axes x, y, z, we get the hydrostatic equations in their traditional form, referred to rectangular coördinates. For us, however, the introduction of artificial systems of coördinates, having no relation to the intrinsic geometry of our problems, will only cause complication. It will, on the contrary, be most convenient for us to have the condition of equilibrium referred as closely as possible to the natural coördinate surfaces, the level or equipotential surfaces. This is obtained if we multiply equation (*b*) by the line element ds, and use the differential formulæ

$$\frac{\partial \phi}{\partial s} ds = d\phi \qquad\qquad \frac{\partial p}{\partial s} ds = dp$$

Between the total increases $d\phi$ and dp of pressure and of potential along the line element ds, we thus get the relation

(*c*) $$d\phi = -\alpha dp$$

This equation gives in its simplest form the intrinsic relation which, in the case of equilibrium, exists between pressure, specific volume, and gravity potential.

35. Equilibrium Relation between the Fields of Potential, of Pressure, and of Specific Volume.— We have considered, independently of each other, the fields of potential, of pressure, and of mass, and the description of each field by means of its proper equiscalar surfaces and sheets. The condition of equilibrium which we have formulated gives a relation between these three fields which can be expressed as a relation between their surfaces and sheets. Expressed in this way the equilibrium relation will contain two distinct principles, the first of which is purely descriptive, dealing with the course of the surfaces, while the other is of metric nature, giving a numerical relation between the unit-sheets.

(I) *Principle of Coincidence of Surfaces.*— The gradients of potential and of pressure being oppositely directed, while the first of them is normal to the equipotential and the second to the isobaric surfaces, we at once conclude that isobaric and equipotential surfaces must coincide.

From this it follows that every isobaric sheet must coincide with an equipotential sheet. Let the two coinciding sheets be infinitely thin. The passage from the one limiting surface of the sheet to the other, then, gives a certain increase of potential $d\phi$, and a corresponding increase of pressure dp; all along the sheet $d\phi$ has the same value, and the same will be the case with dp. Their ratio $d\phi/dp$,

therefore, is constant. But this ratio, taken with the negative sign, is, by equation (c), section 34, equal to the specific volume of the fluid in this sheet. We therefore conclude that the specific volume is constant all along the sheet. This condition being fulfilled for every infinitesimal sheet, it follows that the surfaces of equal specific volume must have the same course as those of equal pressure and of equal potential. Hence:

In the state of equilibrium there is coincidence between the isobaric, the isosteric, and the equipotential surfaces.

This remarkable coincidence of the equiscalar surfaces of three different fields is a necessary but not sufficient condition for equilibrium.

(II) *Principle of the Unit-Sheets.* — From equation (c), section 34, we further conclude that in every direction the variation of potential is α times more rapid than that of pressure. In reference to infinitesimal unit-sheets this means that every isobaric unit-sheet contains α equipotential unit-sheets. For practical reasons it will be important to have this principle formulated not only for infinitely thin sheets, but also for sheets of finite thickness. Integrating, therefore, equation (c), section 34, and denoting by α_m the mean value of the specific volume in the interval between the pressures p_1 and p_2, we get this relation between finite differences of potential and the corresponding finite differences of pressure:

$$(a) \qquad \phi_2 - \phi_1 = -\alpha_m(p_2 - p_1)$$

Applying this to an isobaric unit-sheet, we get $p_2 - p_1 = 1$, and thus

$$(b) \qquad \phi_2 - \phi_1 = -\alpha_m$$

Here $\phi_2 - \phi_1$ is the number of equipotential unit-sheets contained within the considered isobaric unit-sheet. Disregarding the sign, we thus get this numerical law:

In the state of equilibrium the number representing the mean specific volume of the fluid in an isobaric unit-sheet also represents the number of equipotential unit-sheets contained in the isobaric unit-sheet.

These two principles, taken in connection with the rule of signs that increasing potential gives decreasing pressure and *vice versa*, give the full equilibrium relation between the three fields — that of mass, that of pressure, and that of potential.

36. Determination of Heights or Depths of Given Pressures.— In the m.t.s. system of units the thickness of an equipotential unit-sheet is 1 dynamic decimeter. The number of equipotential unit-sheets contained in an isobaric unit-sheet is therefore the number of dynamic decimeters giving the thickness of the sheet. The principle of the unit-sheets, therefore, enables us to find the thickness of any isobaric sheet, adding the thicknesses of the successive unit-sheets and to determine thus the height or depth where the pressure has any given value. This is the dynamic principle of the barometric measurements of heights or of manometric measurements of depths.

Performing this operation practically, it will generally be convenient to pass from the m.t.s. units, dynamic decimeter and centibar, to the greater units, dynamic

meter and decibar, or occasionally also to other decimal parts or multiples of the dynamic meter and the corresponding decimal parts or multiples of the bar.

As a first simple example we can consider pure imcompressible water of unit specific volume. Here there is full coincidence between isobaric and equipotential unit-sheets. The standard isobaric sheets of 1 decibar (section 5) have the thickness of 1 dynamic meter, exactly as the standard equipotential sheets (section 4). Disregarding the atmospheric pressure on the sea's surface, and considering only what we have called the sea-pressure (section 26), we get this simple rule for finding the depth where the pressure has a given value. The number expressing a given sea-pressure in decibars expresses at the same time the depth of this sea-pressure in dynamic meters. This rule, being exact for the case of pure incompressible water, remains a useful approximate rule also for the case of common sea-water.

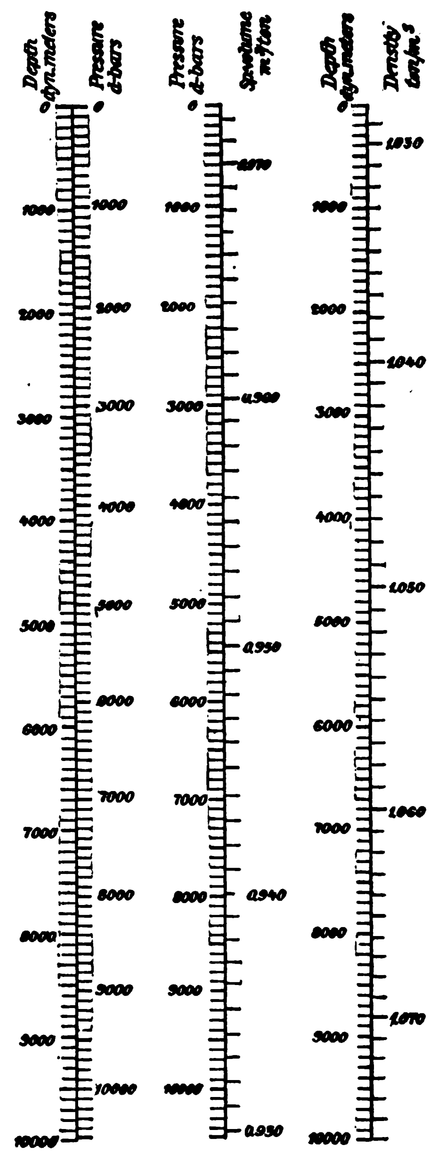


FIG. 1.—State of equilibrium of sea-water of 35 °/₀₀ salinity and 0° C.

As a second example we may consider sea-water of 35 $^0/_{00}$ salinity at 0° C. In table 8 H of Hydrographic Tables we have registered the specific volume of this water for the differences of pressure of 1 bar (10 decibars). The unit of gravity potential corresponding to this difference of pressure is the dynamic decameter. Forming the arithmetic mean of two and two successive numbers in table 8 H, we get the mean specific volume of the water in isobaric sheets of 1 bar, *i. e.*, the thickness of these sheets expressed in dynamic decameters. Adding these thicknesses from the surface downward, we get the depths of all isobaric surfaces for the interval of pressure of 1 bar. The dynamic depths found in this way are given in table 7 H of Hydrographic Tables, the units being again turned into dynamic meters and decibars. The equilibrium relation connecting dynamic depth and pressure according to this table is illustrated by the first vertical of fig. 1. The divisions to the right of this vertical give the pressures in decibars, and the divisions to the left the corresponding depths in dynamic meters. The second vertical of the figure gives in corresponding manner the equilibrium relation between pressure and specific volume, *i. e.*, the relation contained numerically in table 8 H.

If we had constructed complete tables of the specific volume of atmospheric air, we should have been able to determine the heights of given pressures in the atmos-

phere in the same direct way. But as such tables would be very bulky, we have not calculated them, and we shall show later how to proceed without them. On this occasion, therefore, we only remark that the pressure at sea-level is very nearly 10 decibars. In the atmosphere we shall therefore have to count with 10 standard isobaric surfaces of the pressure from 10 to 1 decibars. These surfaces will divide the atmosphere into 10 standard isobaric sheets, the highest of which, however, has only a distinct lower limit, the standard surface of the pressure of 1 decibar, while the existence of the upper limit, the isobaric surface of pressure zero, may be open to discussion. As a consequence of the decrease of the pressure, the thickness of the standard sheets will increase upward. The thickness of each of them will vary with the virtual temperature as shown in table 9 M of Meteorologic Tables. The methods used for calculating this and other tables required for finding the height of given pressures in the atmosphere will be given in Chapter VI.

In the mercury column of a barometer we have the same number of standard sheets as in the atmosphere. The specific volume of the mercury being 0.073554, the thickness of the standard sheets is 0.073554 dynamic meter, or 0.075008 meter for the value of gravity at sea-level at 45° latitude. This is 75 millimeters, practically.

37. Condition of Equilibrium in Terms of Forces per Unit-Volume. — To express the condition of equilibrium we can also use the forces per unit-volume, section 33, (*c*). Equilibrium exists if the forces per unit-volume are equal and oppositely directed.

$$G = -\rho g \tag{a}$$

Proceeding as in section 35, we derive from this

$$\frac{\partial p}{\partial s} = -\rho \frac{\partial \phi}{\partial s} \tag{b}$$

and

$$dp = -\rho d\phi \tag{c}$$

Each of these equations may be formed from the corresponding equation of section 35 simply by multiplying by the density ρ. The difference between the equations is thus the slightest possible, but still important in its further consequences.

38. Equilibrium Relation between the Fields of Potential, Pressure, and Density. — On interpreting geometrically the condition of equilibrium in this form, characterized by the reference of the forces to the unit of volume, we find this difference only, that the field of mass is described by the distribution of density instead of, as previously, by the distribution of specific volume. We thus arrive at the following two slightly changed forms of the principles formulated in section 35 :

(I) *Principle of Coincidence of Surfaces.* — Every isosteric surface being at the same time an isopycnic surface, we immediately get from section 35 (I):

In the state of equilibrium there is coincidence between the isobaric, the isopycnic, and the equipotential surfaces.

(II) *Principle of the Unit-Sheets.* — As is immediately seen, equation (a), section 35, takes the changed form

$$(a) \qquad p_2 - p_1 = -\rho_m(\phi_2 - \phi_1)$$

and thus for an equipotential unit-sheet, $\phi_2 - \phi$ being equal to unity,

$$(b) \qquad p_2 - p_1 = -\rho_m$$

Therefore:

In the state of equilibrium the number representing the mean density of the fluid in an equipotential unit-sheet also represents the number of isobaric unit-sheets contained in the equipotential unit-sheet.

39. **Determination of the Pressures at Given Heights or Depths.** — The principle of the unit-sheets in its first form led to the method of barometric measurements of heights or of manometric measurements of depths (section 36). In its second form it leads to the solution of the inverse problem, namely, the determination of the pressure at given heights or depths. The m.t.s. isobaric unit-sheet represents the difference of pressure of 1 centibar. The number of such sheets contained in the equipotential unit-sheet therefore gives the difference of pressure in centibars between the surfaces limiting the equipotential unit-sheet. Adding these differences of pressure from level surface to level surface, we can determine the pressure at any level if it be known in an initial level. Performing it practically we may as above make use of other units of pressure and of gravity potential than those of the m.t.s. system.

Taking the same examples as above, there will be no difference so long as we consider pure incompressible water at maximum of density. The density being unity, the increase of pressure for each dynamic meter of depth will be 1 decibar, and the number representing the depth in dynamic meters will represent at the same time the sea-pressure expressed in decibars. As a second example we shall determine the pressure in given depths in sea-water of 35 ‰ salinity and 0°C. The hydrographical table 16 H gives the density of this water at all depths for intervals of 1 dynamic decameter. Forming the mean value of two and two successive numbers in this table, we get the average density of the sea-water in equipotential unit-sheets of 1 dynamic decameter, *i. e.*, the increase of pressure in bars from level surface to level surface. Adding these increases of pressure from sea-level downwards, we get the sea-pressure expressed in bars at all dynamic depths for intervals of 1 dynamic decameter. Then, on returning to the smaller units, the dynamic meter and decibar, these pressures are given in table 15 H in our Hydrographic Tables. The equilibrium relation between dynamic depth and pressure contained in this table is intrinsically the same as that contained in table 7 H. Graphically we arrive at the same representation from both tables, given by the first vertical of fig. 1. The third vertical represents the relations between dynamic depth and density contained in table 16 H.

40. **Integral Forms of the Equation of Equilibrium.** — In equations (c), section 34, and (c), section 37, the increase of potential $d\phi$ and the increase of pressure

dp are referred to the displacement along an element of line ds. Forming the sum for any succession of line elements, we get the equations referred to a curve of finite length, namely, from (c), section 34,

$$(a) \qquad \phi_2 - \phi_1 = -\int_{p_1}^{p_2} \alpha dp$$

and from (c), section 37,

$$(b) \qquad p_2 - p_1 = -\int_{\phi_1}^{\phi_2} \rho d\phi$$

The first of these equations gives the difference of potential, *i. e.*, the difference of dynamic height, between the isobaric surfaces of pressures p_2 and p_1. The second gives the difference of pressure between two equipotential surfaces of potentials ϕ_1 and ϕ_2.

The dynamic sense of the integrals forming the second member of equations (a) and (b) is easily found, as we have

$$-\alpha dp = \alpha G_s ds \qquad\qquad -\rho d\phi = \rho g_s ds$$

Thus the integral in (a) is the line-integral of the force of pressure per unit-mass. The integral in equation (b) is the line-integral of the force of gravity per unit-volume. On the other hand, the differences appearing on the left side of the equations $\phi_2 - \phi_1$ and $p_2 - p_1$ are the line-integrals of the force of gravity per unit-mass and of the force of pressure per unit-volume. Equation (a) thus shows that the force of gravity and the force of pressure, both referred to unit-mass, have oppositely equal line-integrals. In the same way, equation (b) shows that the force of gravity and the force of pressure, both referred to unit-volume, have oppositely equal line-integrals. One of the two oppositely equal line-integrals can always be expressed in finite form, namely, that of the force of gravity per unit-mass and that of the force of pressure per unit-volume.

The equations (a) and (b) enable us at once to derive a fundamental property of the integrals appearing on the right side. The values ϕ_1 and ϕ_2 of the potential in the end-points 1 and 2 of the curve depend only upon the situation of these points 1 and 2, and not upon the course of the curve s joining them. The same is the case with the values p_1 and p_2 of the pressure in the same two points. The integrals on the right side, therefore, must have the same property. Hence we conclude:

Under statical conditions the line-integral of the force of pressure per unit-mass

$$(c) \qquad -\int_{p_1}^{p_2} \alpha dp$$

as well as the line-integral of the force of gravity per unit-volume

$$(d) \qquad -\int_{\phi_1}^{\phi_2} \rho d\phi$$

are independent of the course of the curve and dependent only upon the positions of its end-points.

As a corollary we get this other theorem:

Under statical conditions the line-integrals of the force of pressure per unit-mass (c), *as well as the line-integral of the force of gravity per unit-volume* (d) *are zero for every closed curve.*

CHAPTER V.

IDEAL STATES OF EQUILIBRIUM IN THE ATMOSPHERE.

41. Analytical Integration of the Equation of Atmospheric Equilibrium. — The hydrostatic equation

$$(a) \qquad d\phi = -\alpha dp$$

contains three variable quantities, ϕ, p, α. Two of them, p and α, are connected with a third variable ϑ by the equation of state

$$(b) \qquad p\alpha = R\vartheta$$

ϑ being the true temperature of dry or the virtual temperature of moist air. By this equation we may introduce temperature ϑ as a variable in (a) instead of the specific volume α. This will generally be convenient, and the equation of atmospheric equilibrium then takes the form

$$(c) \qquad d\phi = -R\vartheta\frac{dp}{p}$$

Now, supposing a relation between temperature and pressure to be known,

$$(d) \qquad f_1(\vartheta, p) = 0$$

equation (c) is seen to be integrable immediately. To perform the integration we may choose either of two ways. We may use (d) to eliminate the pressure from the second member of (c). The integration then gives a relation between gravity potential and temperature

$$(e) \qquad f_2(\phi, \vartheta) = 0$$

Eliminating afterwards the temperature between (d) and (e), we get the relation of equilibrium connecting gravity potential and pressure

$$(f) \qquad F(\phi, p) = 0$$

Or we may, on the other hand, use equation (d) to eliminate the temperature from the second member of (c). The integration then immediately leads to the equilibrium relation (f) between gravity potential and pressure. The elimination of pressure between equations (f) and (d) will then lead to the relation (e) connecting gravity potential and temperature.

Again, we might have written equation (c) in the form

$$(g) \qquad \frac{d\phi}{\vartheta} = -R\frac{dp}{p}$$

The equation in this form is seen to be integrable at once if a relation between temperature and gravity potential be given, *i. e.*, a relation of the form (e). For the integration we again have the choice of either of two ways. We may use equation (e) to eliminate the gravity potential from the left member of (g). The integration then leads to the relation (d) between temperature and pressure. Then the elimination of the temperature between (d) and (e) leads to the equilibrium relation (f) connecting gravity potential and pressure. Or we might have used (e) to eliminate the temperature from equation (g). The integration would then have led directly to the equilibrium relation (f) between gravity potential and pressure, while elimination between (f) and (e) would have led to the corresponding relation (d) between temperature and pressure.

As will be inferred from the above discussion, we have to notice two cases of immediate integrability, the first characterized by a relation between temperature and pressure (d), the second by a relation between temperature and gravity potential (e). Between these two cases of integrability there is a full correspondence in this sense: that to a given relation between temperature and pressure (d) there will correspond a perfectly definite relation between temperature and gravity potential (e), and *vice versa.*

42. Atmosphere with Constant-Temperature Gradient. — Let us suppose temperature to be a linear function of gravity potential

$$\vartheta = \vartheta_0 - \gamma\phi \tag{a}$$

ϑ_0 being the temperature at sea-level and γ the temperature gradient

$$\gamma = -\frac{d\vartheta}{d\phi} \tag{a'}$$

which is in this case constant.

To find the relation between temperature and pressure, corresponding to the relation (a) between temperature and potential, we eliminate $d\phi$ between equations (a') and section 41 (g). This gives

$$\frac{d\vartheta}{\vartheta} = R\gamma\frac{dp}{p}$$

and hence after integration, p_0 being the pressure at sea-level,

$$\frac{\vartheta}{\vartheta_0} = \left(\frac{p}{p_0}\right)^{R\gamma} \tag{b}$$

We thus arrive at this important result:

If temperature be a linear function of gravity potential, with the temperature gradient γ, it will be proportional to the power $R\gamma$ of pressure, R being the gas constant. And vice versa: If temperature be proportional to any power $R\gamma$ of the pressure, it will be a linear function of gravity potential with the temperature gradient γ.

Eliminating the temperature between the equations (a) and (b), we arrive at the equilibrium relation between gravity potential and pressure, namely,

(c)
$$1 - \frac{\gamma}{\vartheta_0}\phi = \left(\frac{p}{p_0}\right)^{R\gamma}$$

Adding, finally, the equation of state

(d)
$$p\alpha = R\vartheta$$

we can find the corresponding equilibrium values of the specific volume α or of its reciprocal, the density ρ.

The problem is thus fully solved. Summing up the results, we shall choose once the pressure and once the gravity potential as independent variable. In the first case we shall represent the distribution of mass by the specific volume α, in the second by the densitv ρ. Denoting by ϑ_0, p_0, α_0, ρ_0 the values of temperature, pressure, specific volume, and density at sea-level, we easily arrive at the following two schemes of formulæ:

(A)
$$\vartheta = \vartheta_0\left(\frac{p}{p_0}\right)^{R\gamma} \qquad \alpha = \alpha_0\left(\frac{p}{p_0}\right)^{R\gamma-1} \qquad \phi = \frac{\vartheta_0}{\gamma}\left[1 - \left(\frac{p}{p_0}\right)^{R\gamma}\right]$$

(B)
$$\vartheta = \vartheta_0\left(1 - \frac{\gamma}{\vartheta_0}\phi\right) \qquad \rho = \rho_0\left(1 - \frac{\gamma}{\vartheta_0}\phi\right)^{\frac{1}{R\gamma}-1} \qquad p = p_0\left(1 - \frac{\gamma}{\vartheta_0}\phi\right)^{\frac{1}{R\gamma}}$$

each of which represents the full solution of the problem.

43. Limit of the Atmosphere in Case of Constant-Temperature Gradient. — Temperature being a linear function of the gravity potential, and decreasing upwards, absolute zero will be reached at a certain finite height

(a)
$$\phi_L = \frac{\vartheta_0}{\gamma}$$

Substituting this in the two last equations (B), section 43, and remembering that γ is positive when temperature decreases upwards, we get

$$\rho = 0 \qquad p = 0$$

Supposing, thus, the gas laws to be true even at absolute zero, we find the atmosphere to be limited by the level surface determined by (a).

For decreasing values of the temperature gradient γ the height of the atmosphere always increases and converges towards infinity when γ converges towards zero, *i. e.*, in the case of the isothermic atmosphere.

When γ is negative, and thus the temperature rises with the height, ϕ_L also is negative. The atmosphere remains unlimited upwards, while its analytical continuation below sea-level has the limit ϕ_L determined by equation (a).

44. States of Unstable Equilibrium. — In the extreme case $R\gamma = \infty$, *i. e.*, in the case of an infinite decrease of temperature with the height, we get $\phi_L = 0$. The atmosphere is, then, condensed to an infinitely thin sheet. For values of $R\gamma$ decreasing from ∞ to 1, we get values of the temperature gradient γ decreasing from ∞ to 0.00348, this last value representing a fall of temperature of 3.48° C. for every 100 dynamic meters of height. Extreme falls of temperature of this order of magni-

tude may exist locally under extraordinary conditions, as above a hot chimney or above a volcano in action. They may perhaps exist also for a short while over a heated area before the formation of a tornado. But the corresponding state of equilibrinm can not endure. For it is seen from the second equation (B) that as long as $R\gamma$ is comprised between ∞ and 1 there will be increase of density upward. The state of equilibrium is therefore completely unstable.

The limiting case

$$(a) \qquad R\gamma = 1$$

corresponding to a fall of temperature of 3.48° C. for every 100 meters, is interesting from a mathematical point of view. In this case the equations (A) and (B) reduce to the simple forms

$$(\text{A}') \qquad \vartheta = \vartheta_0 \frac{p}{p_0} \qquad \alpha = \alpha_0 \qquad \phi = R\vartheta_0\left(1 - \frac{p}{p_0}\right)$$

$$(\text{B}') \qquad \vartheta = \vartheta_0\left(1 - \frac{1}{R\vartheta_0}\phi\right) \qquad \rho = \rho_0 \qquad p = p_0\left(1 - \frac{1}{R\vartheta_0}\phi\right)$$

These are all linear, those for the specific volume, $\alpha = \alpha_0$, and for the density, $\rho = \rho_0$, showing that specific volume and density are constant. As the pressure and the temperature thus both decrease with the height, they compensate each other in their influence upon the density of the air, the result being a perfectly ***homogeneous atmosphere.***

Also, in the case of the homogeneous atmosphere the equilibrium is unstable. For if a mass of air be moved upwards, the adiabatic cooling will not suffice to bring it down to the temperature of the higher strata, to which it has been moved. Therefore, if once given the slightest displacement upwards, it will continue moving upwards, remaining always lighter than the adjacent air.

The height ϕ_L' of this homogeneous atmosphere has, according to (*a*) and section 43 (*a*), the value

$$(a') \qquad \phi_L' = R\vartheta_0$$

It merits attention that we may introduce the two limiting heights ϕ_L and ϕ_L' as fundamental parameters in our formulæ. To do this we have the expressions

$$(b) \qquad R\gamma = \frac{\phi_L'}{\phi_L} \qquad \frac{\gamma}{\vartheta_0}\phi = \frac{\phi}{\phi_L}$$

The ratios on the right side being independent of the units of gravity potential, we may also write

$$(b') \qquad R\gamma = \frac{H_L'}{H_L} \qquad \frac{\gamma}{\vartheta_0}\phi = \frac{H}{H_L}$$

measuring the height H and the limiting heights H_L and H_L' in dynamic meters. Equations (*b*) or (*b'*) give thus a perspicuous sense to expressions appearing in the equations (A) and (B).

Proceeding to values of $R\gamma$ smaller than 1, we come to states of less pronounced instability. The case $R\gamma = \frac{1}{2}$, corresponding to a decrease of temperature of 1.74° C. for every 100 dynamic meters of height, is interesting mathematically, temperature being in direct and specific volume in inverse proportion to the square root of the pressure, and the density being a linear function of the dynamic height. $R\gamma = \frac{1}{3}$ also gives simple formulæ, representing a state of equilibrium still unstable but greatly approaching the state of indifferent or adiabatic equilibrium.

45. **Indifferent or Adiabatic Equilibrium.** — The state of equilibrium will be indifferent if the adiabatic cooling of a mass of air, which is displaced upwards, will always bring its temperature to that of the air-masses in the new level. For in this case no force will arise tending to favor or to counteract the displacement. The distribution of temperature giving adiabatic equilibrium will be different according to the humidity of the air. Considering first the case of a perfectly dry atmosphere, let κ be the well-known ratio 1.4053 of the two specific heats of an ideal gas. Introducing

$$R\gamma = \frac{\kappa - 1}{\kappa} = 0.2884 \tag{a}$$

we see that the equations (A) and (B) take the forms

$$(\text{A}'')\qquad \vartheta = \vartheta_0\left(\frac{p}{p_0}\right)^{\frac{\kappa-1}{\kappa}} \qquad \alpha = \alpha_0\left(\frac{p}{p_0}\right)^{-\frac{1}{\kappa}} \qquad \phi = \frac{\kappa}{\kappa - 1} R\vartheta_0\left[1 - \left(\frac{p}{p_0}\right)^{\frac{\kappa-1}{\kappa}}\right]$$

$$(\text{B}'')\qquad \vartheta = \vartheta_0\left(1 - \frac{\kappa - 1}{\kappa R\vartheta_0}\phi\right) \qquad \rho = \rho_0\left(1 - \frac{\kappa - 1}{\kappa R\vartheta_0}\phi\right)^{\frac{1}{\kappa-1}} \qquad p = p_0\left(1 - \frac{\kappa - 1}{\kappa R\vartheta_0}\phi\right)^{\frac{\kappa}{\kappa-1}}$$

The two first equations (A″) are the well-known ones connecting temperature and pressure, and specific volume and pressure, respectively, in the case of an adiabatic change of state of an ideal gas. The state of equilibrium defined by equations (A″) or (B″) has therefore the following property: Proceeding upwards to decreasing pressure we find everywhere the temperature which a mass of dry air moved upwards would take on account of its adiabatic cooling. The temperature gradient in this atmosphere is

$$\gamma = \frac{\kappa - 1}{\kappa R} = 0.0010048 \tag{a'}$$

representing a fall of temperature of 1.0048° C. for every 100 dynamic meters of height.

Moist air will have the same adiabatic temperature gradient (a') as dry air, as long as no condensation takes place. But as soon as condensation begins, the heat of condensation will partly compensate for the adiabatic cooling, and the adiabatic gradient will take one of the values given in table D.* While the adiabatic temperature gradient for dry air is constant, that for saturated air varies both with pressure and temperature, decreasing with decreasing pressure and increasing with decreasing temperature. The decrease upward both of pressure and temperature

* The table is taken from Hann's Meteorology (first edition), p. 241, with the difference that the pressure figuring as argument is reduced from millimeters of mercury to m-bars, while the fall of temperature is taken per 100 dynamic instead of per 100 common meters.

therefore counteract each other in their effect upon the fall of temperature, making its variation with the height very gradual. But still it will always increase upward, converging toward the limit 1.0048, which would be reached when all moisture had fallen out. To illustrate this increasing fall of temperature, the values corresponding to the case of a mass of air moved upwards from sea-level with the initial temperature of 15° C. are indicated by heavy-faced figures in the table.

Table D.—*Adiabatic Fall of Temperature per 100 Dynamic Meters for Saturated Air.*

Pressure (millibars).	Temperature (° C.).									
	−10	−5	−0	+0	+5	+10	+15	+20	+25	+30
300	0.52	0.46	0.40	0.42						
400	.58	.52	.45	.47	0.42					
500	**.63**	.57	.49	.51	.46	0.41	0.37			
600	.67	**.60**	.54	.56	.50	.45	.40			
700	.70	.64	**.57**	**.59**	.54	.48	.42	0.39		
800	.72	.66	.59	.61	**.56**	.50	.45	.41	0.38	
900	.75	.69	.62	.64	.59	**.53**	.48	.44	.40	0.37
1000	.77	.70	.64	.66	.61	.55	**.50**	.45	.41	.38

The case of adiabatic equilibrium for saturated air can not, therefore, be comprised in the case of equilibrium with constant-temperature gradients treated here. But as the increase of the gradient upward is gradual, we may with some approximation reckon with constant average values for sheets of moderate thickness. Thus the temperature gradient $\gamma = 0.0005$, corresponding to a fall of temperature of 0.5° C. for every 100 dynamic meters of height, is a value often used by practical meteorologists, and may be taken as an average value of the adiabatic temperature gradient for saturated air in the lower strata of the atmosphere.

46. States of Stable Equilibrium.— Passing to temperature gradients smaller than the adiabatic, we arrive at states of stable equilibrium. If in this case a mass of air be moved upward, the adiabatic cooling will bring it to a lower temperature than that of the surrounding masses and it will sink back again on account of its greater density.

Interesting mathematically is the case $R\gamma = 0$, that is, the case of a temperature-gradient zero,

$$(a) \qquad \gamma = 0$$

or of *isothermic atmosphere*. For greater gradients the atmosphere has been stated to be finite. But in this case it becomes infinite. At the same time the second member of the last equation (A), section 42, and of the two last equations (B), section 42, become indeterminate. But by the theory of indeterminate expressions, or by renewed integration of the equation of equilibrium (c), section 41, after the substitution $\vartheta = \vartheta_0$, we easily arrive at the following set of formulæ representing the state of isothermic equilibrium:

$$(\text{A}''') \qquad \vartheta = \vartheta_0 \qquad \alpha = \alpha_0 \frac{p_0}{p} \qquad \phi = R\vartheta_0 \text{ nat. log.} \frac{p_0}{p}$$

$$(\text{B}''') \qquad \vartheta = \vartheta_0 \qquad \rho = \rho_0 e^{-\frac{\phi}{R\vartheta_0}} \qquad p = p_0 e^{-\frac{\phi}{R\vartheta_0}}$$

Passing to the case of negative temperature gradients, *i. e.*, of increase of temperature upward, the height of the atmosphere remains infinite, the limit determined by formula (*a*), section 43, having only the analytical meaning of the limit of an imaginary continuation of the atmosphere below sea-level. This increase of temperature with the height is meaningless if it be extended to the whole atmosphere.

But "temperature inversion" is well known as a local phenomenon, limited to more or less narrow sheets, occurring specially often in the case of high pressure during winter. The main feature of this state from a dynamic point of view is a pronounced stability which can be overcome only by causes producing different distribution of temperature. Values of $R\gamma$ as $-\frac{1}{2}$ or -1 give very simple forms for equations (A) and (B) and represent increases of temperature with the height which may occur in the sheets of inversion, namely, 1.74 and 3.48° C. for every 100 meters of height.

47. Numerical Representation of the States of Equilibrium.— For the numerical representation of any definite state of equilibrium we have the choice between either of two methods.

TABLE E.—*Ideal States of Atmospheric Equilibrium. Argument, Pressure.*

Pressure (m-bars).	$1000\,\gamma = 3.48°$ C. (Homogeneous atmosphere.)			$1000\,\gamma = 1.0048°$ C. (Dry atmosphere in adiabatic equilibrium.)			$1000\,\gamma = 0.5°$ C.			$1000\,\gamma = 0$. (Isothermic atmosphere.)		
	Height (dynamic meters).	Temperature (° C.).	Specific volume (m³/ton).	Height (dynamic meers).	Temperature (° C.).	Specific volume (m³/ton).	Height (dynamic meters).	Temperature (° C.).	Specific volume (m³/ton).	Height (dynamic meters).	Temperature (° C.).	Specific volume (m³/ton).
0	7835	−273.0	784	27178	−273.0	∞	54600	−273.0	∞	∞	0	∞
100	7054	−245.7	784	13188	−132.5	4034	15368	− 76.8	5632	18047	0	7838
200	6270	−218.4	784	10092	−101.4	2464	11264	− 56.3	3111	12614	0	3919
300	5486	−191.1	784	7973	− 80.1	1846	8666	− 43.3	2198	9436	0	2612
400	4703	−163.8	784	6312	− 63.4	1504	6729	− 33.6	1718	7182	0	1959
500	3919	−136.5	784	4925	− 49.5	1284	5171	− 25.9	1419	5433	0	1568
600	3135	−109.2	784	3723	− 37.4	1127	3861	− 19.3	1214	4004	0	1306
700	2351	− 81.9	784	2656	− 26.7	1010	2726	− 13.6	1064	2796	0	1120
800	1568	− 54.6	784	1694	− 17.0	919	1721	− 8.6	949	1749	0	980
900	784	− 27.3	784	814	− 8.2	845	821	− 4.1	858	826	0	871
1000	0	0	784	0	0	784	0	0	784	0	0	784

We can use the pressure as argument and register temperature, specific volume, and height for suitable integer values of pressure. This method is used in table E, giving temperature, specific volume, and height for each of the standard isobaric surfaces. The four sections of the table correspond to four different temperature gradients: (1) that giving homogeneous atmosphere; (2) that giving adiabatic equilibrium in a perfectly dry atmosphere: (3) the gradient 0.0005 roughly representing in the lower strata adiabatic equilibrium of saturated air and in the higher strata stable equilibrium; (4) the gradient zero giving isothermic atmosphere. In all these cases the temperature is supposed to be zero centigrade at sea-level, and the height is measured in dynamic meters.

On the other hand, we can choose dynamic height as argument and register temperature, density, and pressure for certain standard heights. This is made in table F, the four sections of the table representing the same four cases as in table E.

TABLE F.—*Ideal States of Atmospheric Equilibrium. Argument, Dynamic Height.*

Height (dynamic meters).	$1000\,\gamma = 3.48°$ C. (Homogeneous atmosphere.)			$1000\,\gamma = 1.0048°$ C. (Dry atmosphere in adiabatic equilibrium.)			$1000\,\gamma = 0.5°$ C.			$1000\,\gamma = 0$. (Isothermic atmosphere.)		
	Pressure (m-bars).	Temperature (° C.).	Density (10^{-6} ton/m³).	Pressure (m-bars).	Temperature (° C.).	Density (10^{-6} ton/m³).	Pressure (m-bars).	Temperature (° C.).	Density (10^{-6} ton/m³).	Pressure (m-bars).	Temperature (° C.).	Density (10^{-6} ton/m³).
⋮							⋮	⋮	⋮	⋮	⋮	⋮
30,000	....	....	....	....	....	....	3.9	−150	11	21.8	0	28
29,000	....	....	....	....	....	....	5.1	−145	14	24.7	0	31
28,000	....	....	....	....	....	....	6.7	−140	17	28.1	0	36
27,000	....	....	....	0.00003	−271.2	0.005	8.6	−135	22	31.9	0	41
26,000	....	....	....	0.0188	−261.2	0.56	11.1	−130	27	36.3	0	46
25,000	....	....	....	0.158	−251.1	2.51	14.1	−125	33	41.2	0	53
24,000	....	....	....	0.586	−241.1	6.40	17.7	−120	40	46.8	0	60
23,000	....	....	....	1.514	231.0	12.6	22.2	−115	49	53.2	0	68
22,000	....	....	....	3.186	221.0	21.3	27.5	−110	59	60.4	0	77
21,000	....	....	....	5.877	210.9	33.0	34.0	−105	70	68.6	0	87
20,000	....	....	....	9.889	200.9	47.7	41.7	−100	84	77.9	0	99
19,000	....	....	....	15.54	−190.9	65.9	50.8	− 95	99	88.6	0	113
18,000	....	....	....	23.19	−180.8	87.6	61.6	− 90	117	100.6	0	128
17,000	....	....	....	33.19	−170.8	113	74.4	− 85	138	114.3	0	146
16,000	....	....	....	45.93	−160.7	143	89.3	− 80	161	129.8	0	166
15,000	....	....	....	61.82	−150.7	176	106.7	− 75	188	147.5	0	188
14,000	....	....	....	81.28	−140.6	214	127.0	− 70	216	167.6	0	214
13,000	....	....	....	104.7	−130.6	256	150.4	− 65	252	190.4	0	243
12,000	....	....	....	132.7	−120.5	303	177.5	− 60	290	216.3	0	276
11,000	....	....	....	165.7	−110.5	355	208.7	− 55	334	245.7	0	314
10,000	....	....	....	203.8	−100.4	411	244.4	− 50	382	279.2	0	356
9,000	....	....	....	248.0	− 00.4	473	285.3	− 45	435	317.2	0	405
8,000	....	....	....	298.6	− 80.4	540	331.7	− 40	496	360.3	0	460
7,000	106.9	−243.8	1276	356.1	− 70.3	612	384.5	− 35	563	409.4	0	522
6,000	234.5	−209.0	1276	421.1	− 60.3	689	444.5	− 30	637	465.1	0	593
5,000	362.0	−174.2	1276	494.2	− 50.2	773	512.2	− 25	719	528.4	0	674
4,000	489.6	−139.3	1276	575.8	− 40.2	861	588.6	− 20	810	600.3	0	766
3,000	617.2	−104.5	1276	666.7	− 30.1	956	674.6	− 15	911	682.0	0	870
2,000	744.8	− 69.7	1276	767.2	− 20.1	1057	771.2	− 10	1021	774.8	0	988
1,000	872.4	− 34.8	1276	878.1	− 10.0	1163	879.1	− 5	1143	880.2	0	1123
0	1000	0	1276	1000	0	1276	1000	0	1276	1000	0	1276

The two tables show essentially different features. The first has the important property of being finite, which gives a great practical advantage, while the second continues infinitely to infinite heights. It is important to remark also that the division of the atmosphere into isobaric sheets, as in table E, represents practically a division into sheets of equal mass, and thus, from certain points of view, of equal importance, while the division into equipotential sheets as in table F corresponds to a division into sheets of decreasing masses upward, and thus of decreasing importance.

The states of equilibrium represented by these tables are also illustrated by fig. 2, the method of representation being the same as that used in fig. 1 (p. 45) to illustrate the equilibrium in the sea.

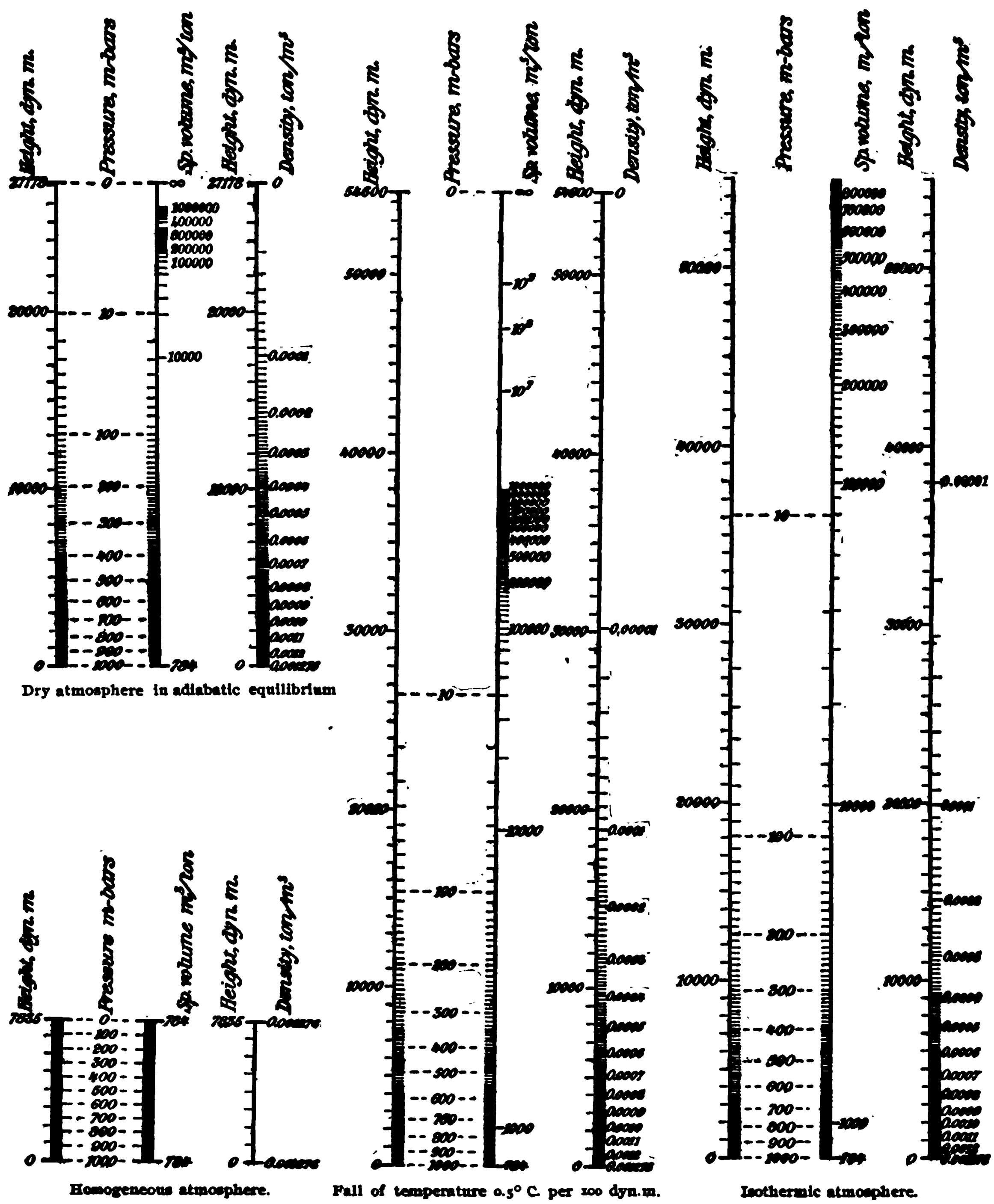


FIG. 2. — States of atmospheric equilibrium.

48. Graphical Representation of the States of Equilibrium. — To get a more comprehensive view of the states of equilibrium than that afforded by the numerical tables and the diagram, fig. 2, we may use a graphic method. Introducing according to (b') section 44, the ratios $\frac{H}{H_L}$ and $\frac{H_L}{H_L'}$ in (B), section 42, this system of equations may be written in the form

$$(a) \qquad \frac{\vartheta}{\vartheta_0} = 1 - \frac{H}{H_L} \qquad \frac{\rho}{\rho_0} = \left(1 - \frac{H}{H_L}\right)^{\frac{H_L}{H_L'}-1} \qquad \frac{p}{p_0} = \left(1 - \frac{H}{H_L}\right)^{\frac{H_L}{H_L'}}$$

To see the content of these equations we may use as ordinates the dynamic heights H, and as abscissæ the ratio H_L/H_L' of the limiting heights. Doing this, we range the different atmospheres according to their heights compared with that of the homogeneous atmosphere. The ratio itself has a real physical meaning only when it is positive. But to every value, positive or negative, of the ratio there corresponds a definite value, positive or negative, of the temperature gradient according to the first equation (b'), section 44, or

$$(b) \qquad \gamma = \frac{1}{R}\frac{H_L'}{H_L}$$

To facilitate the interpretation of the diagram the values of $1000\,\gamma$ according to this equation, *i. e.*, the fall of temperature for every 100 dynamic meters, is also shown along the axis of abscissæ.

In the plane of coördinates thus defined a constant value of the ratio ϑ/ϑ_0 gives an isothermic curve, a constant value of the ratio ρ/ρ_0 an isopycnic curve, and a constant value of the ratio p/p_0 an isobaric curve. Choosing a set of values for these three ratios we get three systems of curves, drawn in fig. 3. The three sets of curves give a full representation of the state of equilibrium for every value of the ratio of the limiting heights H_L/H_L'. Fixing a certain value for this ratio, or for the temperature gradient, we get a definite vertical line in each of the three diagrams. The intersections of this vertical, for instance with the isothermic curve 0.1, give the height H at which the absolute temperature is reduced to one-tenth of the value ϑ_0 which it has at the earth's surface. In the same manner the intersection of the vertical with the isopycnic curve 0.1 gives the height H where the density ρ is reduced to one-tenth of the value ρ_0 which it has at the earth's surface. Finally, the intersection of the vertical with the isobaric curve 0.1 gives the height where the pressure is reduced to one-tenth of its value p_0 at the surface of the earth. Fixing according to the equation of state a consistent set of values ϑ_0, ρ_0, p_0 at the earth's surface, the values of these quantities at any heights are found from the diagram.

As to the course of the curves, it is seen that each diagram contains, on the side of the positive temperature gradients (decrease of temperature upwards), a straight line forming an angle of 45° with the axis and representing respectively temperature, density, and pressure zero. The ordinates of this straight line give the limiting height of the atmosphere for all positive finite values of the temperature gradient,

the value being ∞ for $H_L/H_L' = \infty$, *i. e.*, for gradient zero or isothermic atmosphere. On the negative side of the axis of ordinates no such limiting curve exists, the atmosphere being always unlimited in the case of increase of temperature upward. Both isobaric and isopycnic curves converge at infinity towards horizontal

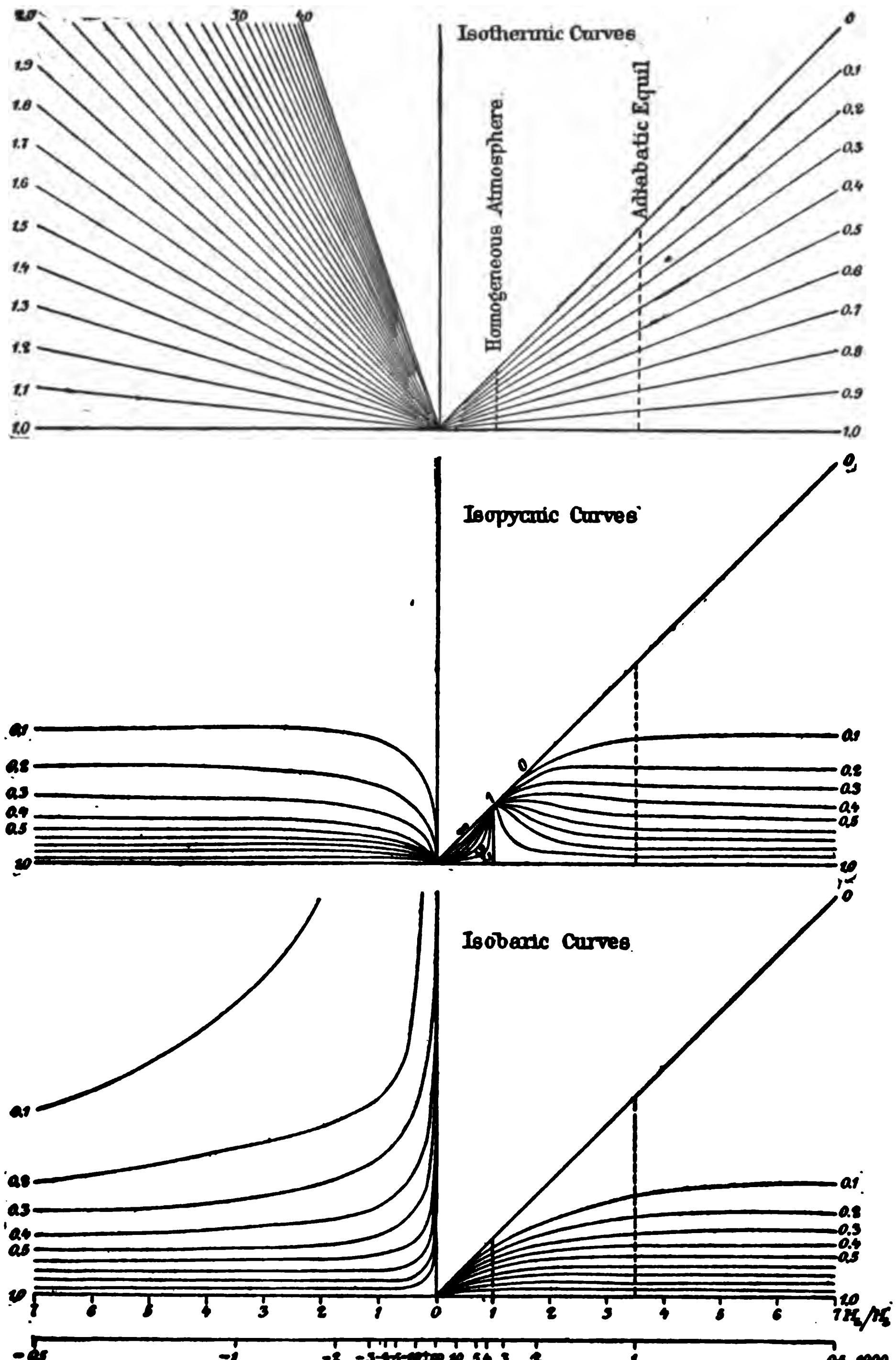


FIG. 3. — States of atmospheric equilibrium for different values of the ratio H_L/H_L' of the limiting heights.

asymptotes, the asymptotic values representing the case of the isothermic atmosphere. The course of the isobaric curves is relatively simple. For positive infinite values of the temperature gradient they all begin at zero. The ordinates increase with decreasing values of the gradient, converging towards infinity for an infinite negative value of this gradient. The course of the isopycnic curves is more complicated. They also all begin at zero for infinite positive values of the temperature gradient. Then they pass through one point — that representing the limit of the homogeneous atmosphere. Afterwards, passing through a maximum, they return to zero for infinite negative values of the gradient, giving as we approach this point an infinitely rapid decrease of the density upward.

CHAPTER VI.

PRACTICAL SOLUTION OF THE HYDROSTATIC PROBLEM FOR THE ATMOSPHERE.

49. **Four Forms of the Problem.**—In the preceding simple cases we have used two different methods of registering numerically the equilibrium relation between pressure and dynamic height. We have registered either ***the height of given pressures*** or ***the pressure at given heights.*** In cases of practical occurrence, when any analytical form to the equilibrium relation can not be given, we shall always try to find the result in one of the same two forms, as a table containing the heights of given pressures or as a table containing the pressures in given heights.

On the other hand, the observed data from which the equilibrium relation may be deduced will generally be given in one of two forms. The observed quantities may be the correlated values of ***pressure***, temperature, and humidity, or of ***height***, temperature, and humidity. From the first set we can calculate the virtual temperature for given values of pressure; from the second the virtual temperature at given heights. The practical problem, therefore, will present itself in one of the following four forms:

(1) To calculate the heights corresponding to given pressures, the virtual temperatures for these values of the pressures being known.

(2) To calculate the heights corresponding to given pressures, the virtual temperatures at given heights being known.

(3) To calculate the pressures at given heights, the virtual temperatures for given values of the pressure being known.

(4) To calculate the pressures at given heights, the virtual temperatures at given heights being known.

Of these four problems the first is by far the simplest, and at the same time practically the most important. We shall therefore first direct our attention to the practical solution of this problem. The others will afterwards easily be solved.

50. **Fundamental Formula.**—As already remarked, the hydrostatic equation in its first integral form, (a), section 40, gives the difference of potential corresponding to a given difference of pressure. Passing from the potential ϕ expressed in dynamic decimeters to the dynamic height H expressed in dynamic meters, and passing simultaneously from centibar to decibar as unit-pressure, the equation takes the form

$$(a) \qquad H_b - H_a = -\int_{p_a}^{p_b} \alpha dp$$

Eliminating the specific volume by the equation of state

(*b*) $$\alpha = \frac{R\vartheta_r}{p}$$

we get

(*c*) $$H_b - H_a = -R\int_{p_a}^{p_b} \vartheta_r \frac{dp}{p}$$

or

(*d*) $$H_b - H_a = -R\int_{p_a}^{p_b} \vartheta_r d \text{ nat. log. } p$$

Considering henceforth nat. log. p as the independent variable, and taking outside the integral sign the average value ϑ_{ab} of the virtual temperature, we get the simple formula

(*e*) $$H_b - H_a = R\vartheta_{ab} \text{ nat. log. } \frac{p_a}{p_b}$$

giving the dynamic height from the isobaric surface p_a to the isobaric surtace p_b.

The defined average value ϑ_{ab} of the variable virtual temperature ϑ_r has a simple meaning: It is that constant temperature which, substituted for the variable temperature ϑ_r gives the sheet between the two isobaric surfaces p_a and p_b its true thickness. This average value is easily found by the virtual-temperature diagram, this diagram being drawn with logarithmic scale for the pressure, as in fig. 5, example 1 below. Here the horizontal lines correspond to constant pressures and the vertical lines to constant temperatures. Three curves running close together are seen in the diagram. The middlemost is that representing the virtual temperatures derived from the observations. The vertical segments of line give the required average values of the virtual temperature of each of the standard isobaric sheets. Each segment is drawn so that the two triangular areas limited by the urve, the segment, and the two standard isobaric lines are equal. These vertical segments may generally be drawn free-hand with a precision exceeding that of the observations from which the curve of virtual temperature has been derived. Of course greater precision, if required, may be obtained by use of a planimeter.

51. Fundamental Tables. — The sheet between the isobaric surfaces p_a and p_b will generally contain a set of isobaric standard sheets. The height $H_b - H_a$ can therefore be calculated as the sum of three terms: (1) the height from the isobaric surface p_a to the nearest standard surface; (2) the height from this standard surface to a certain higher standard surface; (3) the height from the last standard surface to the isobaric surface p_b.

To find height (2), we determine the thickness of any standard sheet. Let n be the pressure in any standard surface. The pressure being measured in decibars, n will have one of the values 1, 2, 3, . . . 10. The thickness $H_{n,n-1}$ of the standard sheet between the surfaces n and $n-1$ is obtained if in the fundamental formula section 50 (*e*) we introduce $p_a = n$, $p_b = n - 1$. Substituting further for R its numerical value when the pressure is expressed in decibars, $R = 28.7$, and writing

$273 + \tau$ instead of ϑ, τ being the virtual temperature counted from the freezing-point of water, we get this expression for the thickness of the standard sheet:

$$(a) \qquad H_{n,n-1} = 28.7\,(273 + \tau_{n,n-1}) \text{ nat. log. } \frac{n}{n-1}$$

Height (2) can evidently be found as the sum of a certain number of heights given by formula (a).

The determinations of the heights (1) and (3) are different forms of one problem, namely, the determination of the distance $H_{n,p}$ from a certain standard surtace of pressure n to any isobaric surface of pressure p. By the fundamental formula this distance is

$$H_{n,p} = 28.7\,(273 + \tau_{n,p}) \text{ nat. log. } \frac{n}{p}$$

$\tau_{n,p}$ being the average virtual temperature for the sheet of air between the isobaric surfaces n and p. This formula containing two continuously variable quantities, $\tau_{n,p}$ and p, is not immediately suited for tabulation. But it may be written as a sum of two terms, a principal term H_0 containing only one variable p, and a correction term ΔH containing two variables, namely, H_0 and τ. We thus write

$$(b) \qquad H_{n,p} = H_0 + \Delta H$$

giving H_0 and ΔH respectively the values

$$b') \qquad H_0 = 7835 \text{ nat. log. } \frac{n}{p}$$

$$(b'') \qquad \Delta H = H_0 \frac{\tau_{n,p}}{273}$$

Thus, tabulating the three formulæ (a), (b'), and (b''), we shall easily be able to calculate the height from any isobaric surface p_a to any isobaric surface p_b, the required values of the virtual temperature being given. Only three tables would therefore be necessary. But for practical reasons, however, we shall give two dittferent tabulations of formula (b'), arranging the table in a special form for the important case of p_a being the pressure at the earth's surface. Thus in the second tabulation of formula (b') the height (1) is the height from the ground to the nearest standard surfaces. We then get the following four tables.

(A) *Table 9 M.—Mutual distances measured in dynamic meters between standard isobaric surfaces.*—This table contains nine small tables in succession, each giving, according to formula (a), the thickness of one of the standard sheets of the atmosphere for practically occurring values of the average virtual temperature. These successive tables are separated the one from the other by horizontal lines representing standard surfaces, the pressures of which are added in millibars.

(B) *Table 10 M.—Distances in dynamic meters, measured from the standard isobaric surfaces to points of given pressure, the average virtual temperature of the sheet being 0° C.*—This table contains ten small tables in succession, calculated according to formula (b'). Each gives the distance from one standard

isobaric surface to all other isobaric surfaces situated below the next higher and above the next lower standard surface. The distance is counted positive upward and negative downward. It gives the first approximation value H_0 of the height $H_{n,p}$ from the standard isobaric surface n to a point of the pressure p.

(C) *Table 11 M.—Distances in dynamic meters from the earth's surface to the nearest standard isobaric surfaces, the average virtual temperature of the sheet being 0° C.*—This table differs from the preceding one only in the arrangement. As argument appears the pressure observed at the earth's surface. The tabulated quantities are the distances to the two nearest standard surfaces above and to the nearest standard surface below the point where the pressure is observed. The distance upward from the earth is counted positive and the distance downward negative. Of course the standard surface below the earth has no real existence. It may, however, for obvious reasons, be useful to bring it under consideration. The table gives the first approximation H_0 to the height $H_{p,n}$ from the earth's surface, where the pressure is p, to the standard surface of pressure n.

(D) *Table 12 M.—Corrections to tables 10 M and 11 M for temperature.*—This table is calculated according to formula (b'') with the two arguments, the average virtual temperature $\tau_{n,p}$ between the two surfaces n and p and the height H_0 found from table 10 M or 11 M. This correction for temperature may be either positive or negative according to the sign of the temperature $\tau_{n,p}$. But as it has equal numerical values for equal numerical values of τ, it will not be necessary to introduce signs in the table. The first part of the table, extending to the value 1100 dynamic meters for height H_0 and to values ± 34° C. for temperature, will be sufficient for most cases practically occurring. The continuation gives the extension to the limiting height of 10,000 dynamic meters and to the values ± 100° C. of temperature.

52. Calculation of the Height Corresponding to a Given Pressure.—If the virtual-temperature diagram be given as a curve connecting virtual temperature and pressure, tables 9 M to 12 M will at once enable us to calculate the height corresponding to any pressure. From the diagram (fig. 5), we take the average virtual temperature, first between the earth's surface, where the pressure is p_a, and the lowest standard surface, then between the successive standard surfaces, and finally between the highest of these and the isobaric surface of the given pressure p_b. By tables 11 M and 12 M we then find the height of the lowest standard surface above the earth, by table 9 M the thickness of the successive standard sheets, and by tables 10 M and 12 M the height of the given isobaric surface above the highest standard surface. Adding these heights we get the height of the isobaric surface p_b above the ground, and adding the height of the ground above sea-level we get the height above sea-level of the given isobaric surface. As all the other heights, that of the ground is to be expressed in dynamic meters. The first of the problems defined in section 49 will then be solved.

Suppose now the virtual-temperature diagram to be given with the heights in dynamic meters as ordinates (fig. 6). The pressure observed at the station at the

earth's surface gives by means of table 11 M an approximation value H_0 of the height above the station of the lowest standard isobaric surface. By means of this approximate value we may, with sufficient precision, take from the diagram the average virtual temperature of the sheet. This temperature used in table 12 M gives the correction ΔH, which added to the approximation value H_0 gives with sufficient correctness the height of the surface above the station. Adding the height of the station we get the height of the surface above sea-level. This height being known, we estimate a value for the height to the next standard surface. This is easily done with fair approximation by the inspection of the virtual-temperature diagram and by comparison with the corresponding heights in table 9 M. For this estimate of height the value of the average virtual temperature is taken from the diagram. Using this value in table 9 M, we generally find the height to the next standard surface with sufficient precision. Otherwise the operation may be repeated, giving for every repetition a more accurate value. The final value of the height found in this manner added to the height of the first standard surface gives the height of the second standard surface. Then the distance to the next standard surface is estimated, the corresponding average virtual temperature determined from the diagram, and this temperature used to find a better value for the distance by means of table 9 M, and so on.

To complete the solution of the problem we finally determine the distance $H_{n,p}$ from one of the standard surtaces, the height of which is found, to a neighboring isobaric surface of the given pressure p_b. An approximation value H_0 of the height is found at once from table 10 M. Using this approximate value we find the average virtual temperature of the sheet from the diagram, and by means of this temperature we find from table 12 M the correction ΔH, which, added to the first approximation value H_0, gives the required height $H_{n,p}$.

The second of the problems defined in section 49 is thus solved.

53. Calculation of the Pressure at a Given Height. — Let H be the given height at which the pressure is to be found. We then determine first, as described in the preceding section, the height of the standard isobaric surfaces. Now, let p_n be the standard surface whose height H_n is nearest the given height H. The problem is then reduced to finding the pressure p at the height $H - H_n$ above the standard surface of pressure n.

Now, the height $H - H_n$ is the quantity tabulated in table 10 M, and the argument is the corresponding pressure p. Then if the average virtual temperature of the sheet of air between the heights H_n and H happens to be 0° C., table 10 M used inversely immediately gives the required pressure.

As a rule, however, this average temperature will have another value, τ. This temperature being known, we can avail ourselves of a simple artifice, determining a difference of height $H' - H_n$ defined by the property of being the height, which, used in table 10 M, gives the required pressure p.

The difference of pressure corresponding to a given difference of height is in inverse proportion to the specific volume of the sheet of air between the two heights,

and therefore also in inverse proportion to the average virtual temperature of this sheet, reckoned from absolute zero. Thus the two heights $H' - H_n$ and $H - H_n$ must be in the proportion

$$H' - H_n = (H - H_n)\frac{273}{273 + \tau}$$

Subtracting $H - H_n$, we find the following value for the required correction:

$$\Delta H = H' - H = -(H - H_n)\frac{\tau}{273 + \tau}$$

This may finally be written in the form

(*a*) $$\Delta H = (H - H_n)\frac{\tau'}{273}$$

the auxiliary quantity τ' having the value

(*b*) $$\tau' = -\frac{273\tau}{273 + \tau}$$

Formula (*a*) has the same form as formula (*b''*), section 51, tabulated in table 12 M. But, to use table 12 M for the determination of the height correction ΔH in the case now treated, we have to use the artificial temperature τ' instead of the real temperature τ. This artificial temperature is tabulated according to formula (*b*) in table 13 M of our Meteorological Tables. Using this table in connection with tables 9 M to 12 M, we can calculate the pressure at any given height.

The practical procedure will turn out somewhat differently according as the virtual temperature is known at a given height or for a given pressure. In both cases we first determine the height of the standard isobaric surface as described in the preceding article. Then, if the virtual temperature be known for a given height, we immediately find from the diagram the average virtual temperature τ for the sheet between the heights H and H_n. On the other hand, if the virtual temperature be known for a given pressure, we use table 10 M to find an approximate value p' of the pressure at the height H. Taking from the diagram the average virtual temperature between the pressures n and p' we get a temperature τ, which with sufficient approximation can be identified with the average virtual temperature of the sheet between the heights H_n and H.

This temperature τ being found, we take the corresponding artificial temperature τ' from table 13 M. Using this and the height $H - H_n$ in table 12 M, we find the required correction ΔH. This correction added to the height $H - H_n$ gives the height $H' - H_n$, which used in table 10 M gives the required pressure p at the height H.

The third and fourth of the problems defined in section 49 are thus solved.

54. Examples of a Complete Interpretation of the Results of a Meteorological Ascent.— On pages 68–75 are given the schemes for the complete hydrostatic application of the observations obtained, under different suppositions, from an ascent

in the air with meteorological instruments. It will be evident that, the hydrostatic results contained in these schemes being once worked out, a set of supplementary results of general meteorological interest might easily have been obtained. We may for instance mention temperatures and humidities at given heights for given pressures, or average values of these quantities for given height-sheets or pressure-sheets. But in order not to complicate the schemes we have taken up only what is of interest for as full an illustration as possible of the developed hydrostatic methods.

The examples are derived from the observations obtained by the celebrated balloon ascent by Berson and Süring from Berlin July 31, 1901, to the greatest height yet attained by man.* The height of the station, Tegel at Berlin, was 40 meters or 39 dynamic meters above sea-level. The observed quantities during the ascent were time, pressure in millimeters of mercury, temperature centigrade, and relative humidity. From the general remarks in the preceding articles and by the small examples added to each table in the Meteorological Tables, the schemes will easily be understood. We shall therefore content ourselves with a few general remarks.

In the first example (page 68) we have made a direct use of the observed data only supposing the pressure to have been observed in millibars instead of in millimeters of mercury.

This first example being worked out, we have constructed the second, considering the calculated heights (column 24 of table J) as observed quantities, column 2 of table K. We have preferred thus to derive example 2 artificially from example 1, instead of taking an independent example, where the heights have been really observed; for the analogy and the contrast of the methods are better illustrated when both are used to work out the same case of atmospheric equilibrium. Comparing the two schemes, we see that the difference amounts mainly to an interchange of the order of the columns, followed by a passage from direct methods to methods of estimation, or *vice versa*.

In connection with this second example it is important to emphasize that a observed heights should be considered only those found according to rational geometrical methods, as for instance when the height of a kite is determined by the angle and the length of the kite-line. The use, on the contrary, of a barometer with height-scale instead of pressure-scale is unscientific. It gives less trustworthy results, and at the same time additional labor; for the working out of the results according to example 2 is more laborious than the corresponding work according to example 1. In some cases both pressure and height may be observed. The observations then give directly the equilibrium relation between pressure and height. But on account of the imperfections of the aneroid barometer, the relation found in this direct way will be much less accurate than that found by one of the above methods, the observations either of pressure or of height being provisionally set aside. The derivation of the results according to both methods, once omitting

* Veröffentlichungen des K. Preussischen Meteorologischen Instituts. R. Assmann und A. Berson: Ergebnisse der Arbeiten am Aeronautischen Observatorium 1900–1901. Berlin, 1902. p. 227.

the observed pressures and once the observed heights, will give a valuable control, especially useful in correcting the records of the barometer.

The common result of examples 1 and 2 is illustrated graphically by the three verticals of fig. 4, the principle of the representation being the same as that used previously in figs. 1 and 2. The first vertical, representing the equilibrium relation between dynamic height and pressure, may be constructed from columns 5 and 8 or 11 and 18 of table J, or from the corresponding columns 6 and 9 or 12 and 18 of table K. The divisions on the second vertical, representing the equilibrium relation between pressure and specific volume, are drawn according to the figures in columns 5, 7, and 10 of table J, or the corresponding columns 6, 8, and 11 of table K. The divisions on the third vertical, giving the equilibrium relation between dynamic height and density, are drawn in accordance with the figures contained in columns 11 and 19 of table J, or 12 and 19 of table K. To obtain greater accuracy the specific volumes in column 10, table J, or column 11, table K, have been changed into densities and used to correct the divisions.

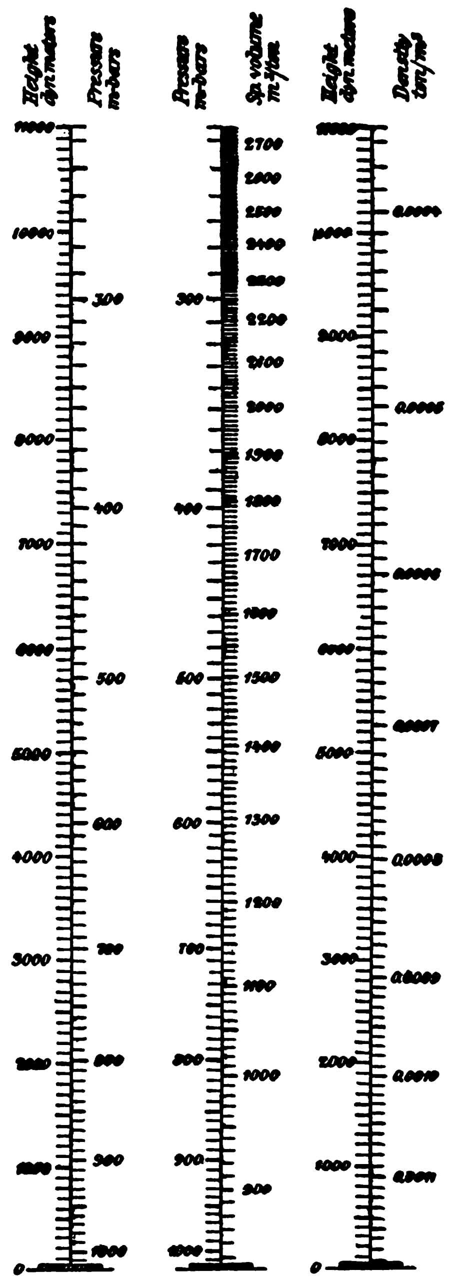


FIG. 4.— State of atmospheric equilibrium above Berlin, July 31, 1901.

We shall later make important practical applications of verticals as those of fig. 4, drawing vertical sections as in figs. 13, 14, 21, 24 below. The most important use will be made, however, of the numbers contained in columns 8 and 7 of table J, respectively, 9 and 8 of table K, *i. e.*, the numbers giving the height of the standard isobaric surfaces and the average specific volume of the air between them, and the numbers contained in columns 18 and 19 of both tables, *i. e.*, the numbers giving the pressure in standard level surfaces and the average density of the air between them. From such numbers as these we shall draw synoptical charts such as those found in Chapter VII, representing in two different ways the distribution of pressure and of mass in the atmosphere.

EXAMPLE 1.—*Observed time, pressure* (*m-bars*), *temperature* (° *C.*), *humidity* (*per cent*). (Table J.)—From the observed pressures and temperatures (columns 2 and 3) the curve of true temperatures is drawn (curve to the left in fig. 5). By means of table 7 M the curve of virtual temperature for saturated air is drawn (curve to the right). Using the percentages of humidity (column 4) the curve of virtual temperature is drawn between the other two curves. (See section 23.)

The vertical segments of line determining the average virtual temperature of the standard sheets are drawn, and the corresponding temperatures read off (column 6). The mutual distances between the standard surfaces (column 7) and the heights of these surfaces (column 8) are determined. In addition the virtual temperatures at the standard surfaces (column 9) may be read off from the diagram, and the corresponding specific volume of the air determined (column 10).

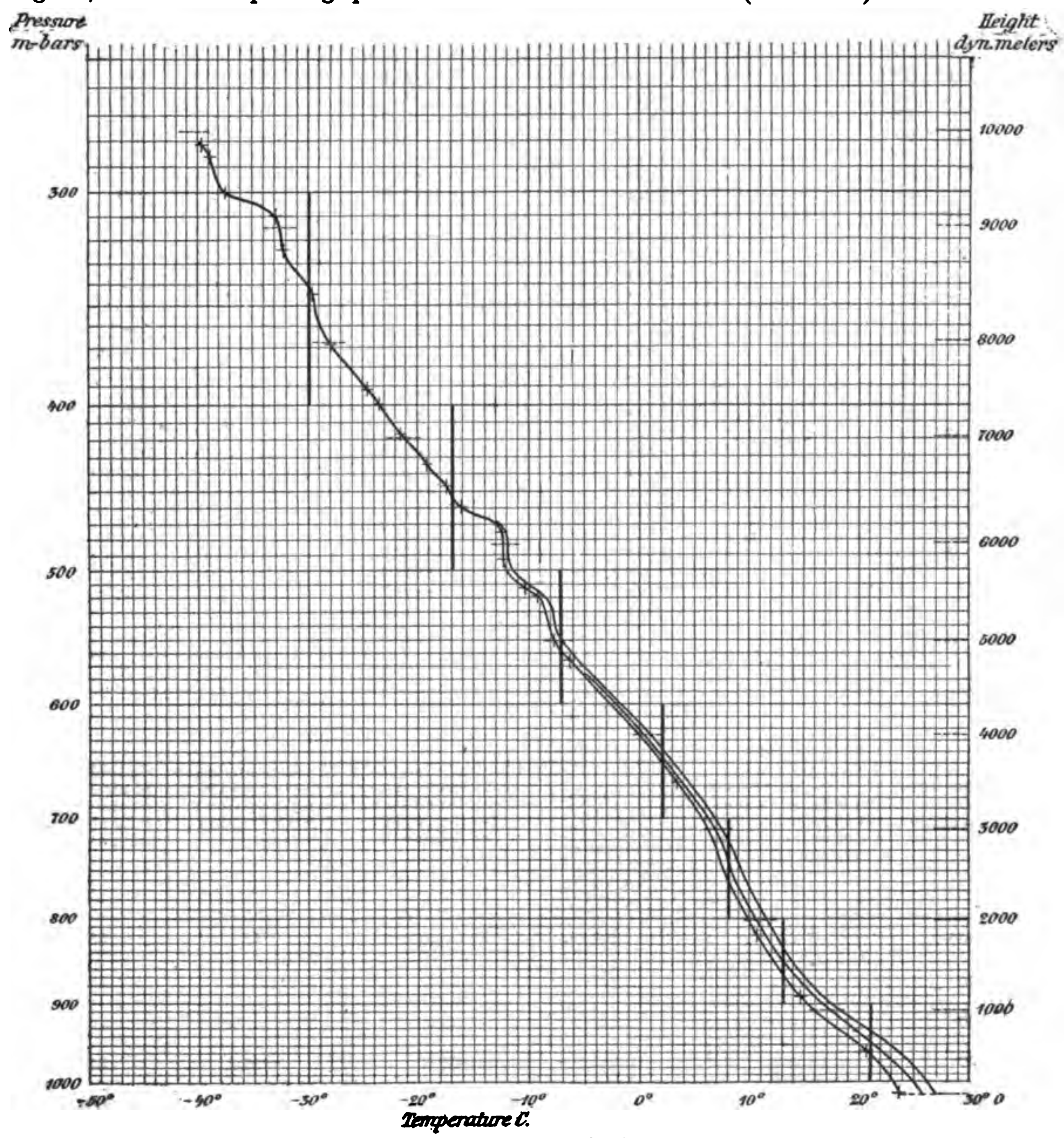


FIG. 5. — Virtual-temperature diagram, with logarithmic pressure-scale.

Columns 11 to 19 give the solution of the inverse problem, the determination of the pressure at a standard height. The dotted horizontal lines in the diagram represent the approximate situation of the level surfaces, these lines being drawn according to the approximate pressures in the heights given in column 14. Not to complicate the figure, the vertical segments giving the virtual temperatures in column 15 are not drawn.

Columns 20 to 24 give the determination of the heights from which the observations in columns 1 to 4 are taken. The horizontal and vertical lines in the diagram required for this determination are not given in fig. 5. It should be emphasized that the determination of these heights is independent of the solution of the inverse problem (columns 11 to 19) and dependent only upon the knowledge of the heights of the standard isobaric surfaces (column 8).

TABLE J (EXAMPLE 1).

1	2	3	4
Observed time.	Observed pressure.	Observed temperature.	Observed relative humidity.
15th meridian.	m-bars.	° C.	Per cent.
10^h 50^m	1015.9	23.4	72
54	957.9	20.4	58
57	889.9	14.6	62
11 00	819.3	10.5	46
16	665.3	3.5	23
30	563.9	− 6.3	42
44	549.3	− 7.8	17
12 16	517.3	− 9.2	84
28	511.9	−10.5	31
40	492.0	−12.5	46
57	473.9	−12.5	38
1 00	460.0	−16.1	51
11	449.0	−17.3	70
20	433.7	−19.2	67
34	421.0	---	--
39	416.4	−21.6	74
46	400.4	−23.3	83
51	392.0	−24.7	85
2 1	383.7	---	--
13	369.3	−27.8	95
25	357.9	---	--
28	345.3	−29.6	--
52	324.6	−32.2	--
55	309.7	−33.0	--
3 3	300.0	−37.3	--
10	285.9	−38.9	--
28	280.6	−39.7	--
32	270.0	---	--
40	257.3	---	--

5	6	7	8	9	10
Standard pressures and pressure (1015.9) at station.	Average virtual temperature in standard sheets and in sheet below lowest standard surface (+25.1) found from diagram (fig. 5).	Mutual distances between standard surfaces, found by table 9 M; height 135, of lowest standard surface above station, found by tables 11 M and 12 M; height, 39, of station above sea-level.	Heights of standard surfaces above sea-level, found by addition of figures of column 7.	Virtual temperature of air at standard surfaces, found from diagram (fig. 5).	Specific volume of air at standard surfaces, found by table 14 M (m^3/ton = cm^3/gr.).
m-bars.	° C.	Dyn. meters.	Dyn. meters.	° C.	m^3/ton.
100	----	----	----	----	----
200	----	----	----	----	----
300	----	----	9365	−37.3	2255
	−29.8	2009			
400	----	----	7356	−23.7	1789
	−17.0	1640			
500	----	----	5716	−11.9	1499
	− 7.2	1391			
600	----	----	4325	− 2.1	1296
	+ 2.2	1218			
700	----	----	3107	+ 5.8	1143
	+ 8.0	1077			
800	----	----	2030	+10.1	1015
	+13.0	967			
900	----	----	1063	+16.5	923
	+21.0	889			
1000	----	----	174	+24.7	855
	+25.1	135			
1015.9	----	----	----	----	----
		39			

TABLE J (EXAMPLE 1)—Continued.

11	12	13	14	15	16	17	18	19
Standard heights.	Nearest standard isobaric surfaces.	Distances from standard isobaric surfaces of column 12, heights of which are given in column 8, to standard heights of column 11.	Approximate pressures in standard heights, obtained by table 10 M as pressures corresponding to distances of column 13.	Average virtual temperatures for sheets between standard isobaric surfaces and isobaric surfaces of column 14, found from diagram (fig. 5).	Corresponding artificial temperature according to table 13 M.	Distances in column 13 artificially changed by addition of height corrections obtained from table 12 M as corresponding to distances in column 13 and temperatures in column 16.	Pressures in standard heights, found from table 10 M as pressures corresponding to artificial distances in column 17.	Average density of air in level sheets between standard heights of column 11. The figures are ten times the differences between pressures of column 18 (ton/m³ = gr/cm³).
Dyn. meters.	m-bars.	Dyn. meters.	m-bars.	° C.	° C.	Dyn. meters.	m-bars.	σ^{-4} ton/m
10000	300	+635	277	−39	45.5	+741	272.9	----
								435
9000	300	−365	314	−34	39	−417	316.4	----
								488
8000	400	+644	368	−26	29	+713	365.2	----
								551
7000	400	−356	419	−22.5	24.5	−388	420.3	----
								611
6000	500	+284	482	−12	12.5	+297	481.4	----
								682
5000	600	+675	550	− 5	5	+687	549.6	----
								759
4000	600	−325	625	− 1	1	−326	625.5	----
								840
3000	700	−107	709	+ 6	− 6	−105	709.5	----
								935
2000	800	− 30	803	+10.5	−10	− 29	803.0	----
								1038
1000	900	− 63	907	+17	−16	− 59	906.8	----
								1138
0	1000	−174	1022	+25	−23	−160	1020.6	----

20	21	22	23	24
Approximate values, found from table 10 M, of distances from nearest standard isobaric surfaces to isobaric surfaces of column 2.	Average virtual temperature of sheets between standard isobaric surfaces and surfaces of column 2, found from diagram (fig. 5).	Distances in column 20, corrected for temperature by table 12 M.	Heights of isobaric surfaces in column 2 = heights where observations of columns 1 to 4 are taken, found from columns 8 and 22.	Heights where observations are taken, reduced from dynamic meters to meters by tables 5 M and 6 M, using $g_0 = 9.8128$.
Dyn. meters.	° C.	Dyn. meters.	Dyn. meters.	Meters.
− 123	25	− 135	39	40
+ 337	23	+ 365	539	549
+ 89	16	+ 94	1157	1180
− 187	10.5	− 194	1836	1872
+ 399	5	+ 406	3513	3583
+ 486	− 4.5	+ 478	4803	4900
+ 692	− 5	+ 679	5004	5104
− 267	−10.5	− 257	5459	5568
− 184	−11	− 177	5539	5650
+ 126	−12	+ 120	5836	5954
+ 420	−12.5	+ 401	6117	6241
+ 653	−13	+ 622	6338	6467
+ 843	−14	+ 800	6516	6648
− 634	−21	− 585	6771	6908
− 401	−22	− 369	6987	7130
− 315	−22.5	− 289	7067	7211
− 8	−23.5	− 7	7349	7499
+ 158	−24	+ 144	7500	7653
+ 326	−24.8	+ 297	7653	7810
+ 626	−26	+ 566	7922	8083
+ 871	−26.5	+ 786	8142	8309
−1102	−32.5	− 971	8394	8567
− 618	−33.5	− 542	8823	9004
− 250	−35	− 218	9147	9336
0	−37	0	9365	9559
+ 377	−38	+ 324	9689	9891
+ 524	−38.5	+ 450	9815	10019
+ 826	−39	+ 708	10073	10283
+1203	−40	+1027	10392	10610

EXAMPLE 2.— *Observed height (meters), temperature (°C.), and humidity (per cent).* (Table K). — The observed geometric height (column 2) is changed into dynamic height (column 5). From these heights and the observed temperatures the curve of true temperature is drawn (curve to the left in fig. 6). By means of table S M the curve of virtual temperature for saturated air is drawn (curve to the right). Using the percentages of humidity (column 4) the curve of virtual temperature is drawn between the two others.

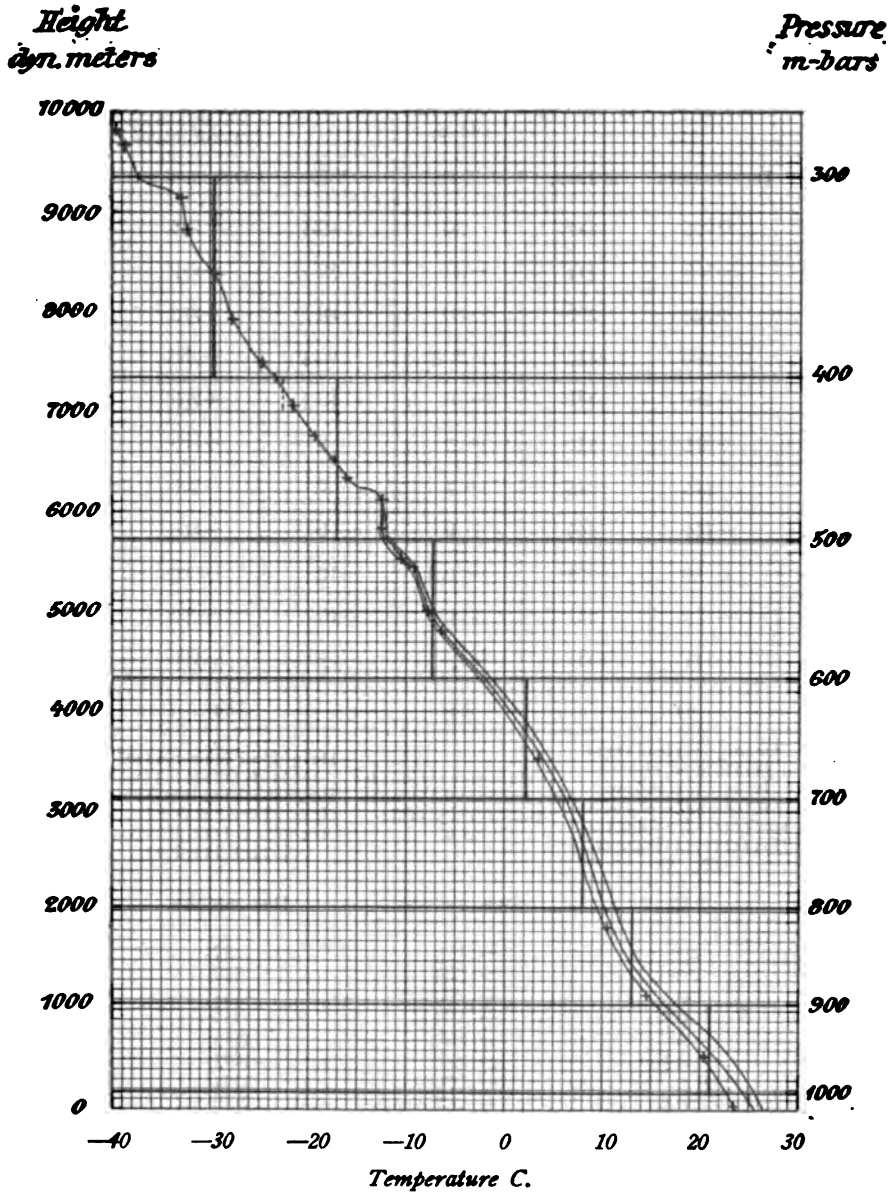


FIG. 6.— Virtual-temperature diagram, with dynamic height as ordinate.

The heavy horizontal lines represent the standard isobaric surfaces, successively drawn according to the estimated thickness of the standard sheets (section 52). The vertical segments of line give the average virtual temperatures of the sheets (column 7) by which the more accurate thickness of the sheets (column 8) and the heights of the surfaces (column 9) are determined.

The solution of the inverse problem, the determination of the pressure at a given height (columns 12 to 19) is found mainly by the same method as in the preceding example, except that one operation less is necessary to find the required virtual temperatures (column 15). Not to complicate the figure, the vertical segments giving these temperatures are not drawn.

Columns 20 to 24 give the determination of the pressures at the heights where the observations were taken. The lines in the diagram required for this determination are not given in fig. 6. The determination of these pressures is independent of the solution of the inverse problem, and dependent only upon the knowledge of the heights of the standard isobaric surfaces (column 9).

The discussion and the comparison of the examples 1 and 2 is important in connection with the practical question as to the choice of method of observation, as well as in connection with the theoretical question as to the choice of method for representing the result of the observations.

It is seen that from the point of view of the computer it is no advantage at all to have the height instead of the pressure as one of the observed quantities. Observed heights being always geometric heights, they have first to be changed into dynamic heights, and then the average virtual temperature of the standard isobaric sheets must be found by the method of conjectures instead of by the direct way which can be followed when the pressure is one of the observed quantities. When some practice is acquired, these conjectures can easily be made with sufficient precision to make repetitions of the operation superfluous. But still the convenience of the direct method can not be attained. Thus, as far as the observations of pressure can be obtained with the same precision as those of height, the observations of pressure should be preferred as those giving least trouble to the computer. In no case the values of pressure should be left out in the publications of the result of the meteorological ascents, as it is unfortunately sometimes done, height being substituted for pressure as the result of the calculations. But especially we must warn, as we have already done, against the use of barographs with height scale instead of pressure scale. For in addition to the increased trouble to the computer, this method will give much less trustworthy results.

On the other hand, it is seen that the calculation of the height of standard pressures is in all cases easier than the calculation of pressure in standard heights. This is equally true whether it is pressure or geometric height which is observed. Even if it may be possible to further simplify the methods developed for calculating pressure in standard dynamic heights, it is not probable that it should be possible to attain the simplicity of the method given for calculating the dynamic height of standard pressures.

In the choice between the two theoretically equivalent methods of representing the distribution of pressure, viz, that of registering the height of standard pressures or that of registering pressure in standard heights, we have thus found an important practical reason for preferring the first method, that of registering the height of standard pressures.

Whichever method of observation be used, and whichever method of representing the results be preferred, it is seen that the fundamental operation remains that of drawing and interpreting the virtual-temperature diagram. What can be done to facilitate this work will therefore be of the highest practical importance. In this respect the hints given in the next section (55) will be useful.

TABLE K (EXAMPLE 2).

1	2	3	4	5
Observed time.	Observed height.	Observed temperature.	Observed relative humidity.	Heights of column 2 reduced to dynamic heights by tables 3 M and 4 M, using $g_0 = 9.8128$.
15th meridian.	Meters.	° C.	Per cent.	Dyn. meters.
10^h 50^m	40	23.4	72	39
54	549	20.4	58	539
57	1180	14.6	62	1157
11 00	1872	10.5	46	1836
16	3583	3.5	23	3513
30	4900	− 6.3	42	4803
44	5104	− 7.8	17	5004
12 16	5568	− 9.2	84	5459
28	5650	−10.5	31	5539
40	5954	−12.5	46	5836
57	6241	−12.5	38	6117
1 00	6467	−16.1	51	6338
11	6648	−17.3	70	6516
20	6908	−19.2	67	6771
34	7130	----	--	6987
39	7211	−21.6	74	7067
46	7499	−23.3	83	7349
51	7653	−24.7	85	7500
2 1	7810	----	--	7653
13	8083	−27.8	95	7922
25	8309	----	--	8142
28	8567	−29.6	--	8394
52	9004	−32.2	--	8823
55	9336	−33.0	--	9147
3 3	9559	−37.3	--	9365
10	9891	−38.9	--	9689
28	10019	−39.7	--	9815
32	10283	----	--	10073
40	10610	----	--	10392

6	7	8	9	10	11
Standard pressures and pressure 1015.9 at station.	Average virtual temperature of standard sheets, and of sheet below lowest standard surface, found from diagram (fig. 6) by use of conjectured thicknesses of sheets.	Mutual distances between standard surfaces, found by table 9 M; height 135 of lowest standard surface above station, found by tables 11 M and 12 M; height 39 of station above sea-level.	Heights of standard isobaric surfaces above sea-level, found by addition of figures in column 8.	Virtual temperature of air at standard surfaces found from diagram (fig. 6).	Specific volume of air at standard surfaces found by table 14 M (m^3/ton = cm^3/gr.).
m-bars.	° C.	Dyn. meters.	Dyn. meters.	° C.	m^3/ton.
100	----	----	----	----	----
200	----	----	----	----	----
300	----	----	9365	−37.3	2255
	−29.8	2009			
400	----	----	7356	−23.7	1789
	−17.0	1640			
500	----	----	5716	−11.9	1499
	− 7.2	1391			
600	----	----	4325	− 2.1	1296
	+ 2.2	1218			
700	----	----	3107	+ 5.8	1143
	+ 8.0	1077			
800	----	----	2030	+10.1	1015
	+13.0	967			
900	----	----	1063	+16.5	923
	+21.0	889			
1000	----	----	174	+24.7	855
	+25.1	135			
1015.9	----	----	----	----	----
		39			

TABLE K (EXAMPLE 2)—Continued.

12	13	14	15	16	17	18	19
Standard heights.	Nearest standard isobaric surfaces.	Distances from standard isobaric surfaces of column 13, heights of which are given in column 9, to standard heights of column 12.	Average virtual temperatures for sheets between standard heights in column 12 and heights of standard isobaric surfaces in column 9, found from diagram (fig. 6).	Corresponding artificial temperatures according to table 13 M.	Distances in column 14, artificially changed by addition of height corrections obtained from table 12 M as corresponding to heights in column 14 and temperatures in column 16.	Pressure in standard heights, found from table 10 M as pressures corresponding to distances in column 17.	Average density of air in level sheets between standard heights of column 12. The figures are ten times the differences between pressures of column 18.
Dyn. meters.	m-bars.	Dyn. meters.	° C.	° C.	Dyn. meters.	m-bars.	10^{-4} ton/m³.
10000	300	+635	−39	+45.5	+741	272.9	----
							435
9000	300	−365	−34	+39	−417	316.4	----
							488
8000	400	+644	−26	+29	+713	365.2	----
							551
7000	400	−356	−22.5	+24.5	−388	420.3	----
							611
6000	500	+284	−12	+12.5	+297	481.4	----
							682
5000	600	+675	− 5	+ 5	+687	549.6	----
							759
4000	600	−325	− 1	+ 1	−326	625.5	----
							840
3000	700	−107	+ 6	− 6	−105	709.5	----
							935
2000	800	− 30	+10.5	−10	− 29	803.0	----
							1038
1000	900	− 63	+17	−16	− 59	906.8	----
							1138
0	1000	−174	+25	−23	−160	1020.6	----

20	21	22	23	24
Distances from standard isobaric surfaces (column 9) to heights in column 5.	Average virtual temperature for sheets in column 20.	Corresponding artificial temperatures according to table 13 M.	Distances in column 20 artificially changed by addition of height corrections obtained from table 12 M, using distances in column 20 and temperatures in column 22.	Pressures in observed heights of column 2, found from table 10 M as pressures corresponding to artificial distances in column 23.
Dyn. meters.	° C.	° C.	Dyn. meters.	m-bars.
− 135	25	−22.9	− 123	1015.9
+ 365	23	−21.2	+ 337	957.9
+ 94	16	−15.1	+ 89	889.9
− 194	10.5	−10.1	− 187	819.3
+ 406	5	− 4.9	+ 399	665.3
+ 478	− 4.5	+ 4.6	+ 486	563.9
+ 679	− 5	+ 5.1	+ 692	549.3
− 257	−10.5	+10.9	− 267	517.3
− 177	−11	+11.5	− 184	511.9
+ 120	−12	+12.6	+ 126	492.0
+ 401	−12.5	+13.2	+ 420	473.9
+ 622	−13	+13.7	+ 653	460.0
+ 800	−14	+14.8	+ 843	449.0
− 585	−21	+22.8	− 634	433.7
− 369	−22	+23.9	− 401	421.0
− 289	−22.5	+24.5	− 315	416.4
− 7	−23.5	+25.7	− 8	400.4
+ 144	−24	+26.3	+ 158	392.0
+ 297	−24.8	+27.3	+ 326	383.7
+ 566	−26	+28.7	+ 626	369.3
+ 786	−26.5	+29.3	+ 871	357.9
− 971	−32.5	+36.8	−1102	345.3
− 542	−33.5	+38.1	− 618	324.6
− 218	−35	+40.1	− 250	309.7
0	−37	+42.8	0	300.0
+ 324	−38	+44.2	+ 377	285.9
+ 450	−38.5	+44.8	+ 524	280.6
+ 708	−39	+45.5	+ 826	270.0
+1027	−40	+46.9	+1203	257.3

55. Remarks on Virtual-Temperature Diagrams. — When many calculations are to be performed, much time may be saved if convenient blanks be prepared for the drawing of the virtual-temperature diagrams. The use of tables 7 M or 8 M may be completely dispensed with if the horizontal lines on these blanks be provided with *divisions showing the distance from the curve of true to that of*

TABLE L.— *Virtual-temperature divisions on lines representing standard isobaric surfaces.* (*Vertical columns in the table correspond to horizontal lines on the virtual-temperature diagram.*)

Pressure (m-bars).							
1000	900	800	700	600	500	400	300
−9.8	−10.7	−11.8	−13.1	−14.5	−16.0	−17.5	−19.7
−9.6	−10.5	−11.6	−12.9	−14.2	−15.7	−17.2	−19.4
−9.4	−10.3	−11.4	−12.6	−13.9	−15.4	−16.9	−19.1
−9.2	−10.1	−11.3	−12.4	−13.6	−15.1	−16.5	−18.8
−9.0	−9.9	−11.0	−12.1	−13.3	−14.8	−16.2	−18.5
−8.8	−9.7	−10.7	−11.8	−13.1	−14.5	−15.9	−18.2
−8.6	−9.5	−10.5	−11.6	−12.8	−14.2	−15.6	−17.9
−8.4	−9.3	−10.2	−11.3	−12.5	−13.8	−15.3	−17.5
−8.2	−9.0	−9.9	−11.0	−12.2	−13.4	−14.9	−17.1
−8.0	−8.8	−9.6	−10.7	−11.8	−13.0	−14.5	−16.7
−7.8	−8.5	−9.3	−10.3	−11.4	−12.6	−14.2	−16.3
−7.6	−8.3	−9.0	−10.0	−11.0	−12.3	−13.8	−15.9
−7.3	−8.0	−8.7	−9.7	−10.5	−11.9	−13.4	−15.5
−7.0	−7.7	−8.4	−9.3	−10.1	−11.5	−13.0	−15.1
−6.7	−7.4	−8.1	−8.9	−9.7	−11.1	−12.6	−14.6
−6.4	−7.0	−7.7	−8.5	−9.3	−10.6	−12.1	−14.1
−6.0	−6.6	−7.3	−8.1	−8.9	−10.1	−11.6	−13.6
−5.6	−6.2	−6.9	−7.7	−8.5	−9.6	−11.0	−13.0
−5.2	−5.8	−6.5	−7.3	−8.0	−9.1	−10.4	−12.3
−4.8	−5.4	−6.0	−6.8	−7.6	−8.5	−9.8	−11.6
−4.4	−4.9	−5.5	−6.3	−7.0	−7.8	−9.2	−10.8
−4.0	−4.5	−5.1	−5.7	−6.4	−7.2	−8.5	−10.0
−3.6	−4.1	−4.6	−5.1	−5.7	−6.5	−7.7	−9.2
−3.2	−3.6	−4.1	−4.5	−5.1	−5.8	−6.9	−8.3
−2.7	−3.1	−3.6	−3.9	−4.4	−5.0	−6.0	−7.3
−2.2	−2.6	−3.0	−3.2	−3.6	−4.2	−5.0	−6.2
−1.7	−2.0	−2.3	−2.5	−2.8	−3.3	−3.9	−5.0
−1.2	−1.4	−1.6	−1.7	−2.0	−2.3	−2.7	−3.7
−0.6	−0.7	−0.8	−0.9	−1.0	−1.2	−1.4	−1.9
0	0	0	0	0	0	0	0
0.6	0.7	0.8	0.9	1.0	1.3		
1.3	1.5	1.7	1.9	2.1	2.7		
2.0	2.3	2.6	2.9	3.3	4.3		
2.7	3.2	3.6	4.0	4.6	6.0		
3.5	4.0	4.6	5.2	6.1	8.0		
4.3	5.0	5.7	6.5	7.7	10.3		
5.1	6.0	6.9	7.9	9.6			
6.0	7.0	8.2	9.5	11.8			
7.0	8.2	9.6	11.2	14.3			
8.0	9.4	11.2	13.2				
9.1	10.8	13.0	15.5				
10.3	12.4	15.0					
11.6	14.1	17.3					
13.1	16.0	19.9					
14.7	18.2						
16.5	20.8						
18.5	23.7						
20.7	27.5						
23.4	31.9						
26.5							
30.3							
35.4							
42.2							

TABLE M.— *Virtual-temperature divisions on lines representing standard levels.* (*Vertical columns in the table correspond to horizontal lines on the virtual-temperature diagram.*)

Height (dynamic meters).							
0	1000	2000	3000	4000	5000	6000	7000
−9.8	−10.9	−12.0	−13.3	−14.5	−15.6	−16.6	−17.5
−9.6	−10.7	−11.8	−13.1	−14.2	−15.3	−16.3	−17.2
−9.4	−10.5	−11.6	−12.8	−13.9	−15.0	−16.0	−16.9
−9.2	−10.3	−11.5	−12.6	−13.6	−14.7	−15.7	−16.5
−9.0	−10.1	−11.2	−12.3	−13.3	−14.4	−15.4	−16.2
−8.8	−9.9	−10.9	−12.0	−13.1	−14.1	−15.1	−15.9
−8.6	−9.7	−10.7	−11.8	−12.8	−13.8	−14.8	−15.6
−8.4	−9.5	−10.4	−11.5	−12.5	−13.5	−14.4	−15.3
−8.2	−9.1	−10.1	−11.2	−12.2	−13.1	−14.0	−14.9
−8.0	−8.9	−9.7	−10.9	−11.8	−12.7	−13.6	−14.5
−7.8	−8.6	−9.4	−10.6	−11.4	−12.3	−13.2	−14.2
−7.6	−8.4	−9.1	−10.2	−11.0	−12.0	−12.9	−13.8
−7.3	−8.1	−8.8	−9.8	−10.5	−11.6	−12.5	
−7.0	−7.8	−8.5	−9.4	−10.1	−11.2	−12.1	
−6.7	−7.5	−8.2	−9.0	−9.7	−10.8	−11.7	
−6.4	−7.1	−7.8	−8.6	−9.3	−10.3	−11.2	
−6.0	−6.7	−7.4	−8.2	−8.9	−9.8	−10.7	
−5.6	−6.3	−7.0	−7.8	−8.5	−9.3	−10.2	
−5.2	−5.9	−6.6	−7.4	−8.0	−8.8	−9.6	
−4.8	−5.5	−6.1	−6.9	−7.6	−8.3	−9.0	
−4.4	−5.0	−5.6	−6.4	−7.0	−7.6	−8.3	
−4.0	−4.6	−5.0	−5.8	−6.4	−7.0	−7.7	
−3.6	−4.2	−4.5	−5.2	−5.7	−6.3	−7.0	
−3.2	−3.7	−4.0	−4.6	−5.0	−5.6	−6.2	
−2.7	−3.2	−3.5	−3.9	−4.4	−4.8	−5.4	
−2.2	−2.7	−2.9	−3.2	−3.6	−4.1	−4.7	
−1.7	−2.1	−2.2	−2.5	−2.8	−3.2	−3.5	
−1.2	−1.4	−1.6	−1.7	−2.0	−2.2	−2.5	
−0.6	−0.7	−0.8	−0.9	−1.0	−1.1	−1.3	
0	0	0	0	0	0	0	
0.6	0.7	0.8	0.9	1.0	1.2		
1.2	1.5	1.7	1.9	2.1	2.6		
1.8	2.3	2.6	2.9	3.3	4.0		
2.5	3.3	3.7	4.1	4.6	5.6		
3.2	4.1	4.7	5.3	6.1	7.5		
3.9	5.1	5.8	6.6	7.7	9.7		
4.7	6.1	7.1	8.0	9.6			
5.5	7.2	8.4	9.7	11.8			
6.4	8.4	9.9	11.4	14.3			
7.3	9.6	11.6	13.5				
8.2	11.1	13.4	15.9				
9.2	12.7	15.5					
10.3	14.5	17.9					
11.6	16.5	20.7					
13.0	18.8						
14.5	21.5						
16.2	24.6						
18.0	28.6						
20.0	33.5						
22.3							
25.0							
28.2							
32.1							

virtual temperature for saturated air. The annexed table L shows how these divisions should be drawn in uninterrupted succession on those horizontal lines which in the diagram represent standard pressures. Table M shows how the corresponding divisions should be drawn on those horizontal lines which in the diagram represent standard heights.

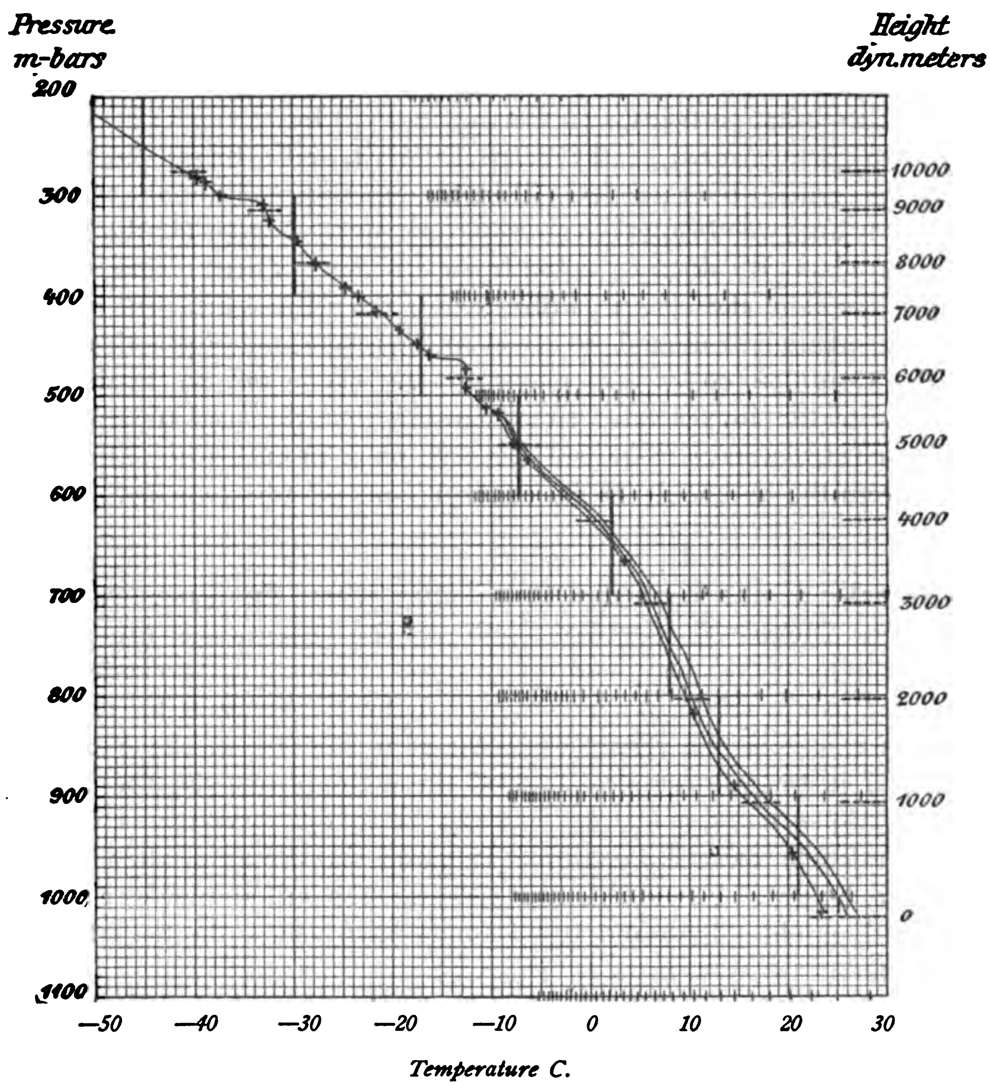


FIG. 7. — Virtual-temperature diagram with common pressure-scale and with virtual-temperature divisions.

In both tables the figures give the distances of the points of division from that ordinate, which in the diagram represents 0° C. They are found by a simple process of summation of the numbers contained in the tables of virtual temperature, respectively 7 M and 8 M. Examples of these divisions taken from table L and introduced on the lines representing standard pressures are shown in fig. 7 as well as in several of the following diagrams.

Theoretically it is correct to use a logarithmic scale of pressure. Practically, however, *the common pressure-scale* may be used without introducing error of any importance. Fig. 7 gives the same virtual-temperature diagram as fig. 5, but with a difference of scale. It is seen that the average virtual temperatures deduced from this diagram are practically the same as those deduced from fig. 5.

In reality the temperatures will be found a little too high from the diagrams with the common pressure-scale. The amount of error can be determined theoretically if we suppose the curve to be a straight line in the one of the two diagrams. It will then run up to $0.008\,(\tau_{10} - \tau_9)$ for the isobaric sheet between the 1000 and the 900 m-bar surfaces, and to $0.057\,(\tau_2 - \tau_1)$ for the sheet between the 200 and the 100 m-bar surface, τ_{10} and τ_9 respectively, τ_2 and τ_1 being the temperatures at the limiting surfaces of the sheet. Thus a temperature difference of 10° between the limiting surfaces will bring the error up to about 0.1 degree for the lower sheets, and somewhat above 0.5 degree for the highest sheet. These errors will generally be much smaller than the errors of observation from these different sheets.

While the errors introduced are thus unimportant, many practical advantages are gained. Common coordinate paper can be used, and we avoid the special inconvenience of the logarithmic scale, namely, that the best observations, those from the lower strata, have to be worked out on a minute scale, and the inferior ones, those from the higher strata, by constructions on a large scale.

56. Examples of the Method of Calculation when the Pressure is Given in Millimeters or Inches of Mercury. — When the observations are given in irrational units, the most direct method is to change at once all observations to rational units using the tables of the Appendix. And this change will be necessary, if it be desired to work out the results with the completeness of the examples given above. But if it be only required to find the main result, viz, the height of the standard isobaric surfaces, a shorter way can be followed, which is illustrated by the two examples below. No example is given for the case when the observed quantity is the height. For in this case the first step will always be a change from geometric to dynamic height, and it is immaterial whether the height is observed in meters or feet.

It is to be hoped that the time will soon come when all observations obtained from the higher strata are recorded in rational units. But as a vast amount of such observations has already been produced and recorded according to the different systems of irrational units, it will for some time to come be a question of great importance to be able to work out with as little waste of labor as possible the most important results in rational units from the data given in irrational units.

The simplest method of doing this, under the supposition that no other auxiliaries are at hand than our tables and common coordinate paper, is illustrated by the examples 3 and 4 below. As will be seen immediately from these examples, several operations will drop out, and no little amount of time will be saved, if special blanks be prepared, containing, besides the common coordinate lines, also

some special auxiliary lines and auxiliary divisions. These are also shown on figs. 8 and 9 belonging to the examples. When extended work of this kind is to be performed, the best method of saving time will therefore be to print such special blanks.

When the observations of pressure are recorded in millimeters of mercury and those of temperature in centigrade degrees, the blanks should contain (see fig. 8): (1) horizontal lines representing the standard isobaric surfaces; (2) virtual-temperature divisions on each of these lines. These divisions are obtained by using table L, page 76, as explained in the preceding section. If these blanks be used the somewhat time-wasting work of drawing by hand the lines representing the standard isobaric surfaces drops out. Further, the virtual-temperature divisions allowed us to draw the virtual-temperature diagram without using table 11 A of the Appendix. It is thus seen that in using these special blanks, the height of the standard isobaric surfaces can be determined with practically the same ease as if the observations of pressure had been taken in rational units. Some supplementary results, as for instance the specific volume of the air at the standard isobaric surfaces, are also obtained with the same ease. But if the working out of the example should be carried still further, if it be required, for instance, to determine pressure in given heights, or to find the heights at which the observations were taken, it will be the best plan to change from the beginning the observed pressures from millimeters of mercury to millibars, and to proceed as in example 1.

When the observations of pressure are given in inches of mercury, and those of temperature simultaneously in Fahrenheit degrees, the blanks should contain (see fig. 9): (1) special divisions along the axis of abscissæ representing the centigrade degrees, while the main divisions are used to represent the Fahrenheit degrees; (2) horizontal lines representing the standard isobaric surfaces; (3) virtual-temperature divisions on each of these lines. These divisions are found by using table L, p. 76, in connection with the centigrade divisions along the axis of abscissæ. If these blanks be used, the following facilitations are obtained: The special drawing by hand of each line representing a standard isobaric surface is no more required. The virtual-temperature divisions allow us to draw the virtual-temperature diagram without being obliged to refer to table 12 A of the Appendix. The use of table 9 A of the Appendix for the transition from Fahrenheit to centigrade degrees is no more required. In this way column *6a*, table O, drops out, the centigrade temperature recorded in column *6b* being found directly from the diagram. It is seen that in this way, by the use of these special blanks, the height of the standard isobaric surfaces are found with practically the same ease as if the observations ot pressure had been recorded in m-bars and those of temperature in centigrade degrees. As in the preceding case, some supplementary results are also easily obtained, such as the specific volume of the air at the standard isobaric surfaces. Even these supplementary calculations are simplified by the centigrade divisions along the axis of abscissæ, column *9a*, of table O, dropping out when these divisions are at hand. But if the example should be worked out still more in detail, if it be required to determine pressure in given heights, or to find the heights at which the

different observations were taken, it will be the best plan, exactly as in the preceding case, to change at once the given observations to rational units, and proceed as in example 1.

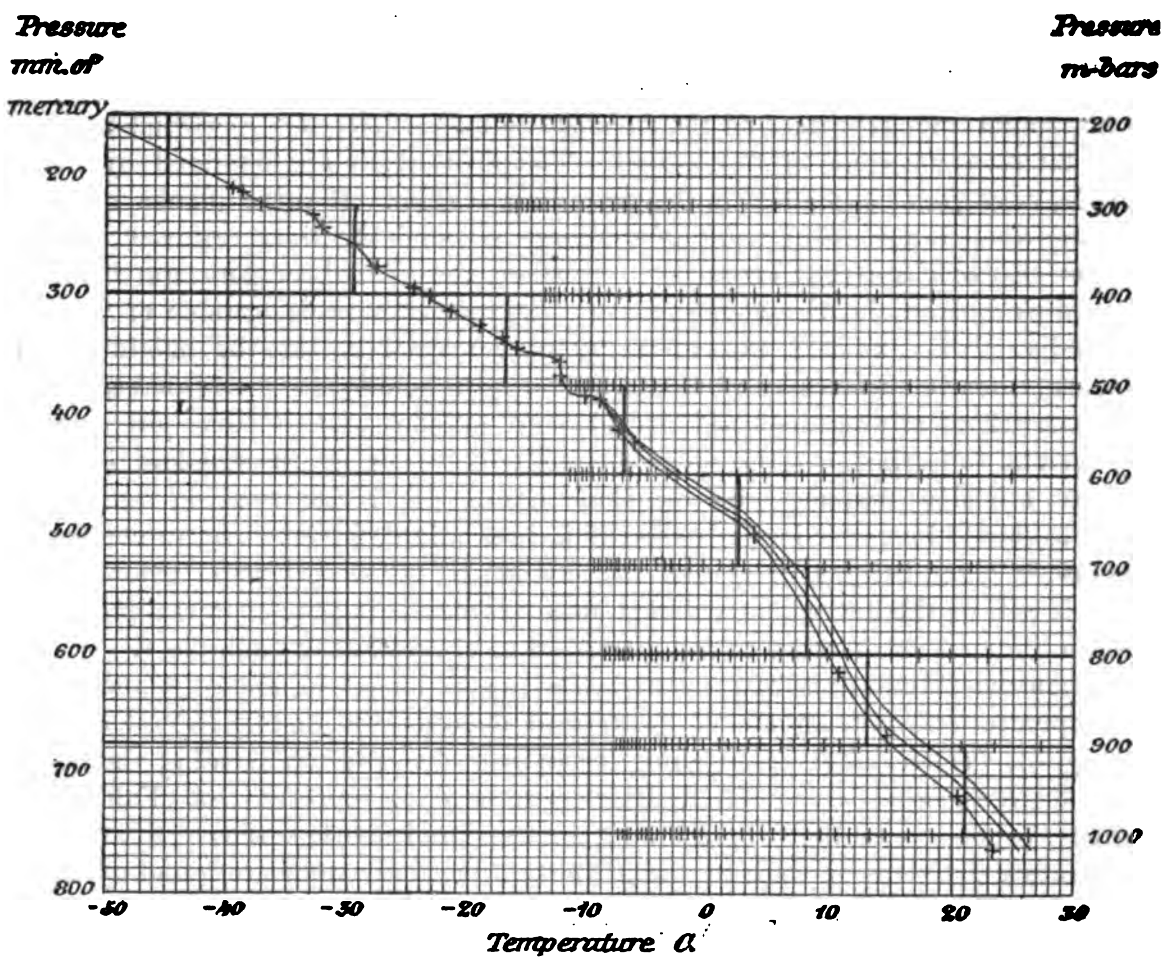


FIG. 8. — Virtual-temperature diagram, pressure in millimeters of mercury.

EXAMPLE 3. — *Observed time, pressure (millimeters of mercury), temperature (°C.), and humidity (per cent).* (Table N.) — From the observed pressures and temperatures (columns 2 and 3) the curve of true temperature is drawn (the curve to the left in fig. 8). The curve of virtual temperature for saturated air (curve to the right) is drawn by means of table 11 A of the Appendix.

TABLE N (EXAMPLE 3).

1	2	3	4	5	6	7	8	9	10
Observed time.	Observed pressure.	Observed temperature.	Observed humidity.	Standard pressures and pressure 1015.9 at the station, the latter found by table 7 A of Appendix.	Average virtual temperature in standard sheets and in sheet below lowest standard surface (+25.1) found from diagram (fig. 8).	Mutual distances between standard surfaces, found by table 9 M; height 135 of lowest standard surface above station, found by table 11 M and 12 M; height 39 of station above sea-level.	Heights of standard surfaces above sea-level, found by addition of figures of column 7.	Virtual temperature of air at the standard surfaces, found from diagram (fig. 8).	Specific volume of air at standard surfaces, found by table 14 M.
15th meridian.	mm. Hg.	°C.	Percent.	m-bars.	°C.	Dyn. meters.	Dyn. meters.	°C.	m³/ton.
10h 50m	762.0	23.4	72	100	....	....	....	....	....
54	718.5	20.4	58						
57	667.5	14.6	62	200	....	....	....	....	....
11 00	614.5	10.5	40						
16	499.0	3.5	23	300	....	....	9365	−37.3	2255
30	423.0	− 6.3	42		−29.8	2009			
44	412.0	− 7.8	17	400	....	....	7356	−23.6	1790
12 16	388.0	− 9.2	84		−17.0	1640			
28	384.0	−10.5	31	500	....	....	5716	−11.9	1499
40	369.0	−12.5	46		− 7.2	1391			
57	355.5	−12.5	38	600	....	....	4325	− 2.0	1296
1 00	345.0	−16.1	51		2.2	1218			
11	336.8	−17.3	70	700	....	....	3107	+ 6.0	1144
20	325.3	−19.2	67		8.0	1077			
34	315.8	....	..	800	....	....	2030	+10.2	1016
39	312.3	−21.6	74		13.0	967			
46	300.3	−23.3	83	900	....	....	1063	+16.7	924
51	294.0	−24.7	85		21.0	889			
2 1	287.8	....	..	1000	....	....	174	+24.7	854
13	277.0	−27.8	95		25.1	135			
25	268.5	....	..	1015.9	....	....	....	....	....
28	259.0	−29.6	..			39			
52	243.5	−32.2	..						
55	232.3	−33.0	..						
3 3	225.0	−37.3	..						
10	214.5	−38.9	..						
28	210.5	−39.7	..						
32	202.5	....	..						
40	193.0	....	..						

Observing the percentages of humidity in column 4, the curve of virtual temperature is drawn between the two other curves. Then the horizontal lines representing the standard pressures are drawn according to the following table of the values of the standard pressures in millimeters of mercury:

m-bars	1000	900	800	700	600	500	400	300	200	100
Millimeters of mercury	750	675	600	525	450	375	300	225	150	75

The standard isobaric sheets being thus marked in the diagram, their average virtual temperature is determined by drawing the vertical segments of line in the usual way. Then the determination of the thickness of the standard sheets and the height of the standard surfaces follow as before (columns 5 to 8), as well as the determination of the virtual temperature and the specific volume of the air at the standard surfaces (columns 9 and 10).

EXAMPLE 4. — *Observed time, pressure (inches of mercury), temperature (° F.), and humidity (per cent).* (Table O). — From the observed pressures and temperatures (columns 2 and 3) the curve of true temperature Fahrenheit is drawn (the curve to the left in fig. 9). Then the curve of virtual temperature for saturated air is drawn (curve to the right) by means of table 12 A of the Appendix. Finally, the curve of virtual temperature is drawn between the two others in accordance with the percentages of humidity (column 4). Then the horizontal lines representing the standard surfaces are drawn in accordance with the following table giving the value of the standard pressures in inches of mercury:

m-bars	1000	900	800	700	600	500	400	300	200	100
Inches mercury	29.53	26.58	23.62	20.67	17.72	14.77	11.81	8.86	5.91	2.95

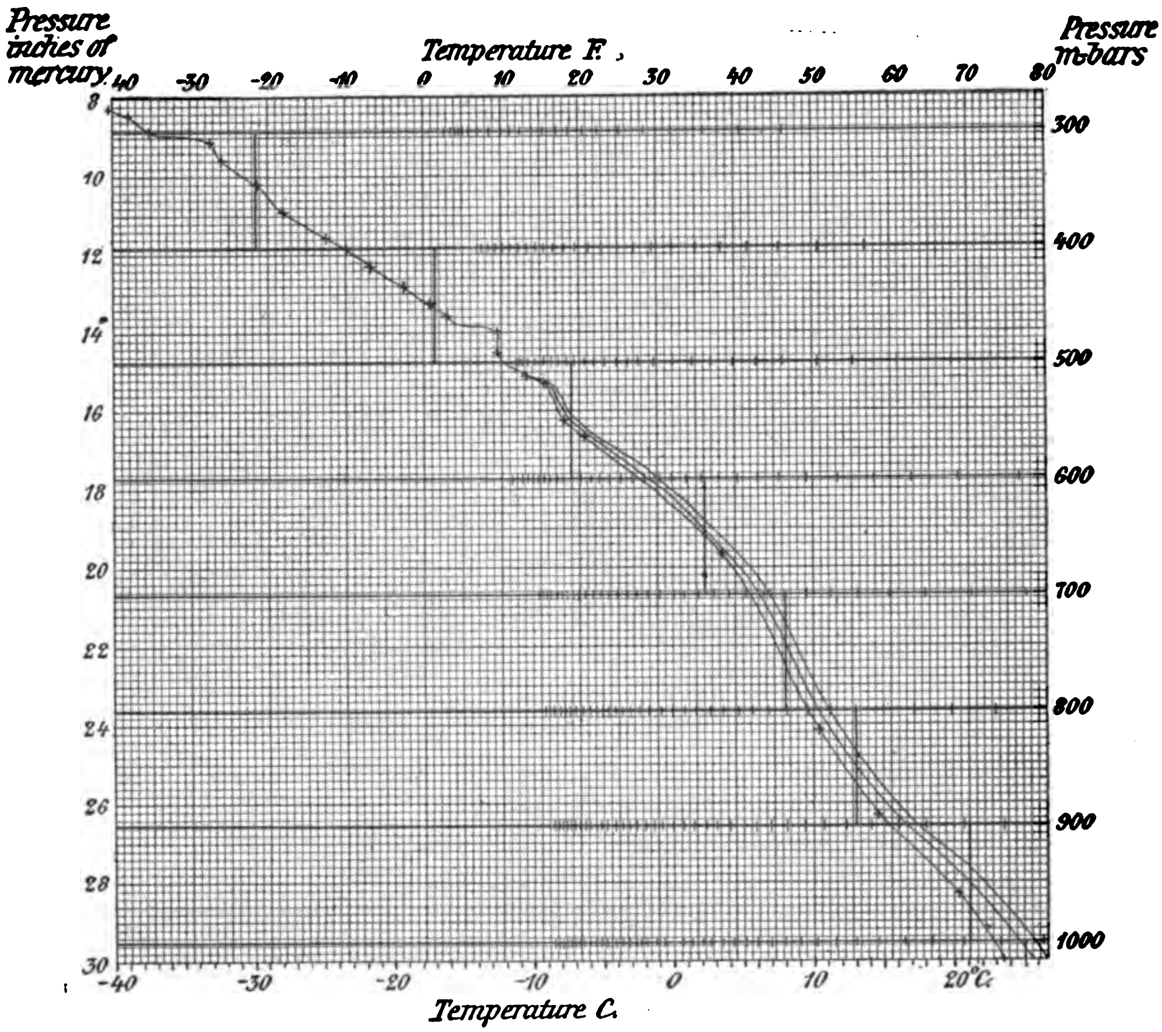


FIG. 9. — Virtual-temperature diagram; pressure in inches of mercury, temperature in degrees Fahrenheit.

The standard isobaric sheets being thus marked in the diagram, their average virtual temperatures are determined in the usual way by drawing the vertical segments of line. The diagram gives these temperatures in degrees Fahrenheit (column 6*a*), but by table 9 A of the Appendix they are changed into degrees centigrade. Afterwards the thickness of the standard sheets (column 7), and the height of the standard surfaces (column 8), are determined as in the preceding examples. From the diagram also the temperature Fahrenheit at the standard surfaces can be read off, and from this temperature, changed into centigrade (column 9*b*), we find the specific volume of the air at the standard surfaces (column 10).

TABLE O (EXAMPLE 4).

1	2	3	4	5	6a	6b	7	8	9a	9b	10
Observed time.	Observed pressure.	Observed temperature.	Observed humidity.	Standard pressures and pressure 1015.9 at station, the latter found by table 8 A of Appendix.	Average virtual temperatures in standard sheets and in sheet below lowest standard surface (+77.0), found by diagram (fig. 9).	Corresponding temperatures centigrade, found by table 9 A of Appendix.	Mutual distances between standard surfaces, found by table 9 M; height 135 of lowest standard surface above earth, found by tables 11 M and 12 M; height 39 of station above sea-level.	Heights of standard surfaces above sea-level, found by addition of figures of column 7.	Virtual temperature of air at standard surfaces found from diagram (fig. 9).	Corresponding temperatures centigrade, found by table 9 A of Appendix.	Specific volume of air at standard surfaces found by table 14 M.
15th meridian.	Ins. Hg.	° F.	Per cent.	m-bars.	° F.	° C.	Dyn. met.	Dyn. met.	° F.	° C.	m³/ton.
10h 50m	30.00	74.2	72	100	....	....	....	....	....	....	....
54	28.29	68.7	58								
57	26.28	58.3	62	200	....	....	....	....	....	....	....
11 00	24.19	50.8	46								
16	19.65	38.3	23	300	....	....	....	9365	−35.2	−37.3	2255
30	16.65	20.7	42		−21.5	−29.7	2009				
44	16.22	18.0	17	400	....	....	....	7356	−10.5	−23.6	1790
12 16	15.28	15.4	84		+ 1.5	−17.0	1640				
28	15.12	13.2	31	500	....	....	....	5716	+10.5	−11.9	1499
40	14.53	9.5	46		+19.0	− 7.2	1391				
57	14.00	9.5	38	600	....	....	....	4325	+28.5	− 2.0	1296
1 00	13.59	3.0	51		+36.0	+ 2.2	1218				
11	13.26	0.8	70	700	....	....	....	3107	+42.8	+ 6.0	1144
20	12.81	− 2.6	67		+46.5	+ 8.0	1077				
34	12.43	....	..	800	....	....	....	2030	+50.3	+10.2	1016
39	12.30	− 6.8	74		+55.5	+13.0	967				
46	11.82	−10.0	83	900	....	....	....	1063	+62.0	+16.7	924
51	11.58	−12.5	85		+70.0	+21.1	889				
2 1	11.33	....	..	1000	....	....	....	174	+76.5	+24.7	854
13	10.90	−18.0	95		+77.0	+25.0	135				
25	10.57	....	..	1015.9	....	....	....	....	....	....	....
28	10.20	−21.3	..				39				
52	9.59	−26.0	..								
55	9.14	−27.3	..								
3 3	8.86	−35.2	..								
10	8.44	−38.0	..								
28	8.29	−39.5	..								
32	7.97	....	..								
40	7.60	....	..								

57. Example of Rapid Derivation of the Main Hydrostatic Results of a Meteorological Ascent. — We have given examples above of the derivation hydrostatically of the results of a meteorological ascent in as complete a form as possible. We have shown how to calculate all quantities with pressure and with height as independent variable, and have also taken up problems of a more secondary interest from a meteorological point of view, such as the calculation of the heights at which the different readings of the instruments were taken.

We shall now show how with the smallest loss of time we may deduce the most important hydrostatic results. This rapid reduction of the data of a meteorological ascent may soon be of practical importance. It is already proved possible to carry out meteorological ascents every day, in all kinds of weather.*

* Compare, for instance, R. Assmann: Ergebnisse der Arbeiten des K. Preussischen Aeronautischen Observatoriums bei Lindenberg im Jahre 1905. Braunschweig, 1906. List of ascents during the year 1905, pp. xxvi–xxix.

Simultaneous ascents from a system of stations for aeronautical meteorology may therefore be organized, and the problem will present itself, how to use the results of these ascents for the daily forecasts of the weather. The meteorologist at each station must therefore be able, as soon as possible, to send off a telegram giving the main result of his ascent. We shall here take under consideration the preparatory work for sending this telegram as far as the hydrostatic state of the atmosphere above the station is concerned. The further work at the central bureau after the reception of the telegrams will be discussed in the next chapter.

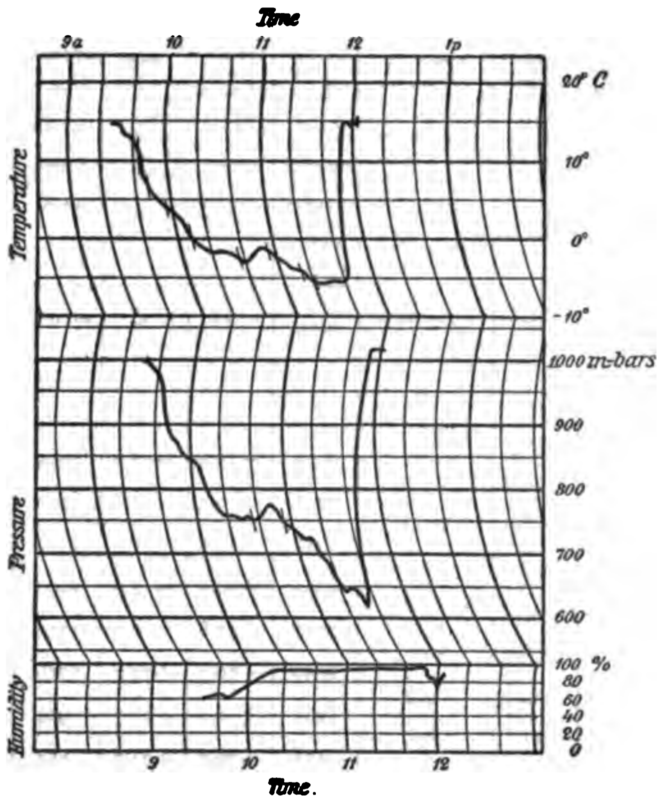


FIG. 10.— Meteorogram, Berlin, August 28, 1901.

The hydrostatic state of the atmosphere above a station is given if we know the height above sea-level of the standard isobaric surfaces. To enable the central bureau to find these heights it will be sufficient if the telegram contain (1) the height of the lowest standard isobaric surface; (2) the average virtual temperature of the standard isobaric sheets.

We have thus to show what the meteorologist at the station has to do from the moment he receives the meteorogram representing the results of the ascent on his table, in order to find the results (1) and (2) to be telegraphed.

Discussions of instrumental technics will not be taken up in this treatise. But it is important to remind the reader of the existence of two kinds of registering meteorographs. The first, which is most commonly used, contains a clock, and all the meteorological elements are registered as functions of the time. The second contains no clock. The barometer produces the motion of the paper on which the curves for the other meteorological elements are thus registered as functions of the atmospheric pressure. We shall show how to interpret a meteorogram obtained by each of these two kinds of instruments.

(A) *Meteorological elements registered as a function of time.* — Fig. 10 represents a meteorogram obtained by a kite flight from Tegel at Berlin, August 28, 1901.* The circle-arcs are coordinate curves of equal time. The curve in the

* R. ASSMANN und A. BERSON: Ergebnisse der Arbeiten am Aeronautischen Observatorium 1900–1901, p. 259. The figure is changed, in as much as the coordinate curves on the barogram are drawn for m-bars instead of for millimeters of mercury.

first section of the diagram represents the variation of temperature, that in the second the variation of pressure, and that in the third the variation of humidity, all as functions of time. If there are instrumental errors, the corrections are supposed to be introduced graphically upon the diagram, the curves of fig. 10 being such corrected curves.

In order to derive from this meteorogram the curve of virtual temperature, we have to determine sets of corresponding values of pressure, temperature, and humidity. To do this most conveniently, we start with the points where the barometer-curve cuts the lines for 1000, 950, 900, . . . m-bars. Using a pair of compasses, we mark the corresponding points on the thermometer and hygrometer curves. Reading the temperature corresponding to the marked points on the temperature curve, we draw the curve of true temperature in the diagram (fig. 11). Then, using the virtual-temperature divisions, we draw the curve of virtual temperature for saturated air. Finally, using the humidities corresponding to the marked points on the hygrometer curve of fig. 10 we draw in the diagram fig. 11 the curve of virtual temperature between the two other curves. The vertical segments of the line giving the average virtual temperatures of the standard sheets are drawn, as well as the segment (invisible on account of its shortness), giving the average virtual temperature (+ 16) of the air between the lowest standard surface and the earth. By means of this temperature and the pressure 1001.2 at the station, we find the height 10 dynamic meters of the 1000 m-bars surface above the earth, using tables 11 M and 12 M as described previously. Adding the height (39) of the station, we get the height (49) of this standard surface above sea-level. The figures to be telegraphed are then

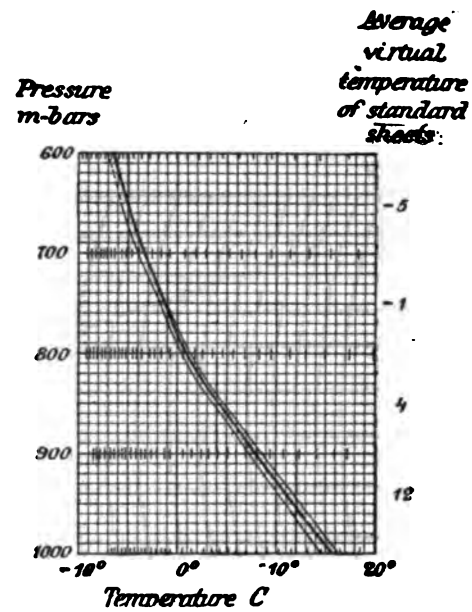


FIG. 11.—Virtual-temperature diagram, Berlin, August 28, 1901.

(a) 49, 12, 4, −1, −5

the first, 49, being the height of the 1000 m-bars surface, and the four other numbers the virtual temperature of the standard sheets.

The interval of time from the moment the meteorologist has obtained the meteorogram (fig. 10) on his desk until he has found the figures (*a*) to be telegraphed ought not to exceed ten to fifteen minutes.

(B) *Meteorological elements registered as function of pressure.*—The curve to the left in fig. 12 is recorded by Professor Assmann's baro-thermograph* at an ascent with a registering balloon on July 4, 1901.†

* R. Assmann und A. Berson, *l. c.*, p. 42.

† R. Assmann und A. Berson, *l. c.*, p. 209. The original figure is changed by introduction of coordinate curves representing the pressure in m-bars, and by change of positive direction on the axis of temperatures.

This recorded curve is the curve of true temperature. Using the virtual-temperature divisions, we draw the curve to the right, the curve of virtual temperature for saturated air. No humidity having been registered, we suppose the relative humidity to have had the average value of 70 per cent and draw the curve of virtual temperature at a distance from the curve of true temperature equal to 70 per cent of the distance between the two curves. Then the circle arcs are drawn, which, in this system of coordinates, represent the average virtual temperatures of the standard sheets, as well as the arc (invisible in the figure on account of its shortness) representing the average virtual temperature (+12) of the air between the lowest standard surface and the earth. Using this temperature and the pressure (1005.2) at the station, we find by tables 11 M and 12 M the height (42 dynamic meters) of the lowest standard surface above the station, and adding the height (39) of the station, we find the height (81 dynamic meters) of this surface above sea-level. The figures to be telegraphed

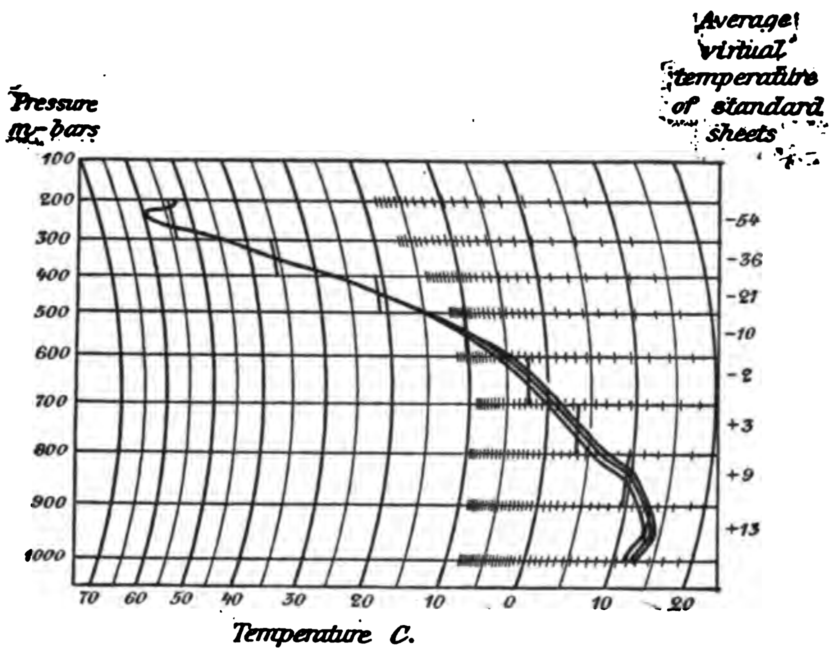


FIG. 12.—Meteorogram, Berlin, July 4, 1901.

(*b*) 81, 13, 9, 3, −2, −10, −21, −36, −54

are thus found. The time from the moment the meteorologist has obtained the meteorogram until he has found the figures (*b*) ought not to exceed five minutes.

58. Extrapolation of the Virtual-Temperature Diagrams.—The virtual-temperature diagram obtained from the observations of a meteorological ascent may be prolonged some distance upward, so as, for instance, to attain the next standard isobaric surface. If the prolongation be not too long, a slight deviation from the course which real observations would have given will have no great influence upon the calculated heights or pressures. Short extrapolations of this kind have been used occasionally in the examples given above.

A special kind of extrapolation will be of great importance, namely, those from the earth's surface. Complete observations along a vertical in the atmosphere will always remain rare, while we may get abundant observations from stations at the earth's surface. The observation of pressure, temperature, and humidity gives one

point of the virtual-temperature diagram for a vertical in the atmosphere passing through the station. If the curve could be continued upwards somewhat from this point, we should be able to solve the hydrostatic problem for a vertical of moderate height. The solution would give a perfectly satisfactory accuracy in sufficiently small heights above the station, but of course decreasing accuracy with increasing height. Experience would gradually show to what height the extrapolation might be ventured.

As a guide for extrapolations of the virtual-temperature diagram, table 16M of Meteorological Tables has been constructed. It has been obtained by a statistical study of the results of the international balloon and kite ascents for the three years 1901, 1902, and 1903. The table gives the correction, which should be added to the virtual temperature at the station in order to give the virtual temperature at the heights above the station figuring as argument. The little table headed "Under the earth's surface" gives the correction for extrapolations downwards, based upon the common supposition of a decrease of the temperature of 0.5 degree per each 100 meters, used generally at present for "reductions to sea-level" of barometric records.

An observation being given, taken at a station at the earth's surface, table 16M thus enables us to draw the virtual-temperature diagram for a vertical through the station. The ordinates being the heights, we have to use the method shown in example 2, page 72, for calculating from this diagram the heights corresponding to given pressures or the pressures at given heights, negative heights below the earth's surface being also for theoretical reasons included.

We emphasize that table 16M should serve only as a guide in extrapolating virtual temperatures, and that it must be used with caution. Preferably a table of this kind should be made for each meteorological station, based solely upon data from ascents in air from this station. Great differences, dependent on the situation of the station, would probably be found. Thus for stations situated on high isolated mountains the temperature inversions (positive temperature corrections) given in table 16M in case of high pressure during winter, would probably not be found, and the gradients under ordinary conditions would probably be found smaller than above low land. When these gradients are determined by ascents undertaken from mountains, the value of meteorological observations at stations on mountains will be very much increased.

59. **Extrapolation of Average Virtual Temperatures.**—The method developed in the preceding article is important, because it enables us to find the hydrostatic state of the atmosphere near the earth's surface every day by common meteorological observations, quite independently of ascents in the air. Since it therefore furnishes methods which at once might be introduced into the daily meteorological work for the forecast of the weather, it will be important to simplify the operations to be performed as much as possible.

If the problem be to find completely the hydrostatic state, both as to the height corresponding to any given pressure, and as to the pressure at any given height, no

simplification is possible. In such a case the virtual-temperature diagram must be drawn. But limiting the problem to the determination of the heights of the lowest standard isobaric surfaces, a convenient short cut is easily found. Instead of using table 16 M, giving the local values of the virtual temperatures, we use table 15 M, which gives in a corresponding manner the average virtual temperatures for the sheets of air between the station and the heights figuring as argument. This table is deduced from the preceding one by a process of integration, the principle of which will be clear in itself.

This table being given, we may proceed as follows in order to find the heights of the nearest standard isobaric surfaces: Using the observed pressure we find from table 11 M approximate values of the heights of these surfaces above the station. Using these approximate height values and table 15 M, we find the average virtual temperatures of the corresponding sheets. These virtual temperatures enable us to correct the approximate heights already found by means of table 12 M. The complete procedure is seen by the example annexed to the table.

This method of calculating the height above or the depth below the earth of standard isobaric surfaces is analogous to the method of "reduction to sea-level" of the barometric observations taken at stations situated at heights above sea-level. We emphasize some important differences however — our aim is always to find the heights of isobaric surfaces really existing in the air. The main reductions are therefore made upward and not downward. Consequently the result of the reduction is capable of being controlled by actual observations made in the open air, while reductions to the interior of the earth involved in the reduction to sea-level can not be made the subject of any kind of test by actual observations made at the place for which the pressure is calculated. Further, the reductions to sea-level are made generally according to a schematic method, using under all conditions the same temperature gradient. We have retained this gradient for small reductions downwards, while in working out the part of table 15 M to be used for reductions upwards, we have tried to introduce individual temperature gradients according to the different types of weather. In this direction probably much progress could be made by a statistical study of the results of ascents in the air, as remarked above.

CHAPTER VII.

SYNOPTIC REPRESENTATION OF THE FIELDS OF PRESSURE AND OF MASS IN THE ATMOSPHERE.

60. Quasi Static State. — Setting aside extraordinary phenomena, as, for instance, waterspouts, we may characterize atmospheric motions as slow motions going on near a state of equilibrium. Comparing simultaneous barometric records taken from two different places, we find the conditions of equilibrium apparently fulfilled, if the two places are at small or moderate distances from each other. Only as the distance increases do we find a gradual departure from the fulfillment of these conditions. As long, therefore, as the distance is small, the correct result will be produced if we use the barometric records for calculating the difference of height between the two barometers. The instruments will especially show the same pressure if they are placed on the same level. But if we sufficiently increase the distance between them, they must finally be placed on distinctly different levels in order to show the same pressure.

Thus, in reality, there is a deviation from the principle of the coincidence of surfaces (sections 35, 38). But the angle of intersection is so small that we must follow the surfaces over great distances in order to find an appreciable separation. On the other hand, proceeding along the plumb-line, we can not attain distances sufficiently great in order to prove an unquestionable deviation from the principle of the unit-sheets (sections 35, 38). Owing to the great lateral and small vertical extent of the atmosphere, we have therefore this peculiar relation, characterizing what we may call the quasi static state of the atmosphere:

The condition of equilibrium is apparently fulfilled along every vertical line. But as we proceed in a horizontal direction, there is a gradual change from vertical to vertical in this apparent state of equilibrium.

This important principle forms the basis of all practical investigations in atmospheric dynamics. In making use of it, it is important to remark that we need not take the expression "vertical" in the narrow sense of the word. We can consider the greatest angle of inclination of the isobaric surfaces as a kind of critical angle. Every curve whose angle of inclination is everywhere great in comparison with this critical angle will be called a *quasi vertical* curve, while curves whose angle of inclination is of the same order of magnitude or smaller than this critical angle will be called a *quasi horizontal* curve. The latter curves may attain lengths comparable to the lateral extent of the atmosphere, while the first remain short in the same sense as the true vertical curves are short. Following a quasi vertical curve, we can not therefore attain sufficient distances to be able to observe any appreciable departure from the hydrostatic conditions, and the principle stated above can therefore at once be extended from true vertical to quasi vertical curves.

61. Consequences of the Principle of the Quasi Static State.—In the preceding chapter we have shown how the results of meteorological ascents could be worked out according to the principles of hydrostatics.

In the case of true equilibrium, one ascent would be sufficient to give the state of the whole atmosphere. For according to the principle of coincidence of surfaces, the state will be the same at all points contained in the same level surface. Therefore, if we know the state at the points of a curve cutting a set of level surfaces, we also know the state at all points of these level surfaces.

Now, the actual state of the atmosphere is not one of true equilibrium. But owing to the principle of the quasi static state, the hydrostatic methods may still be used to a certain extent. The curve along which the ascent of a kite or balloon has taken place is always a quasi vertical curve. Along every curve of this kind the conditions of equilibrium are fulfilled with sufficient approximation to entitle us to use the principles of hydrostatics. The states recorded by the instruments at the different points of this curve may be interpreted as if recorded at points of corresponding heights in a true vertical. By means of the developed hydrostatic methods we therefore find the distribution of pressure and of mass along this vertical. Although calculated upon a supposition not strictly fulfilled, the distributions of pressure and of mass found in this way will be very nearly the true ones.

This does not, however, entitle us to draw any conclusion as to the distribution of pressure and of mass along other verticals. For verticals of sufficient mutual separation, the distributions will generally be distinctly different, and must be found by independent observations. But this being done, we can easily calculate by interpolation the distribution of pressure and of mass also for all interjacent verticals, and thus find this distribution in the whole atmosphere.

Before concluding the consideration of atmospheric statics, we shall develop the geometrical methods of representing synoptically the results obtained by this method.

62. Method of Drawing Charts Representing Scalar Fields.—It will be useful first to exemplify some practical methods of drawing charts representing scalar fields in a plane.

If the values of a scalar quantity be known at a number of points in a plane, we always know how many equiscalar curves will pass between any two of these points, the curves being drawn for fixed intervals, say for unit-differences of the scalar quantity. By this condition the course of the equiscalar curves is determined to some extent, the more accurately so the greater the number of points in which the value of the scalar quantity is known. Therefore, knowing the value of such a quantity at a sufficient number of points, we can draw the equiscalar curves with sufficient accuracy, and thus arrive at the graphic representation of the scalar field in the plane.

This is the well-known method of drawing isothermic charts from the observations of temperature, isobaric charts from the observations of pressure, topographic charts from measurements of heights, and so on.

63. Arithmetical and Graphic Methods of Adding and Subtracting Scalar Fields. — Let α_1 and α_2 be scalar quantities of the same kind, say both temperatures, both pressures, or both heights above sea-level. From the fields α_1 and α_2 we shall often have to deduce the field of their sum

(*a*) $$\alpha_1 + \alpha_2$$

or the field of their difference

(*b*) $$\alpha_1 - \alpha_2$$

For the solution of this problem two methods offer themselves.

(A) *Arithmetical method.* — We may form arithmetically the sum (*a*), likewise the difference (*b*), at a certain number of points. The numbers representing these sums or differences are noted in the plane, and the equiscalar curves drawn according to the method mentioned in the preceding article.

This method will be especially convenient when the values of both quantities α_1 and α_2 are observed at the same points. The sums or differences used in drawing the chart can then be derived directly from the observed quantities.

(B) *Graphic method.* — The curves are supposed to be drawn *for the same interval* in the field of α_1 as in that of α_2. Superimposed upon each other, the curves divide the plane into a set of curvilinear parallelograms. We can then draw two sets of diagonal curves, and it will easily be verified that the one set represents the field of the sum $\alpha_1 + \alpha_2$, and the other that of the difference $\alpha_1 - \alpha_2$.

This graphic method of forming the sum or difference of two scalar fields is most convenient, and will be much used below. It will generally prove practicable to draw the curves of the different systems on different papers laid upon each other. Transparent paper may be used, or, more conveniently, common paper placed on a sheet of glass and strongly illuminated from below.

The arithmetical and the graphic methods supplement each other in a valuable manner. The latter gives both systems of diagonal curves sharply, if the two sets of originally given curves cut each other at nearly right angles. But if the angles approach 0° or 180°, only the one system, that containing the long diagonals of the parallelograms, will be sharply defined. The other, that containing the short diagonals, will be very indeterminate, greatly varying with small errors in the course of the originally given curves. This second set is therefore obtained better by the arithmetical method, especially if the arithmetical sums or differences can be formed from originally measured and not from interpolated values of the two scalar quantities α_1 and α_2.

64. Charts of Absolute and of Mutual Topography of Isobaric Surfaces. — The synoptic representations of the fields of pressure and of mass are worked out on charts containing the situation of the stations from which the observations have been made.

On a chart of this description we can mark the height of a certain isobaric surface above each station. Then, by aid of these heights, we draw, as explained in section 62, a topographic chart showing the configuration of the isobaric surface. In this

way we can draw a topographic chart for every standard isobaric surface reached by the ascents. A set of such charts gives a perspicuous representation of the distribution of pressure in the investigated part of the atmosphere. Examples are given in figs. 13 and 19.

To find the correlative representation of the distribution o mass, we have to remember that the figures representing the mutual distances from one standard surface to the next at the same time represent the average specific volume of the air in the sheets between the surfaces. A chart of *mutual topography* of two successive standard isobaric surfaces will therefore also represent the field of average specific volume in the sheet between the two surfaces. We may draw these charts directly from the figures representing the thickness of the sheets (example 1, column 7, table J, p. 70; example 2, column 8, table K, p. 74) or indirectly from the charts of absolute topography, using the method of graphic subtraction, the latter method being, however, less accurate. Charts of this description are given in figs. 14 and 20.

Of course we might also have represented the distribution of mass by topographic charts of the isosteric surfaces. But, however interesting these might be, the above representation, obtained in immediate connection with the distribution of pressure, will generally be found the more useful, apart from the greater facility with which it is found.

In order better to conceive the topography represented by the charts it will be usetul to draw profile curves of the isobaric surfaces. A set of such verticals as the second of fig. 4 are drawn at horizontal distances corresponding to the distances between the stations. Joining the points belonging to the same isobaric surface, we get the profile curves. Taking the other set of divisions on the same verticals, we may also get the profile curves of the isosteric surfaces. If both sets of profile curves be drawn on the same diagram, as in figs. 17 and 21, they intersect each other, showing the deviation from the hydrostatic principle of the coincidence of the surtaces. These vertical sections are not of the same practical interest as the charts of absolute and relative topography, but have still interesting theoretical properties, and enable us to get a more complete conception of the content of the charts. An important geometrical relation between the section containing the profile curves and the charts of relative topography will be given presently (section 73).

65. Charts of Absolute Pressure and of Mutual Pressure Differences in Level Surfaces. — On the chart containing the stations we can then note the numbers representing the pressure found at a certain level, and guided by these numbers draw an isobaric chart for this level, in the same manner as such charts are drawn for sea-level in the daily weather service. A set of such charts, drawn for a set of standard levels, will give as complete a representation of the distribution of pressure as the preceding one by topographic charts of isobaric surfaces. Examples of such isobaric charts at different levels are given in figs. 15 and 22. As too many charts would be acquired if drawn for every standard level, *i. e.*, for every dynamic meter of height, we have only drawn them for intervals a thousand times greater, *i. e.*, for level differences of 1,000 dynamic meters. The pressures represented by the isobaric curves are added in m-bars.

To find the correlative representation of the distribution of mass, we have to remember that the difference of pressure from one standard equipotential surface to another is equal to the average density of the air in the sheet between them. By arithmetical or graphic subtraction of the fields of pressure in the level surfaces limiting an equipotential sheet we therefore get a chart representing the average distribution of density in this sheet. Such charts are given in figs. 16 and 23. The figures added to the curves give the mutual pressure differences in m-bars. As they refer to level sheets of 1000 dynamic meters interval they will, after division by 10^5, give the average densities of the sheets.

A valuable complement to these charts of absolute pressure and of pressure difference are vertical sections like those of figs. 18 or 24. These are obtained by means of verticals like the third of fig. 4. A set of such verticals being drawn at proper mutual distances, points representing the same dynamic height are united by curves, and in like manner points representing the same value of density. In this way we obtain the profile curves of the equipotential and isopycnic surfaces, those of the equipotential surfaces being drawn simply as horizontal equidistant lines.

An important relation between these vertical sections and the corresponding charts will be developed below (section 73).

66. Construction for Lower Levels of Charts of Absolute and of Mutual Topography from Observations Made at the Earth's Surface.—In drawing the charts described in principle in the preceding articles, it is important to make as complete a use as possible of the observations from the stations at the earth's surface. For these observations are abundantly at hand, while those from the open air will always remain relatively scarce. By means of the method of extrapolation developed in sections 58 and 59, it will be possible from the observations at the earth's surface to draw charts for the lower sheets of the atmosphere.

From stations near sea-level the heights of the three lowest standard surfaces may be found, and from many mountain stations the heights also of the fourth and fifth and even higher surfaces may be determined with satisfactory accuracy. The common meteorological observations will therefore enable us to draw topographic charts of the three, four, or even five lowest standard isobaric surfaces. The three first charts of fig. 19 are obtained in this way, only slightly corrected and extended afterwards by the results obtained by ascents in the air. It is important to remark that charts of this kind can be obtained every day from the regular meteorological observations, and with the same ease as the charts for sea-level now in use.

It is of course always desirable to derive the charts directly from the original observations, and not from these observations after they have been "reduced to sea-level." But often, when past atmospheric states must be worked out from published observations, these are accessible only in the distorted form of isobaric charts for sea-level. Re-reductions to higher levels are thereby made more troublesome and less trustworthy. But it is important to notice that it is very easy to change an isobaric chart for sea-level into a topographic one for the 1000 m-bar surface, provided the isothermic chart be known besides the isobaric.

To perform this change when the isobaric chart is drawn for millimeters of mercury and the isothermic for degrees of the centigrade thermometer, table 18 A

of the Appendix is used. The table shows that the level curve of the height zero coincides with the isobaric curve of a pressure of 750 mm. mercury at sea-level, independently of the temperature. The level curve of a height of 50 dynamic meters coincides almost completely with the isobaric curve for a pressure of 755 mm. mercury, deviating for high temperatures towards the isobaric curve of 754 mm. mercury, and for low towards the isobaric curve of 756 mm. mercury. In the same way the curve of 100 dynamic meters of height closely follows the isobaric curve of 760 mm. mercury, with small deviations towards higher pressure for low temperature and towards lower pressure for higher temperature, and so on. Using this table and the isothermic chart, slight changes are easily made in the isobaric curves, giving thus the level curves representing the topography of the 1000 m-bars surface.

Table 19 A of the Appendix serves the same purpose, in the case of the isobaric chart being drawn for inches of mercury and the isothermic for Fahrenheit degrees. This table has been used to draw the topographic chart for the 1000 m-bars surface in fig. 13 from the corresponding isobaric chart for sea-level published by the U. S. Weather Bureau.

The principle for the calculation of tables of this kind is explained in the next article, where tables serving an analogous purpose are described.

From the charts of absolute topography, obtained by extrapolation from below, those of relative topography, representing the distribution of mass in the sheets between the standard surfaces, may be deduced at once by the method of arithmetical or graphic subtraction. The arithmetical method will generally be found preferable on account of the acuteness of the angles of intersection of the curves of absolute topography (section 63). Two charts obtained in this way are given in fig. 20.

67. Construction for Lower Levels of Charts of Absolute Pressure and of Pressure Differences from Observations at the Earth's Surface. — Drawing the extrapolated virtual-temperature diagram as explained in section 58, and calculating the pressure in standard levels, we can draw the isobaric charts of absolute pressure in these levels. Afterwards, by the method of arithmetical or graphic subtraction (the first being generally preferable), the charts of relative pressure, representing the distribution of density in the level sheets, can be drawn.

On the other hand, if the charts of absolute topography of standard isobaric surfaces be drawn, it is easy to change them into isobaric charts for corresponding standard levels. To see this we remark that the level curves on isobaric surfaces and the isobaric curves on level surfaces belong to one family, the curves of intersection between isobaric and level surfaces. The level curves on an isobaric surface and the isobaric curves on a level surface from about the same height in the atmosphere will therefore resemble each other. Further, the standard isobaric surfaces of pressures 1000, 900, 800, 700, 600, 500, 400, and 300 m-bars are nearly in the levels of 0, 1000, 2000, 3000, 4000, 5000, 7000, and 9000 dynamic meters, and therefore only a small correction is required to change the level curves of these isobaric surfaces into isobaric curves at the corresponding levels.

The principles for finding these corrections are easily seen. The isobaric curve 700 m-bars in the level surface 3000 dynamic meters is identical with the given level curve 3000 dynamic meters on the isobaric surface 700 m-bars. The isobaric

curve 705 m-bars will run where the sheet of air between the isobaric and the level surface exerts the pressure of 5 m-bars. In order to exert this pressure the sheet must have the thickness of 56 dynamic meters if it has the temperature of 0° C., the thickness of 58 dynamic meters if it has the temperature of 10° C., and so on. This is seen at once from tables 10 M and 12 M. The required isobaric curve of 705 m-bars will thus coincide with the given level curve of 3056 dynamic meters where the sheet has the temperature of 0° C., with the given level curve 3058 dynamic meters where the sheet has the temperature of 10° C., and so on. These temperatures and the level curves being given, the isobaric curves can thus be drawn.

To avoid the laborious use of tables 10 M and 12 M, table 17 M has been derived from them ; as one argument appears the pressures along any isobaric curve to be drawn, and as the other the virtual temperatures in the given isobaric surface. These are always known (see example 1, column 9, table J, p. 70, example 2, column 10, table K, p. 74). To these temperatures at the surface will correspond a definite average temperature of the sheet if we make the common supposition of a 'fall of temperature of 0.5° C. for every 100 dynamic meters of height. On account of the smallness of the reductions a greater accuracy than that obtained under this simple supposition will never be required. Using this supposition, the tabulated numbers are derived from tables 10 M and 12 M. They indicate with which level curves the required isobaric curves should coincide. Using these tables and the given topographic chart and temperature chart for the isobaric surfaces, the required isobaric charts can be drawn with great ease.

68. Correction of Charts for Lower Levels and Construction of Charts for Higher Levels by Means of Observations Obtained from Ascents.— If results from simultaneous ascents in the air were available in sufficient number, charts of absolute and of mutual topography, or of absolute pressure and pressure differences, could be drawn directly and independently of each other for every level. But as long as these ascents remain comparatively rare, it will be advisable first to draw all charts which can be obtained by extrapolation from the stations at the earth's surface as completely as possible.

This being done, our first task will be to correct the charts according to the absolute values obtained from the ascents. This is easily done for charts of absolute topography or of absolute pressure. The values obtained from the ascents are noted on the charts, and the whole set of curves displaced or changed so as to suit these values. As a rule this is easily done without any noticeable change in the qualitative course of the curves. These corrections have been made on the charts of figs. 19 and 22.

Greater difficulty will be found in correcting charts of mutual topography or of pressure differences, because their curves have a very complicated course, evidently in great measure depending upon the topography of the land and the distribution of land and sea. It is not easy to see how to change the course of such curves so as to suit the small number of correct values obtained by the ascents. In the examples worked out below we have therefore desisted from making this correction.

In fig. 20 are given side by side two charts of mutual topography obtained by extrapolation from 219 stations at the earth's surface, and two as obtained from the results of ascents in the air from 5 stations. If ascents had been made at a sufficient number of places the curves of the latter charts would probably have had mainly the same course as those of the extrapolated charts, but with slightly changed situations of the different curves, and it would have involved no difficulty to correct the extrapolated charts by the fundamental values obtained by the ascents.

As to the charts for higher levels, those of mutual topography or of pressure differences are drawn directly from the results of the ascents. Afterwards we use the following method for drawing the charts of absolute topography not obtained by extrapolations from below: The chart of mutual topography of two surfaces is placed upon that of the absolute topography of the lower one. Then the absolute topography of the upper one is obtained by graphic addition. The chart thus obtained is then corrected in accordance with the absolute heights found from the ascents and from the observations on mountains of a sufficient height. For the present, however, the latter observations must be used with caution because of our ignorance of temperature gradients above mountains (section 58). This chart being drawn, we place upon it the next chart of mutual topography, proceed in the same manner, and so on.

The charts of absolute pressure in the higher standard levels are found by a completely analogous procedure.

In drawing charts in this way, one after the other by graphic addition, there is this advantage — that the characteristic feature of the distribution of pressure as known from the numerous observations from the earth's surface does not disappear as we proceed upward, as would have been the case if each chart had been drawn independently of the others by means of the small number of calculated values.

69. Remarks on the Rapid Work Essential for Daily Weather Service. — In the preceding articles we have shown in detail how to find and represent as completely as possible the distribution of pressure and mass in the atmosphere. Nothing would prevent the use of these methods in the daily meteorological service for the forecasts of the weather. But then it becomes a question of vital importance how to be able to draw the whole system of charts with as short a delay in time as possible.

We have, then, first to make a choice between the two methods, developed side by side — that of representing the absolute and the relative topography of isobaric surfaces, or that of representing the absolute and relative pressure in level surfaces. There is no doubt as to what choice to make; the charts of absolute and relative topography can be found by a smaller number of operations, and therefore be ready within a shorter time. It may be possible that the method of constructing the isobaric charts in level surfaces might be developed to a greater degree of simplicity than is done here. But it is not probable that the simplicity of the other method could be reached. The preference in favor of the first method is due to the greater theoretical simplicity of the problem of determining the height corresponding to a given pressure compared with that of determining the pressure at a given height.

The superiority of the charts of absolute and relative topography being admitted, the meteorologists at the central bureau have to work out such charts from two sets of telegrams, giving (1) the observations from the common meteorological stations, (2) the height of the lowest standard surface and the virtual temperature of the standard sheets above the aeronautical stations from which ascents have been made (section 57).

From the first set of observations the charts of absolute and relative topography are drawn as described above for the lower levels. They can be drawn independently of each other, and accordingly simultaneously by different workers. As the drawing of each chart is of precisely the same nature as the drawing of an isobaric chart for sea-level, nothing prevents the whole set from being ready within an interval of time not exceeding that required for drawing the single isobaric chart for sea-level.

From the telegraphed values of the virtual temperatures of the standard isobaric sheets the higher-level charts of relative topography are drawn. In doing this it is not necessary first to change by table 9 M the telegraphed virtual temperatures into heights. The curves for constant thickness of a sheet are curves for certain constant values of the virtual temperature. We may therefore note these temperatures on the chart and draw the curves for constant thickness of the sheet directly from them, table 9 M showing which virtual temperature corresponds to a required value of the vertical distance.

The charts of relative topography being drawn, the corresponding charts of absolute topography are found by the method of graphic addition. Finally, if required, the charts are corrected according to the absolute heights, which to save time may have been calculated by another computer. But as long as observations from the open air are rare, one man will probably be able to perform all the work for the higher-level charts during the time required by the other workers to draw the lower-level charts. Thus, by good organization there is nothing to prevent the whole system of charts giving the distribution of pressure and mass in the atmosphere for all heights reached by extrapolations from below and by direct ascents in the air, being ready within an interval of time not greatly exceeding that required for drawing such charts as are now used for sea-level.

70. Example 1. — Atmospheric Conditions over North America, September 23, 1898. — The first simultaneous meteorological kite ascents were organized by the U. S. Weather Bureau during the summer of 1898.* September 23 seven ascents succeeded, five of which were fairly simultaneous, between 7 and 11 o'clock in the morning, and thus simultaneous also with the common meteorological observations at 8 o'clock, time of the seventy-fifth meridian. Two of the ascents, from North Platte and from Dodge City, came between 2 and 5 in the afternoon. During the days September 21 to 24, kite ascents were made also from the Blue Hill Meteorological Observatory near Boston.† None of them were simultaneous with

* See H. C. Frankenfield: Vertical Gradients of Temperature, Humidity, and Wind Direction. A preliminary report on the kite observations of 1898. Weather Bureau Bulletin F. Washington, 1899. The original results of the kite ascents have not been published. Those used below have been kindly communicated by the Weather Bureau.

† H. Helm Clayton: Studies of Cyclonic and Anticyclonic Phenomena with Kites. Bulletin No. 1, 1899, of Blue Hill Meteorological Observatory.

those of the Weather Bureau. But by a method of interpolation to be explained below they have been reduced to simultaneousness with the others. Besides the results of these kite ascents we have had at our disposal the synoptic charts of the Weather Bureau for this day, but not the original observations from the stations at the earth's surface.

Table P contains for each of the kite-flights the calculated dynamic heights of the three lowest standard isobaric surfaces (first column under each station), and the mutual distances between these surfaces (second column under each station).

TABLE P.—*Dynamic heights of standard isobaric surfaces and mutual distances between them, United States, September 23, 1898.*

Station:	Blue Hill.		Cleveland.		Dodge City.		Knoxville.		North Platte.		Omaha.		Pierre.		Topeka.	
Dynamic height:	188		210		739		296		840		370		477		291	
Pressure (m-bars).																
800	1956		1882		1953		1997		1916		1958		1905		1958	
		975		970		1011		974		991		996		996		996
900	981		912		942		1023		925		962		909		962	
		886		888				894				888		879		904
1000	95		24				129				74		30		58	

From the figures contained in table P and from the charts of the Weather Bureau, the charts in figs. 13 and 14 have been drawn. The level curves used to represent the absolute topography of the standard isobaric surfaces (fig. 13) are drawn continuously where these surfaces run in the open air, while they are dotted where they represent only the ideal continuation of these surfaces below the earth. From a topographic chart the curves of intersection of the isobaric surfaces with the earth (heavy curves in fig. 13) have been obtained. The curves representing the mutual topography of successive standard isobaric surfaces are drawn continuously only as long as both surfaces run in the open air, while they are dotted as soon as the lower surface cuts the earth. The curves of intersection both of the upper and the lower surface are drawn as heavy curves, and the portion of land rising above the upper surface is shaded.

The topography of the 1000 m-bar surface (first chart of fig. 13) is derived from the isobaric chart of the Weather Bureau, table 19 A of the Appendix being used as explained in section 66. The charts of mutual topography (fig. 14) have been drawn directly from the figures of table P. The situation of the kite stations is marked on the first chart of fig. 13. In drawing the curves the observations from the two stations North Platte and Dodge City, where the ascents came 6 to 8 hours too late, have also been used, only with less attention given to them than to the others. The chart of absolute topography of the 900 m-bar surface (second chart of fig. 13) has been obtained by graphic addition of the first chart of fig. 13 and the first of fig. 14, and the chart of the 800 m-bar surface in the same manner by graphic addition of the second of fig. 13 and the second of fig. 14. Afterwards they have been corrected according to the absolute heights given in table P. As we have not had, as already mentioned, at our disposal the original observations from the stations at the earth's surface, we have refrained from every extrapolation from below.

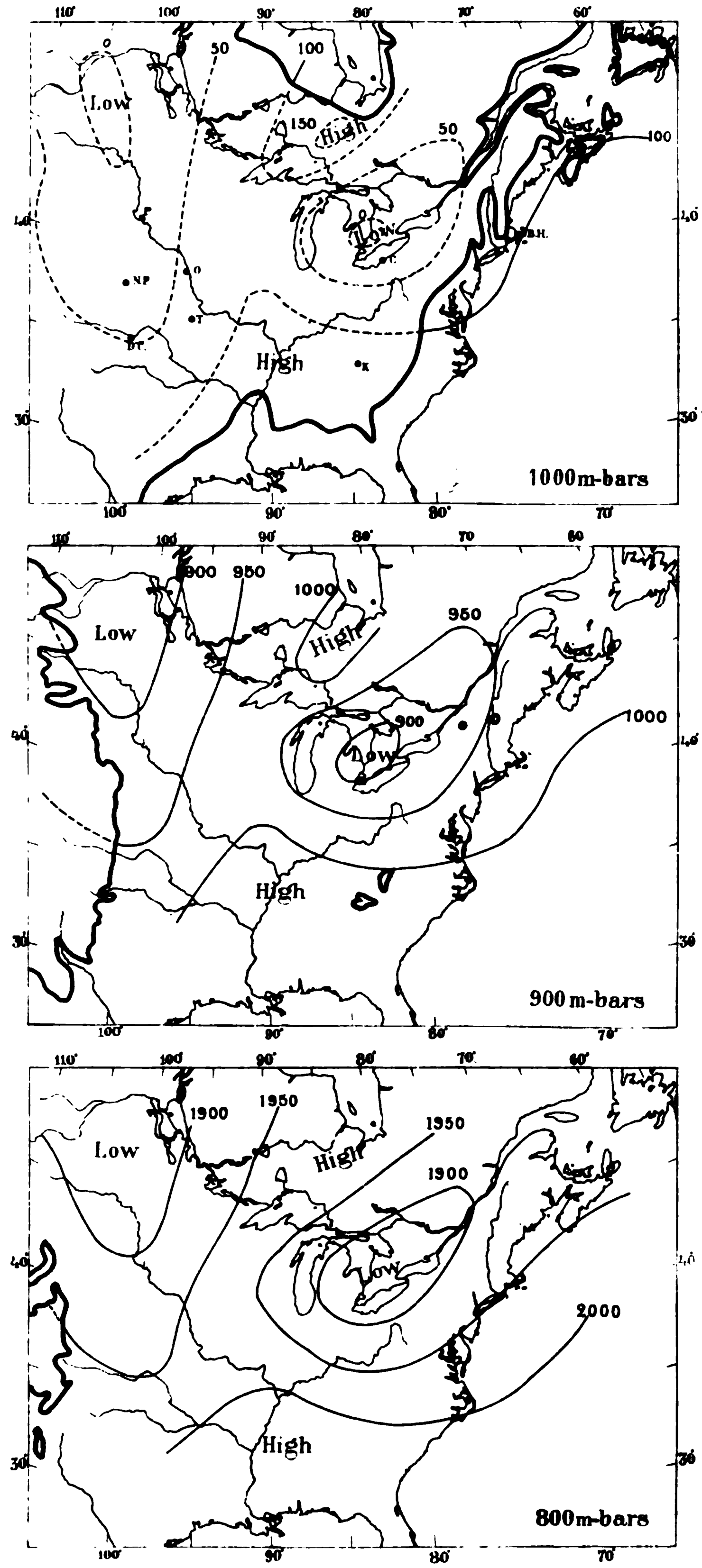


FIG. 13. Field of pressure, Sept. 23, 1898, represented by topographic charts giving the height of isobaric surfaces in dyn. meters.

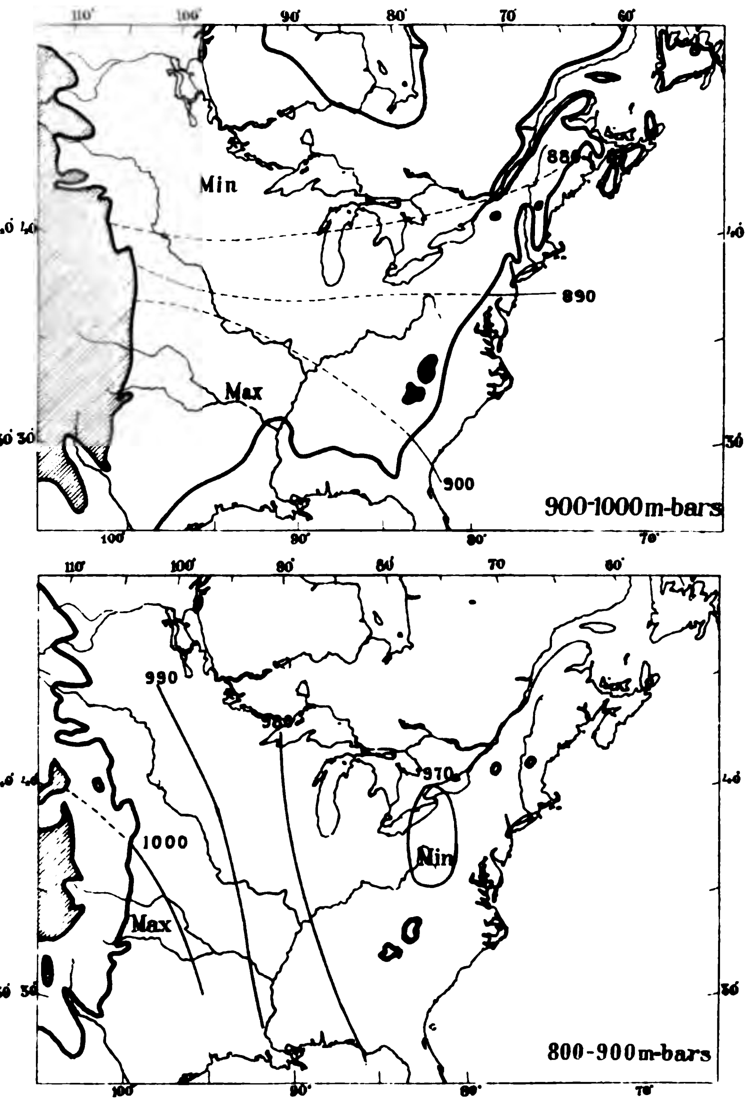


FIG. 14. Field of mass, Sept. 23, 1898, represented by charts giving the average specific volume (m^3/ton) of the air in isobaric sheets, or the thickness of these sheets in dyn. meters.

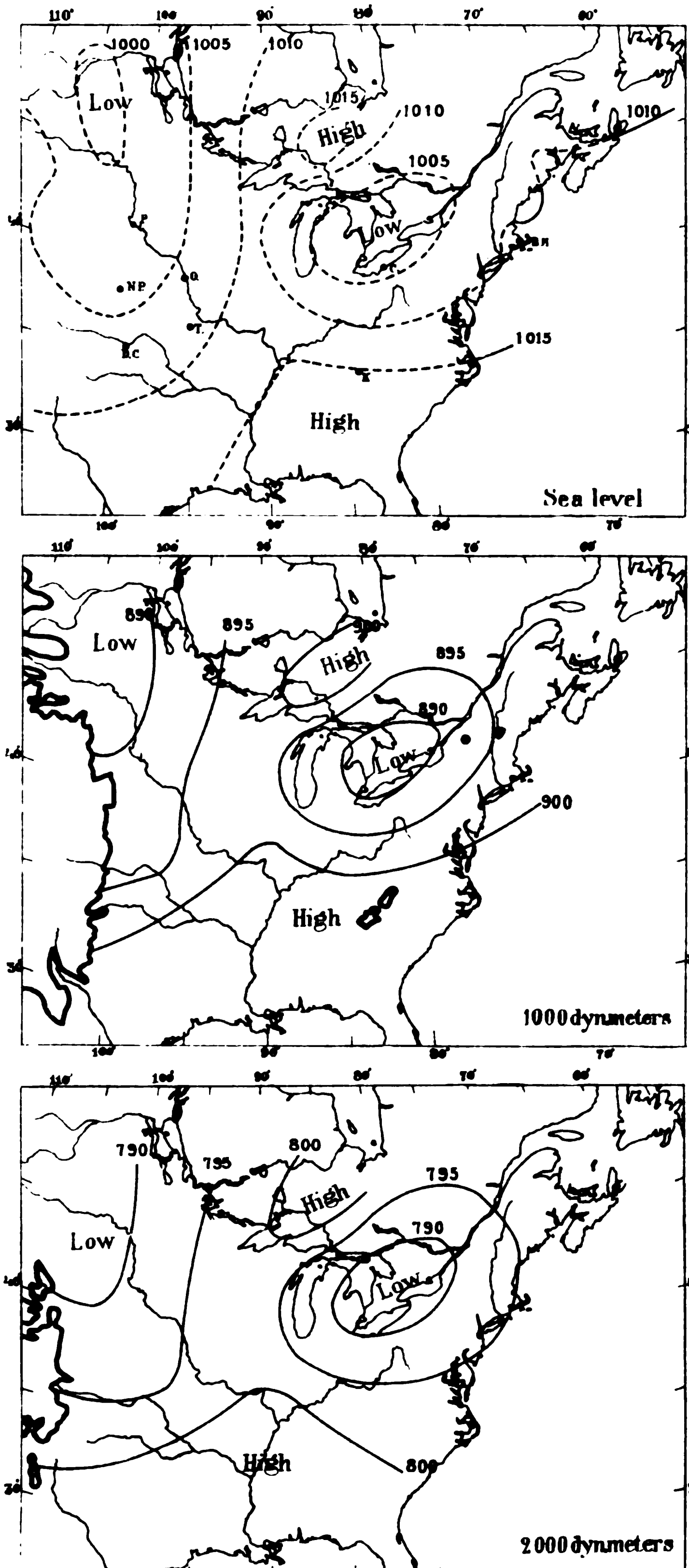


FIG. 15. Field of pressure, Sept. 23, 1898, represented by isobaric charts giving the pressure in level surfaces in m-bars.

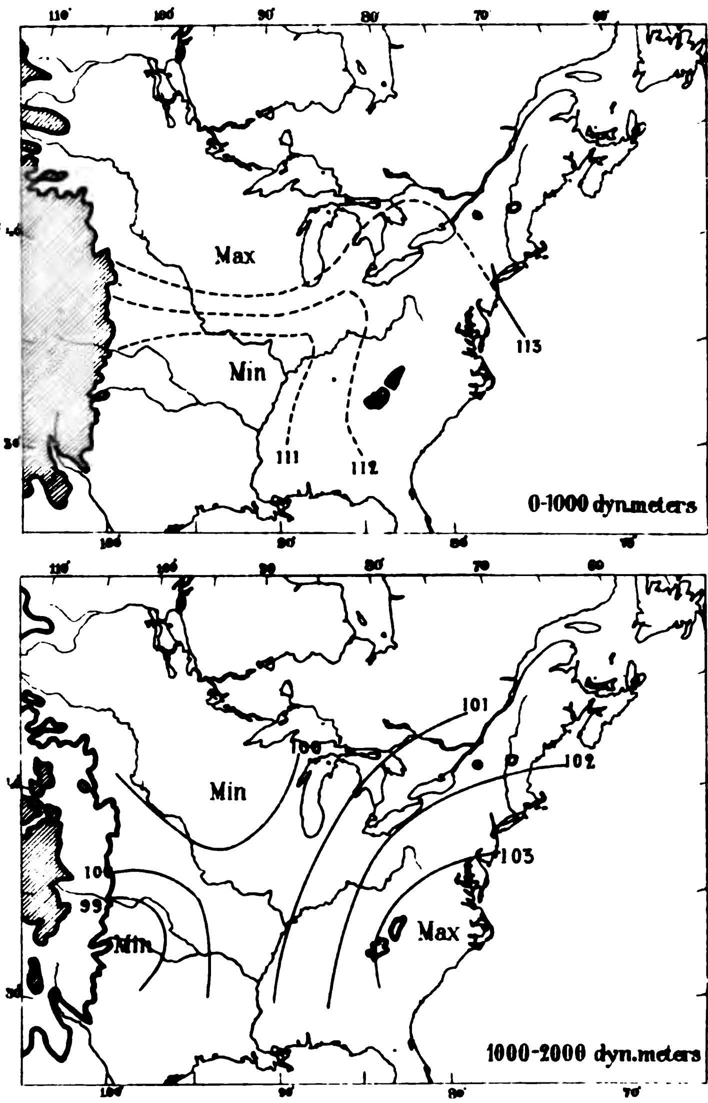


FIG. 16. Field of mass, Sept. 23, 1898, represented by charts giving the average density (10^{-6}ton/m³) of the air between level surfaces, or the difference of pressure between these surfaces in m-bars.

The 1000 m-bar surface rises to a height of 100 dynamic meters or more above the Atlantic Ocean, Hudson Bay, and the nearest parts of the coast; it cuts the earth's surface along a line running approximately parallel to the coast, and shows a marked depression in the region about the Great Lakes, where it goes down to or below sea-level. The 900 m-bar surface runs at an average height of 950 dynamic meters, and cuts the earth along the lower slope of the Rocky Mountains as well as about a few of the higher peaks of the Appalachian Mountains, which rise as islands above the surface. The 800 m-bar surface runs at a height of 1950 dynamic meters and cuts the earth only along the upper slope of the Rocky Mountains. Both show the same depression as the 1000 m-bar surface in the region about the Great Lakes.

The curves of the first chart, fig. 14, giving the mutual topography of the 900 m-bar and the 1000 m-bar surface, run mainly east and west, indicating a decreasing distance between the surfaces as we proceed from south to north. The curves of the next chart, giving the mutual topography of the 800 m-bar and the 900 m-bar surfaces, have a very different course, running mainly north and south, and indicating a decreasing distance between the surfaces as we proceed from west to east. Interpreted as charts of the distribution of mass in the standard isobaric sheets, the first shows decreasing specific volume, *i. e.*, increasing concentration of mass, as we proceed from south to north, while the second indicates a corresponding concentration of mass as we proceed from west to east, the greatest concentration apparently being found a little south of the greatest depression of the isobaric surfaces.

Fig. 17 is a vertical section showing the profile curves of the isobaric and the isosteric surfaces. This section is not, however, derived from the observations used in drawing the charts, but from the successive ascents performed at Blue Hill on each of the four days September 21 to 24. Supposing the cyclone to have moved during these days without undergoing any change in its interior constitution, the section obtained in this way would have given the same result as a set of simultaneous ascents from four properly chosen stations on any of these days. We waive the question as to the invariability of the cyclone during this time, and therefore also as to what approximation the four successive ascents from Blue Hill may be interpreted as four simultaneous ascents from different stations. The sections obtained by successive ascents from the same stations will always be of value in themselves, and in this case in enabling us to find by interpolation the state of the atmosphere above Blue Hill at the time of the Weather Bureau ascents September 23. The point marked *B. H.* indicates the vertical of the moving cyclone which was above Blue Hill at the time, and from its sections with the isobaric and the isosteric curves the numbers under the column Blue Hill in table P have been derived.

Table Q contains the result of the same kite ascents as table P, but worked out for the absolute pressures at given levels and the pressure differences from level to level. The corresponding synoptical representation of the state of the atmosphere is given in figs. 15, 16, and 18. The isobaric curves are drawn continuously or dotted according as they represent real pressure in the open air or ideal pressure below the earth's surface. The curves of intersection of the different levels with the earth's surface are drawn heavy.

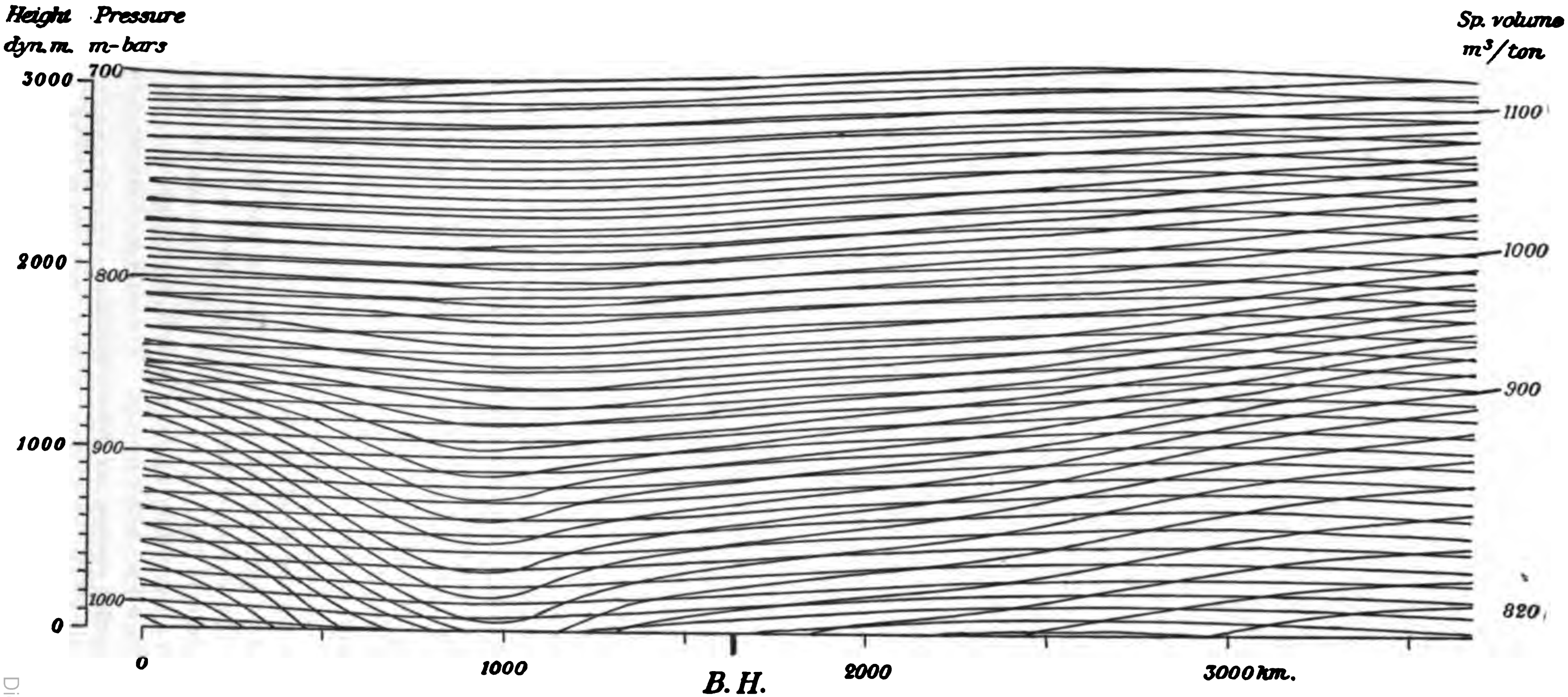


FIG. 17.—United States, September 23, 1898. Profile curves of isobaric and of isosteric surfaces. Every parallelogram represents 10 m. t. s. isobaric-isosteric unit-tubes.

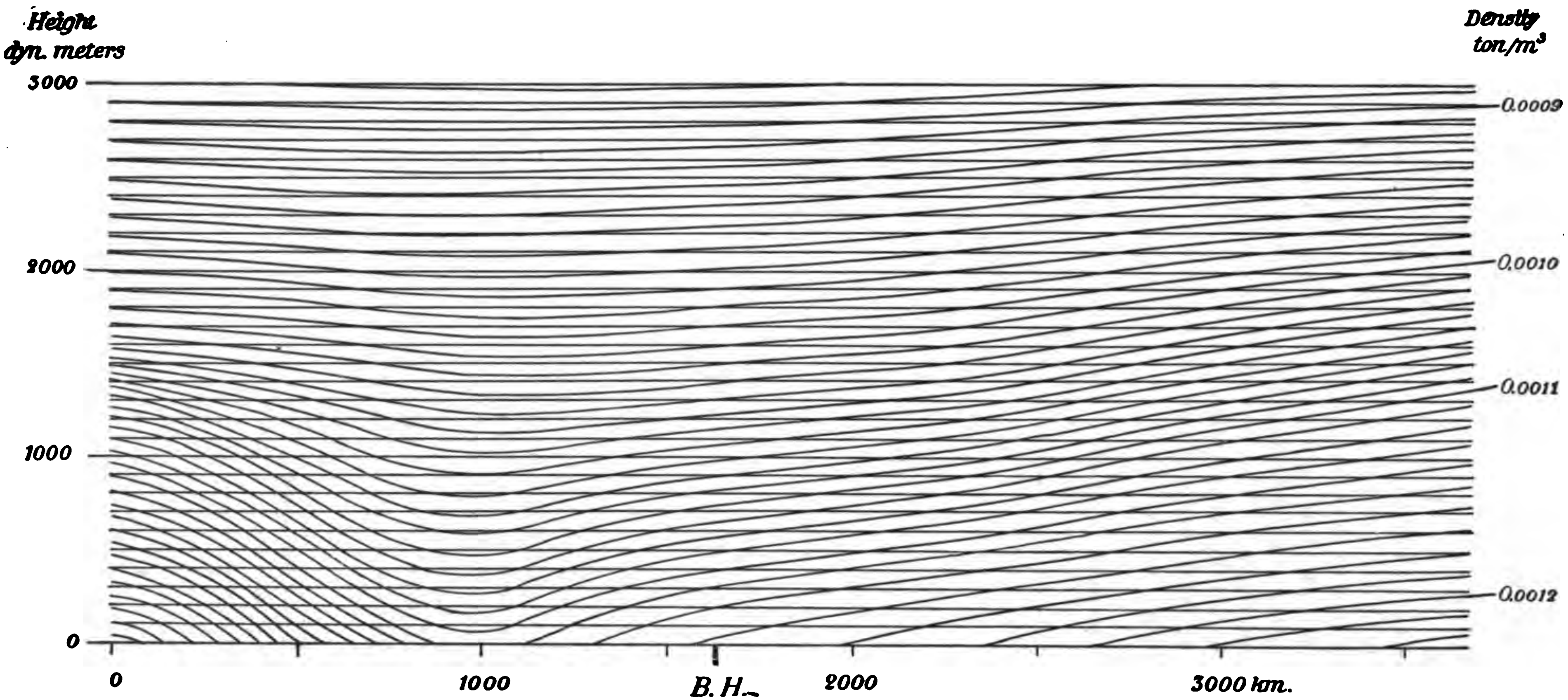


FIG. 18. — United States, September 23, 1898. Profile curves of equipotential and of isopycnic surfaces. Every parallelogram represents 10^{-8} m. t. s. equipotential-isopycnic unit-tubes.

The first chart of fig. 15 gives the pressure at sea-level. It is derived from the isobaric chart of the Weather Bureau, only changed by table 8 A of the Appendix from inches of mercury to millibars. The two charts of pressure-differences (fig. 16), from level 0 to that of 1000, and from this level to that of 2000 dynamic meters, are drawn from the figures in table Q. The curves of these charts are dotted from the point where the lower limiting surface of the sheet cuts the ground and stop where the upper surface cuts the ground, the part of the earth rising above this upper surface being shaded. The second chart of absolute pressure (fig. 15) is obtained by graphic addition of the first charts of fig. 15 and the first of fig. 16, and in the same way the third chart of absolute pressure is obtained by graphic addition of the second chart of fig. 15 and the second of fig. 16.

TABLE Q.—*Pressure (m-bars), in standard level surfaces, and differences of pressure between them. United States, September 23, 1898.*

Station:	Blue Hill.	Cleveland.	Dodge City.	Knoxville.	North Platte.	Omaha.	Pierre.	Topeka.
Dynamic height:	188	210	739	296	840	370	477	291
Height (dynamic meters).								
2000	795.8	788.6	795.6	799.7	791.9	796.0	790.9	796.0
	102.2	102.0	98.4	102.9	100.2	100.0	99.5	100.0
1000	898.0	890.6	894.0	902.6	892.1	896.0	890.4	896.0
	113.4	112.3		112.4		113.0	113.3	110.7
0	1011.4	1002.9		1015.0		1009.0	1003.7	1006.7

The three charts of absolute pressure show barometric depressions in the region about the Great Lakes. Interpreting the charts of pressure differences as charts of mass distribution, we get mainly the same results as from the corresponding charts of relative topography. Where these indicate a minimum of specific volume, those of relative pressure give a maximum of density, and *vice versa*.

Fig. 18 is a vertical section showing the profile curves of the level and the isopycnic surfaces. As in the corresponding section of fig. 17, these profile curves are drawn not from the simultaneous kite ascents at different stations, but from the successive ascents at Blue Hill. Thus if the cyclone has passed without undergoing any interior change (which can not be asserted), these profile-curves will correspond exactly to the same atmospheric state as that represented by the chart. But the construction of the section in this way was of special importance in enabling us to find graphically the absolute and relative pressures in the column Blue Hill in table Q.

71. Example 2. — Atmospheric Conditions over Europe, November 7, 1901. — In further illustration we shall consider a second example. It will differ from the preceding one by the greater completeness of the observations. On the one hand, the ascents have reached much greater heights, and on the other the observations from numerous stations at the earth's surface have been available to us in their original form, and not only after reduction to sea-level.

On November 7, 1901, in the morning and forenoon there ascended* from Paris one registering balloon with two instruments; from Strassburg two registering and one manned balloon; from Berlin two registering and one manned balloon; from Vienna one registering and two manned balloons. From St. Petersburg one registering balloon ascended on the following morning, November 8. This ascent has been treated as simultaneous with the others, our aim being only to exemplify the technics of our methods, not to discuss the true state of the atmosphere on this particular occasion.

TABLE R.—*Dynamic heights of standard isobaric surfaces and mutual distances between them, computed from ascents, Europe, November 7, 1901.*

Station: Latitude: Longitude: Dynamic height:	Paris. 48° 48′ 2° 29′ 48	Strassburg. 48° 35′ 7° 46′ 141	Berlin. 52° 30′ 13° 23′ 48	Wien. 48° 15′ 16° 21′ 198	Petersburg. 59° 56′ 30° 16′ 5
Pressure (m-bars).					
200	11427		11303		
	2491		2502		
300	8936	9057	8801	8947	8136
	1891	1941	1850	1883	1759
400	7045	7116	6951	7064	6377
	1563	1576	1563	1563	1428
500	5482	5540	5388	5501	4949
	1335	1340	1335	1345	1251
600	4147	4200	4053	4156	3698
	1159	1173	1155	1177	1106
700	2988	3027	2898	2979	2592
	1027	1047	1023	1035	993
800	1961	1980	1875	1944	1599
	926	937	923	933	899
900	1035	1043	952	1011	700
	838	841	841	850	820
1000	197	202	111	161	—120

The results of these ascents have been worked out according to the methods developed in the preceding chapter. In cases where two or more balloons have ascended from the same place, the observations from all the ascents have been introduced in the same diagram and the curve of virtual temperatures drawn so as to suit all observations as closely as possible, attention being paid to the different observations in proportion to their probable value. Thus as long as observations from manned balloons are available, the curve is drawn through points representing these observations. The distance of the observations of the registering balloons from this curve gives valuable corrections to the records of the registering balloon, which may be applied for the greater heights not reached by the manned balloons.

Table R contains the results of the ascents worked out as absolute heights of the standard isobaric surfaces and as distances from surface to surface, while table T contains the same results in the form of absolute pressures at standard levels and differences of pressure from level to level.

* Publications de la Commission Internationale pour l'Aérostation scientifique. Observations des ascensions internationales simultanées et des stations de montagne et de nuages 1901, pp. 390–410. Strassburg, 1903. The use of this important publication is very much impeded by the fact that the observations taken at the common meteorological stations on the days of the ascents are not accessible till many years afterwards, according as the meteorological year books of the different countries appear. This circumstance has forced us to choose our example from the first year of the international ascents, when the aeronautical stations were less numerous and the self-registering instruments less trustworthy than they are at present.

TABLE S.—*Dynamic heights of standard isobaric surfaces and mutual distances between them, computed by extrapolation from observations taken at the earth's surface, Europe, November 7, 1901.*

Station:	Angmagsalik.	Upernivik.	Jakobshavn.	Godthaab.	Ivigtut.	Vestmannö.	Styckisholm.	Grimsey.	Berufjord.	Thorshavn.
Latitude:	65° 36½'	72° 47'	69° 13'	64° 10½'	61° 12'	63° 26'	65° 5'	66° 33'	64° 40'	62° 2½'
Longitude:	37° 33½' W.	56° 7' W.	51° 2' W.	51° 43½' W.	48° 10' W.	20° 15' W.	22° 46' W.	18° W.	14° 19' W.	6° 45' W.
Dynamic height:	31	13	12	9	5	8	11	7	18	9
Pressure (m-bars).										
800	1791	1851	1812	1721	1700	1896	1857	1838	1855	1896
	903	883	903	909	919	928	917	919	936	934
900	888	968	909	812	781	968	940	919	919	962
	816	855	817	823	830	838	830	830	847	845
1000	72	113	92	—11	—49	130	110	89	72	117

Station:	Vardö.	Alten.	Bodö.	Brönnö.	Christiansund.	Florö.	Bergen.	Skudesnes.	Mandal.	Färder.
Latitude:	70° 22'	69° 58'	67° 17'	65° 28'	63° 7'	61° 36'	60° 23'	59° 9'	58° 2'	59° 2'
Longitude:	31° 8'	23° 15'	14° 24'	12° 13'	7° 45'	5° 2'	5° 21'	5° 16'	7° 27'	10° 32'
Dynamic height:	10	10	7	10	22	8	21	4	5	9
Pressure (m-bars).										
800	1534	1573	1664	1718	1805	1853	1856	1874	1847	1810
	915	909	909	919	920	925	925	931	925	928
900	619	664	755	799	885	928	931	943	922	882
	830	826	826	831	832	837	837	843	837	842
1000	—211	—162	—71	—32	53	91	94	100	85	40

Station:	Christiania.	Dovre.	Karesuando.	Gellivara.	Iockmock.	Haparanda.	Stensele.	Östersund.	Hernösand.	Falun.
Latitude:	59° 55'	62° 5'	68° 26'	67° 8'	66° 36'	65° 50'	65° 4'	63° 11'	62° 38'	60° 37'
Longitude:	10° 43'	9° 7'	22° 30'	20° 40'	19° 51'	24° 9'	17° 11'	14° 38'	17° 57'	15° 38'
Dynamic height:	24	631	325	358	253	9	321	308	15	114
Pressure (m-bars).										
700		2784	2565	2621	2630		2667	2718		
		1014	992	998	1006		1020	1024		
800	1771	1770	1573	1623	1624	1597	1647	1694	1646	1700
	916	908	895	897	903	919	914	918	918	916
900	855	862	678	726	721	678	733	776	728	784
	827	820	812	816	822	817	826	823	830	830
1000	28	42	—134	—90	—101	—139	—93	—47	—102	—46

Station:	Upsala.	Askersund.	Visby.	Vexiö.	Skagen.	Vestervig.	Fanö.	Kjöbenhavn.	Hammershus.	Nikolaistad.
Latitude:	59° 52'	58° 53'	57° 39'	56° 33'	57° 44'	56° 47'	55° 27'	55° 41'	55° 17'	63° 4'
Longitude:	17° 38'	14° 55'	18° 18'	14° 49'	10° 38'	8° 20'	8° 24'	12° 36'	14° 38'	21° 40'
Dynamic height:	23	94	16	164	3	25	5	13	15	
Pressure (m-bars).										
800	1704	1738	1723	1775	1827	1860	2008	1825	1815	1605
	925	912	929	920	935	935	968	933	934	909
900	779	826	794	855	892	925	1040	892	881	696
	826	828	841	833	846	845	873	845	845	826
1000	—47	—2	—47	22	46	80	167	47	36	—130

Station:	Kajana.	Tammerfors.	Hangö.	Valencia.	Aberdeen.	Falmouth.	Kew.	Deerness.	Landale.	Scarborough.
Latitude:	64° 14'	61° 30'	59° 46'	51° 56'	57° 10'	50° 9'	51° 28'	58° 56'	56° 41'	54° 18'
Longitude:	27° 44'	23° 45'	22° 57'	10° 15' W.	2° 6' W.	5° 4' W.	0° 19 W.	2° 45' W.	5° 41' W.	0° 24' W.
Dynamic height:	131	89	5	13	26	55	10	52	7	38
Pressure (m-bars).										
800	1596	1618	1650	1966	1958	1988	1964	1963	2012	1999
	908	910	913	925	930	937	927	947	953	948
900	688	708	737	1041	1028	1051	1037	1016	1059	1051
	827	828	836	831	855	844	835	854	858	855
1000	—139	—120	—99	210	173	207	202	162	201	196

TABLE S.—*Dynamic heights of standard isobaric surfaces and mutual distances between them, computed by extrapolation from observations taken at the earth's surface, Europe, November 7, 1901*—Continued.

Station:	Croukbourne.	Hillington.	Churchstoke.	St. David's Peirbroke.	Birr Castle.	Ben Nevis.	Parc du St. Maur.	Tour Eiffel.	Nantes.	Besançon.
Latitude:	54° 10′	52° 48′	52° 31′	51° 53′	53° 6′	56° 48′	48° 48′	48° 52′	47° 15′	47° 15′
Longitude:	4° 29′ W.	0° 33′	3°.5′ W.	5° 16′ W.	7° 55′ W.	5° W.	2° 29′	2° 17′	1° 34′ W.	5° 59′
Dynamic height:	42	28	164	66	53	1318	48	329	41	305
Pressure (m-bars).										
700						2995				
						1044				
800	2030	1971	2000	2036	2012	1951	1934	2022	1957	1937
	956	934	937	954	938	928	914	947	920	908
900	1074	1037	1063	1082	1074	1023	1020	1075	1037	1029
	862	841	843	859	846		818	849	829	815
1000	212	196	220	223	228		202	226	208	214

Station:	Puy de Dôme.	Puy de Dôme.	Lyon.	Pic du Midi.	Perpignan.	Saint Honorine du Fay.	Toulouse.	Marseille.	Brest.	Langres.
Latitude:	45° 46′	45° 46′	45° 42′	42° 56′	42° 42′	49° 5′	43° 37′	43° 18′	48° 23′	47° 52′
Longitude:	3° 5′	2° 58′	4° 47′	0° 8′	2° 53′	0° 30′ W.	1° 27′	5° 23′	4° 30′ W.	5° 20′
Dynamic height:	380	1438	293	2793	31	116	190	73	64	457
Pressure (m-bars).										
600				4182						
				1183						
700		3038		2999						
		1059		1030						
800	1938	1979	1974	1969	2000	1947	1966	1976	1969	1982
	911	931	932		957	916	941	930	927	937
900	1027	1048	1042		1043	1031	1025	1046	1042	1045
	817		827		862	824	844	867	835	838
1000	210		215		181	207	181	179	207	207

Station:	Mont Ventoux.	Bagnères de Bigorre.	Mont Aigoual.	Helder.	Vlissingen.	Bern.	Genf.	Zürich.	Rigi Kulm.	Sils-Maria.
Latitude:	44° 10′	43° 4′	44° 7′	52° 58′	51° 26′	46° 57′	46° 12′	47° 23′	47° 3′	46° 26′
Longitude:	4° 57′	0° 9′	3° 35′	4° 44′	3° 36′	7° 26′	6° 9′	8° 33′	8° 30′	9° 46′
Dynamic height:	1862	536	1524	5	8	560	397	483	1751	1773
Pressure (m-bars).										
600	4148								4224	4169
	1172								1198	1165
700	2976	2953	3028			2992			3026	3004
	1025	1011	1055			1026			1051	1023
800	1951	1942	1973	1999	1980	1966	1989	1976	1975	1981
	922	915	931	961	932	918	937	926	931	906
900	1029	1027	1042	1038	1048	1048	1052	1050	1044	1075
		830		865	857	822	832	831		
1000		197		173	191	226	220	219		

Station:	Castasegna.	Lugano.	Basel.	Säntis.	Memel.	Neufahrwasser.	Swinemünde.	Hamburg.	Margrabowa.	Breslau.
Latitude:	46° 20′	46° 0′	47° 33′	47° 15′	55° 43′	54° 24′	53° 56′	53° 33′	54° 2′	51° 7′
Longitude:	9° 31′	8° 57′	7° 35′	9° 20′	21° 7′	18° 40′	14° 16′	9° 59′	22° 30′	17° 2′
Dynamic height:	686	269	273	2451	12	4	10	25	159	144
Pressure (m-bars).										
600				4218						
				1193						
700	3031			3025						
	1065			1038						
800	1966	1973	1958	1987	1725	1772	1839	1893	1773	1876
	928	929	919		936	930	939	937	929	923
900	1038	1044	1039		789	842	900	956	844	953
	830	834	826		847	835	843	847	840	838
1000	208	210	213		−58	7	57	109	4	115

TABLE S.—*Dynamic heights of standard isobaric surfaces and mutual distances between them, computed by extrapolation from observations taken at the earth's surface, Europe, November 7, 1901*—Continued.

Station:	Ratibor.	Nordhausen.	Helgoland.	Aachen.	Eichberg.	Schneekoppe.	Wasserleben.	Brocken.	Potsdam.	Strassburg.
Latitude:	50° 6′	51° 30′	54° 10′	50° 47′	50° 55′	50° 44′	51° 56′	51° 48′	52° 23′	48° 35′
Longitude:	18° 13′	10° 48′	7° 51′	6° 6′	15° 48′	15° 44′	10° 45′	10° 37′	13° 4′	7° 46′
Dynamic height:	197	214	41	201	332	1579	153	1126	83	141
Pressure (m-bars).										
600						4075				
						1166				
700						2909		2926		
						1037		1031		
800	1886	1930	1928	1965	1882	1872	1916	1895	1886	1946
	918	938	950	937	926	924	935	921	933	919
900	968	992	978	1028	956	948	981	974	953	1027
	833	841	855	841	839		847		845	822
1000	135	151	123	187	117		134		108	205

Station:	Mülhausen.	Gr. Belchen.	Karlsruhe.	Villingen.	Höckenswand.	Hohenheim.	Pilatus.	Untersberg.	Zugspitze.	Schmittenhöhe.
Latitude:	47° 45′	47° 53′	49° 1′	48° 4′	47° 44′	48° 43′	46° 59′	47° 43′	47° 25′	47° 20′
Longitude:	7° 10′	7° 6′	8° 27′	8° 27′	8° 10′	9° 13′	8° 16′	12° 2′	11° 59′	12° 44′
Dynamic height:	237	1366	124	701	986	304	2027	1632	2907	1928
Pressure (m-bars).										
500									5572	
									1373	
600							4227	4195	4199	4214
							1197	1197	1180	1191
700		3041		2916	2998		3030	2998	3019	3023
		1056		992	1028		1046	1052	1028	1043
800	1952	1985	1949	1924	1970	1841	1984	1946	1991	1980
	919	939	921	889	921	914		929		926
900	1033	1046	1028	1035	1049	927		1017		1054
	825		829	805	836	820				
1000	208		199	230	213	107				

Station:	Schneeberg.	Kuttenplan.	Budweis.	Lemberg.	Czernowitz.	Bregenz.	Innsbruck.	Salzburg.	Sonnblick.	Obir.
Latitude:	47° 45′	49° 54′	48° 58′	49° 50′	48° 17′	47° 30′	47° 16′	47° 48′	47° 3′	46° 30′
Longitude:	15° 50′	12° 43′	14° 28′	24° 1′	25° 56′	9° 45′	11° 24′	13° 2′	12° 57′	14° 29′
Dynamic height:	1414	513	381	301	237	431	560	420	3044	2001
Pressure (m-bars).										
500									5554	
									1380	
600									4174	4223
									1180	1198
700	3047	2924					2975		2994	3025
	1067	1015					1020			1046
800	1980	1909	1930	1876	1875	1984	1955	1944		1979
	927	909	930	919	911	927	910	927		
900	1053	1000	1000	957	964	1057	1045	1017		
		823	833	828	818	829	816	829		
1000		177	167	129	146	228	229	188		

Station:	Wien.	Riva.	Beirut.	Moskwa.	Kola.	Mesen.	Kem.	Arkangelsk.	Valaam.	Povenetz.
Latitude:	48° 15′	45° 53′	33° 54′	55° 49′	68° 53′	65° 50′	64° 57′	64° 33′	61° 23′	62° 51′
Longitude:	16° 21′	10° 50′	35° 29′	37° 34′	33° 1′	44° 16′	34° 39′	40° 32′	30° 57′	34° 49′
Dynamic height:	198	88	33	167	6	14	12	6	36	42
Pressure (m-bars).										
800	1904	2000	1982	1765	1533	1575	1565	1574	1618	1618
	909	935	951	915	909	884	908	894	922	913
900	995	1065	1031	850	624	691	657	680	696	705
	835	847	856	828	825	804	827	815	836	830
1000	160	218	175	22	−201	−113	−170	−135	−140	−125

TABLE S.—*Dynamic heights of standard isobaric surfaces and mutual distances between them, computed by extrapolation from observations taken at the earth's surface, Europe, November 7, 1901*—Continued.

Station:	Kargopol.	Nikolsk.	Vologda.	Pernov.	Velikie Louki.	Vichni Volotchek.	Viatka.	Sarapoul.	Bogoslowsk.	Ekaterinburg.
Latitude:	61° 30′	59° 32′	59° 14′	58° 23′	56° 21′	57° 35′	58° 36′	56° 28′	59° 45′	56° 50′
Longitude:	38° 57′	45° 27′	39° 53′	24° 30′	30° 31′	34° 34′	49° 41′	53° 49′	60° 1′	60° 38′
Dynamic height:	123	148	119	9	102	164	158	116	186	280
Pressure (m-bars).										
800	1619	1683	1676	1671	1712	1704	1709	1731	1653	1725
	909	899	909	920	916	916	881	851	827	852
900	710	784	767	751	796	788	828	880	826	873
	825	814	826	837	830	830	799	768	749	770
1000	—115	—30	—59	—86	—34	—42	29	112	77	103

Station:	Vilno.	Smolensk.	Nischni Novgorod.	Zlatoounst.	Oufa.	Orenburg.	Vlotslavsk.	Novaja Alexandria.	Vasilevitchi.	Pinsk.
Latitude:	54° 41′	54° 47′	56° 20′	55° 10′	54° 43′	51° 45′	52° 40°	51° 25′	52° 16′	52° 7′
Longitude:	25° 18′	32° 4′	44° 0′	59° 41′	55° 56′	55° 6′	19° 4′	21° 57′	29° 48′	26° 6′
Dynamic height:	145	216	155	449	171	111	64	144	135	139
Pressure (m-bars).										
700				2749						
				973						
800	1738	1763	1765	1776	1767	1843	1818	1846	1792	1811
	925	914	912	872	857	891	925	927	912	914
900	813	849	853	904	910	952	893	919	880	897
	831	825	824	790	776	807	837	838	825	827
1000	—18	24	29	114	134	145	56	81	55	70

Station:	Orel.	Elatma.	Penza.	Polibino.	Ploti.	Ounan.	Loubny.	Koursk.	Kharkow.	Sagonny.
Latitude:	52° 58′	54° 58′	53° 11′	53° 44′	47° 57′	48° 45′	50° 1′	51° 45′	50° 0′	50° 36′
Longitude:	36° 4′	41° 45′	45° 1′	52° 56′	29° 10′	30° 13′	33° 2′	36° 12′	36°.14′	39° 43′
Dynamic height:	179	137	214	106	140	212	162	231	137	202
Pressure (m-bars).										
800	1819	1776	1846	1780	1847	1874	1903	1845	1895	1865
	907	903	907	867	886	901	923	905	913	898
900	912	873	939	913	961	973	980	940	982	967
	820	818	822	784	804	818	830	821	820	807
1000	92	55	117	129	157	155	150	119	162	160

Station:	Saratow.	Rostrow am Don.	Akktouba.	Astrakhan.	Totaikoi.	Magaratch.	Obdorsk.	Sourgout.	Tioumen	Ouralsk.
Latitude:	51° 32′	47° 13′	48° 18′	46° 21′	44° 54′	44° 32′	66° 31′	61° 17′	57° 10′	51° 12′
Longitude:	46° 3′	39° 43′	46° 9′	48° 2′	34° 11′	34° 13′	66° 35′	73° 20′	65° 32′	51° 22′
Dynamic height:	58	47	11	—14	297	78	26	43	81	37
Pressure (m-bars).										
800	1900	1922	1908	1929	1903	1982	1581	1619	1710	1882
	920	901	895	899	892	933	814	839	856	906
900	980	1021	1013	1030	1011	1049	767	780	854	976
	827	811	809	811	799	839	740	760	776	818
1000	153	210	204	219	212	210	27	20	78	158

Station:	Omsk.	Stavropol.	Novorossisk.	Goudaour.	Tiflis.	Novo Bajazet.	Choucha.	Leukoran.	Askhabad.	Taschkent.
Latitude:	54° 58′	45° 3′	44° 44′	42° 28′	41° 43′	40° 20′	39° 46′	38° 46′	37° 57′	41° 20′
Longitude:	73° 23′	41° 59′	37° 49′	44° 28′	44° 48′	45° 7′	46° 45′	48° 52′	58° 23′	69° 18′
Dynamic height:	88	563	36	2160	396	1940	1340	—20	221	469
Pressure (m-bars).										
600				4168		4212				
				1170		1185				
700		2929		2998		3027	3039			
		1004		1022		1035	1044			
800	1688	1925	1941	1976	1987	1992	1995	2014	2018	1999
	854	899	914		927	920	923	942	944	969
900	834	1026	1027		1060	1072	1072	1072	1074	1030
	773	805	822		829			848	847	849
1000	61	221	205		231			224	227	181

TABLE S.—*Dynamic heights of standard isobaric surfaces and mutual distances between them, computed by extrapolation from observations taken at the earth's surface, Europe, November 7, 1901*—Continued.

Station:	Samarkand.	Derkoulskoe Verderie.	Marionpolskoe.	Kobi.	Krestovaja.	Madrid.	Coimbra.	San Fernando.	Turin.	Riposto.
Latitude:	39° 39′	49° 3′	47° 39′	42° 34′	42° 30′	40° 28′	40° 12′	36° 28′	45° 5′	37° 41′
Longitude:	66° 57′	39° 48′	37° 30′	44° 31′	44° 27′	3° 41′ W.	8° 25′ W.	6° 12′ W.	7° 42′	15° 14′
Dynamic height:	704	152	274	1957	2332	729	138	28	271	14
Pressure (m-bars).										
600				4173	4143					
				1171	1171					
700	3084			3002	2972	3062				
	1071			1025	1019	1041				
800	2013	1893	1915	1977	1953	2021	1956	1932	1951	2069
	964	899	892	900		930	954	955	910	971
900	1049	994	1023	1077		1091	1002	977	1041	1098
	858	811	808			849	862	864	828	876
1000	191	183	215			242	140	113	213	222

Station:	Aetna.	Prag.	Trieste.	Lizza.	Punta d'Ostro.	Mostar II.	Bjelasnica.	Sarajevo II.	Bihac.	Kupres.
Latitude:	37° 41′	50° 5′	45° 39′	43° 5′	42° 7′	43° 20′	43° 42′	43° 52′	44° 49′	44° 0′
Longitude:	15° 0′	14° 25′	13° 46′	16° 14′	18° 34′	17° 29′	18° 15′	18° 26′	15° 52′	17° 17′
Dynamic height:	2890	193	25	23	63	58	2026	548	222	1166
Pressure (m-bars).										
600	4272						4179			
	1206						1177			
700	3066						3002			3018
	1048						923			1047
800	2018	1908	1996	2027	2029	1986	2079	1959	1913	1971
		923	945	963	958	932		906	915	926
900		985	1051	1064	1071	1054		1053	998	1045
		837	850	867	862	840		826	821	
1000		148	201	197	209	214		227	177	

Station:	Kolozvar.	O'Gyalla.	Sepsi Szt-Gyorgy.	Turkeve.	Ungvár.	Zsombolya.	Pola.	Jerusalem.	Oran.	Alger.
Latitude:	46° 46′	47° 52′	45° 53′	47° 7′	48° 36′	45° 47′	44° 51′	31° 48′	35° 42′	36° 47′
Longitude:	23° 36′	18° 12′	25° 48′	20° 45′	22° 18′	20° 43′	13° 51′	35° 11′	0° 39′ W.	3° 4′
Dynamic height:	331	117	517	86	116	80	31	733	59	38
Pressure (m-bars).										
700			2979					3056		
			1033					1082		
800	1934	1907	1946	1932	1943	1930	1968	1974	1921	1926
	910	912	925	914	930	910	920	964	964	967
900	1024	995	1021	1018	1013	1020	1048	1010	957	959
	823	822	826	824	835	820	843	865	871	875
1000	201	173	195	194	178	200	205	145	86	84

Station:	Bizerte.	El-Djem.	Saida.	Fort National.	Geryville.	Laghouat.	Ouargla.	Le Krey.	Ismailia.
Latitude:	37° 17′	35° 21′	34° 51′	36° 38′	33° 41′	33° 48′	31° 55′	33° 49′	30° 36′
Longitude:	9° 50′	10° 38′	0° 10′	4° 12′	1° 00′	2° 53′	5° 10′	35° 40′	32° 16′
Dynamic height:	9	162	848	898	1278	737	153	995	9
Pressure (m-bars).									
700			2985	2982	2990	2967		3087	
			1061	1074	1063	1068		1077	
800	1991	2028	1924	1908	1927	1899	2009	2010	2034
	972	988	958	960	950	948	990	956	980
900	1019	1040	966	948	977	951	1019	1054	1054
	880	889	863	870		862	879	859	883
1000	139	151	103	78		89	140	195	171

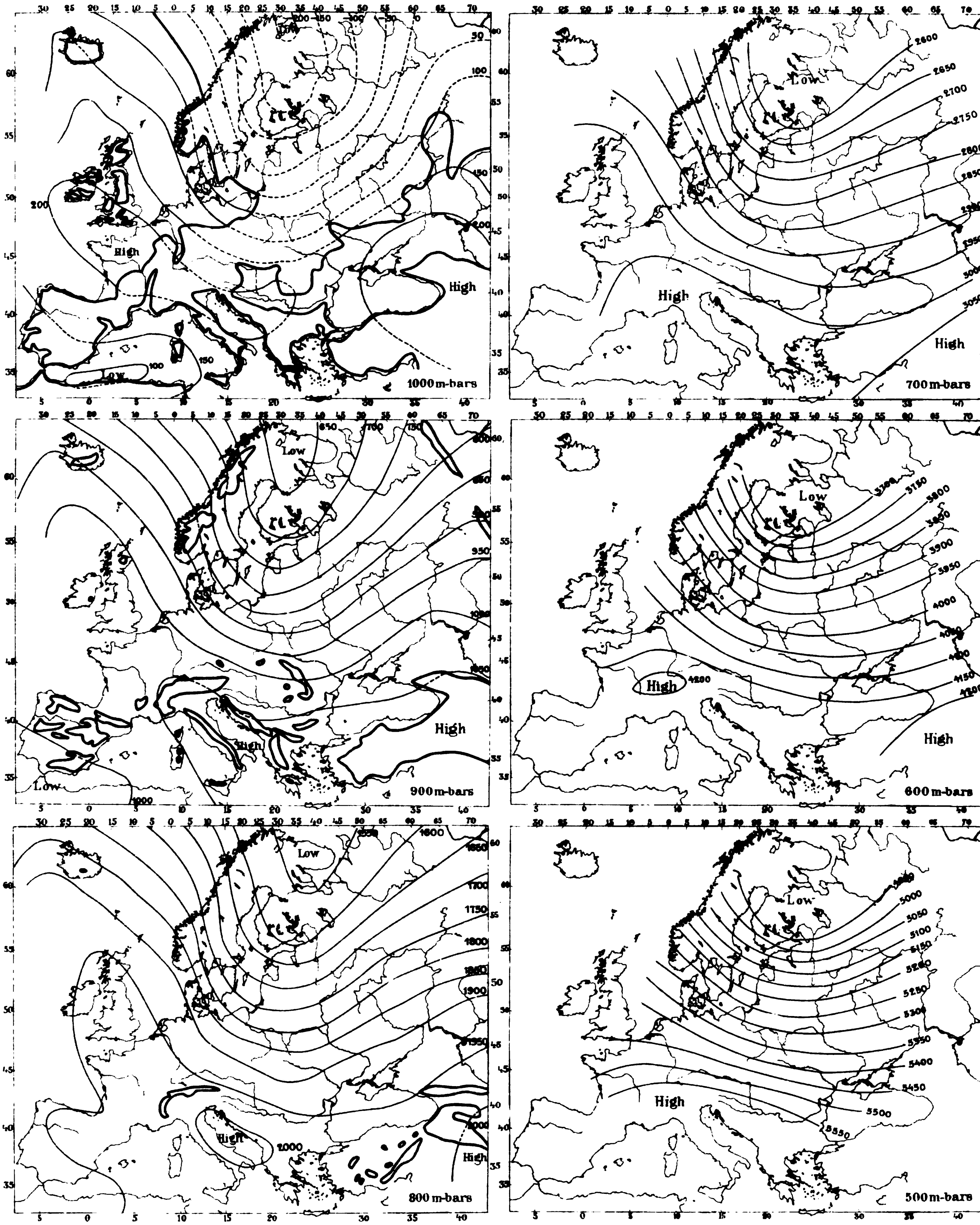


FIG. 19. Field of pressure, Nov. 7, 1901, represented by topographic charts giving the height of isobaric surfaces in dyn. meters.

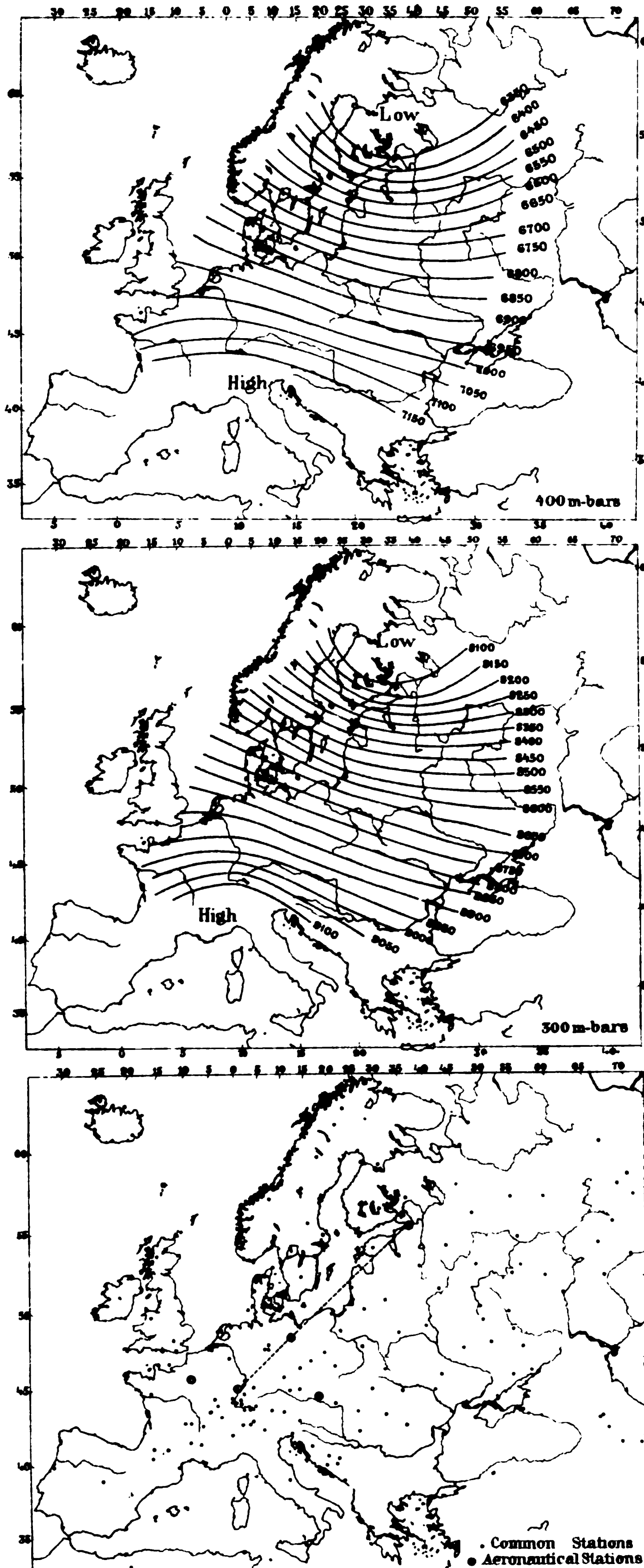


FIG. 19. (Continued). Field of pressure, Nov. 7, 1901. The last chart gives the situation of the meteorological stations.

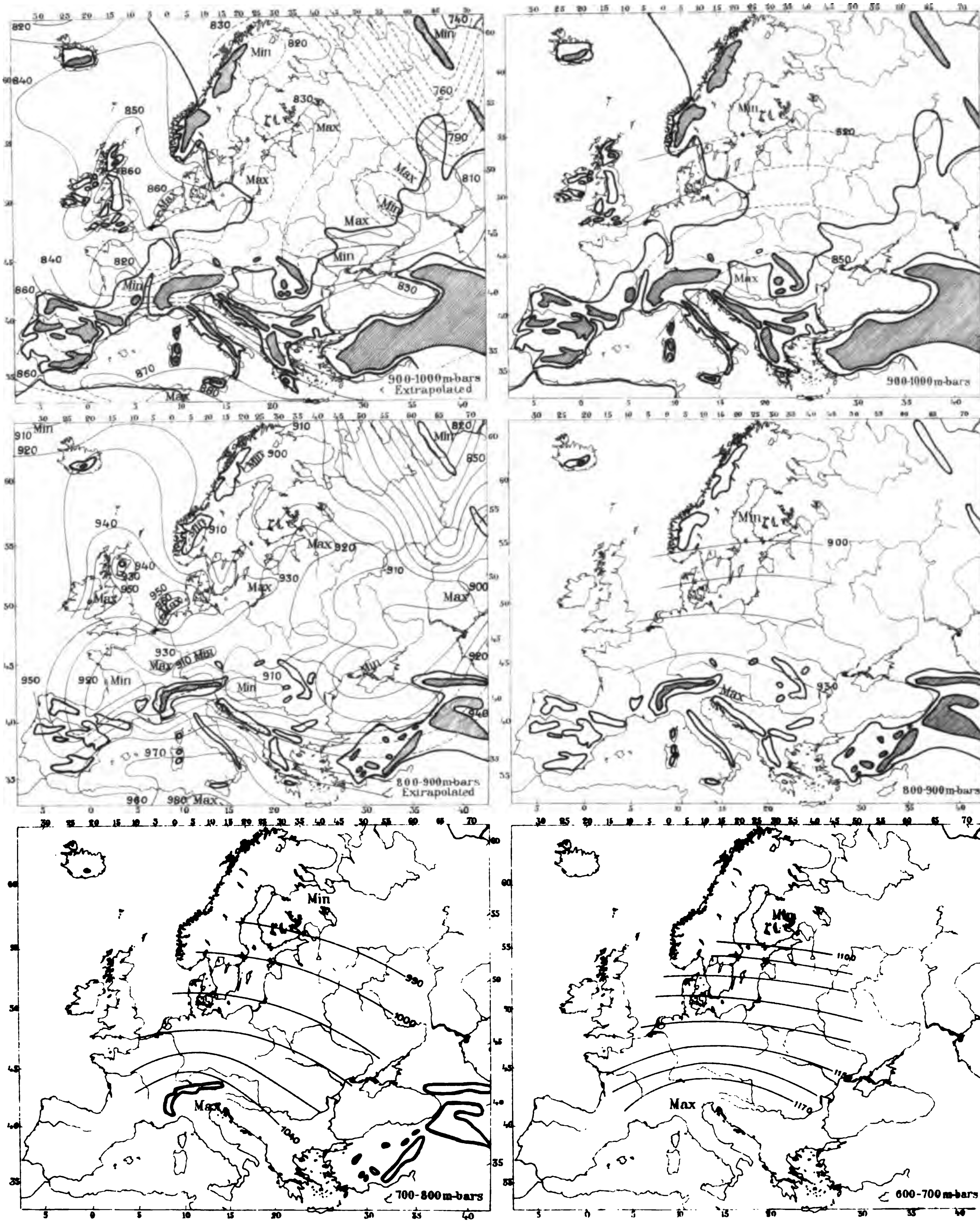


FIG. 20. Field of mass, Nov. 7, 1901, represented by charts giving the average specific volume (m^3/ton) of the air in isobaric sheets, or the thickness of these sheets in dyn. meters.

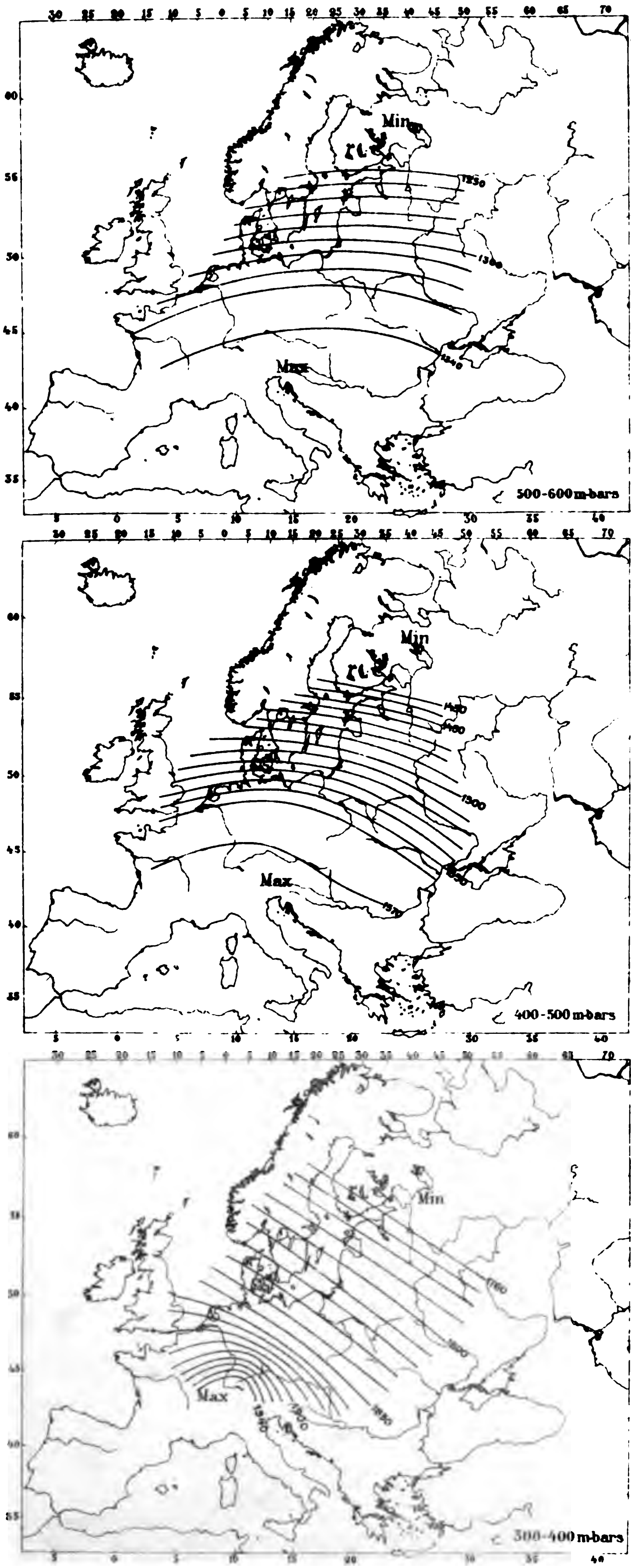


FIG. 20. (Continued). Field of mass, Nov. 7, 1901

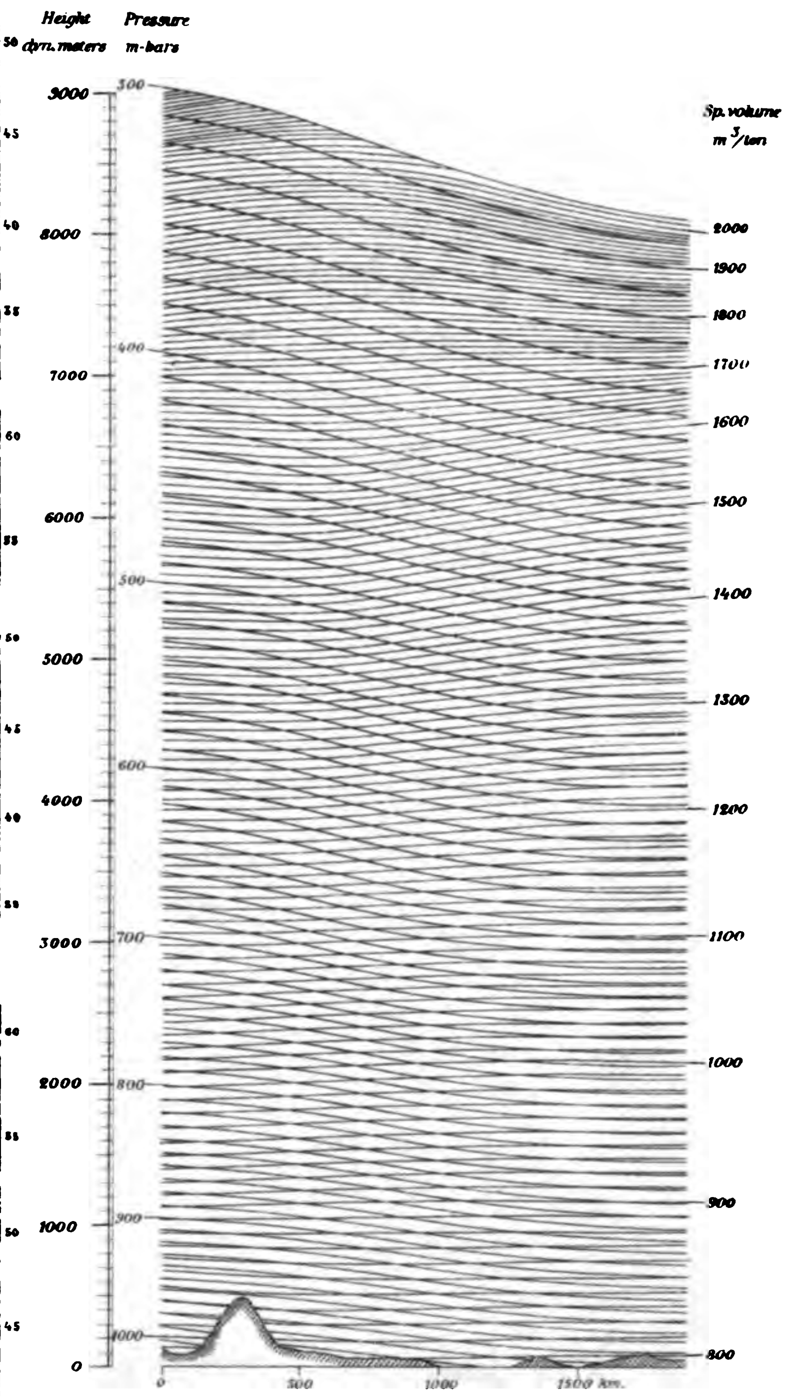


FIG. 21. Profile curves of isobaric and of isosteric surfaces. Each parallelogram represents 10 m.t.s. isobaric-isosteric unit tubes.

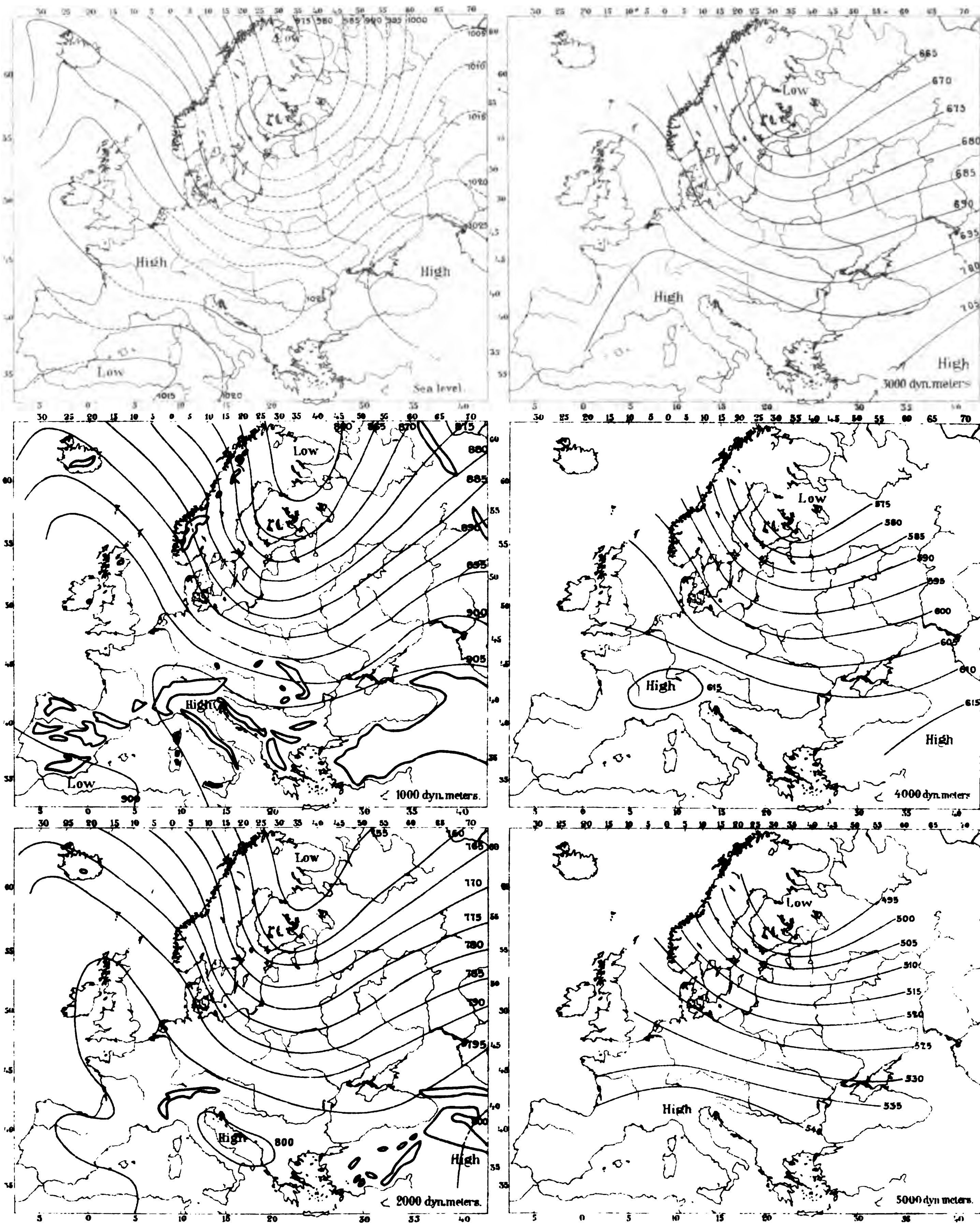


FIG. 22. Field of pressure, Nov. 7, 1901, represented by isobaric charts giving the pressure in level surfaces in m-bars.

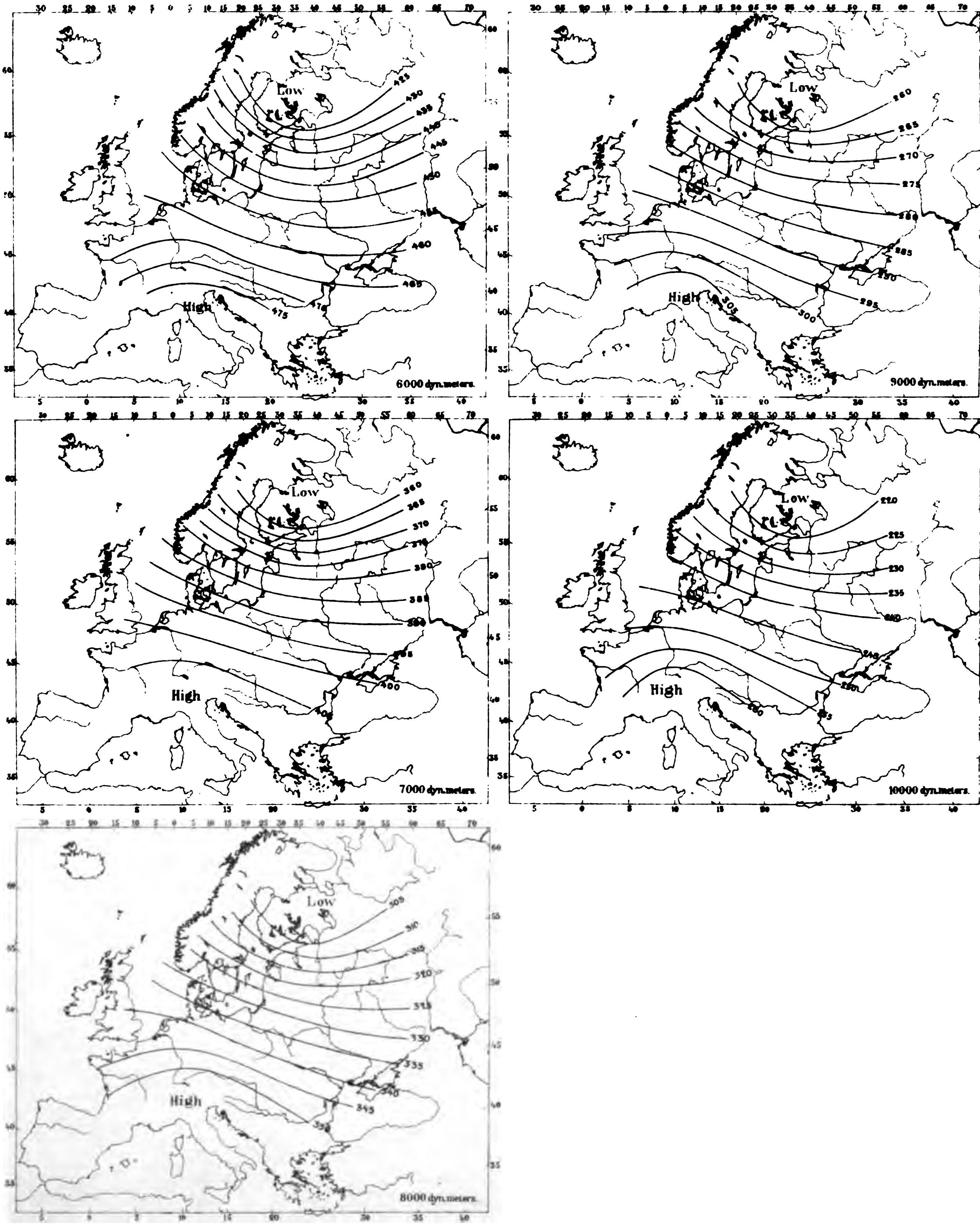


Fig. 22 (Continued). Field of pressure, Nov. 7, 1901, represented by isobaric charts giving the pressure in level surfaces in m-bars.

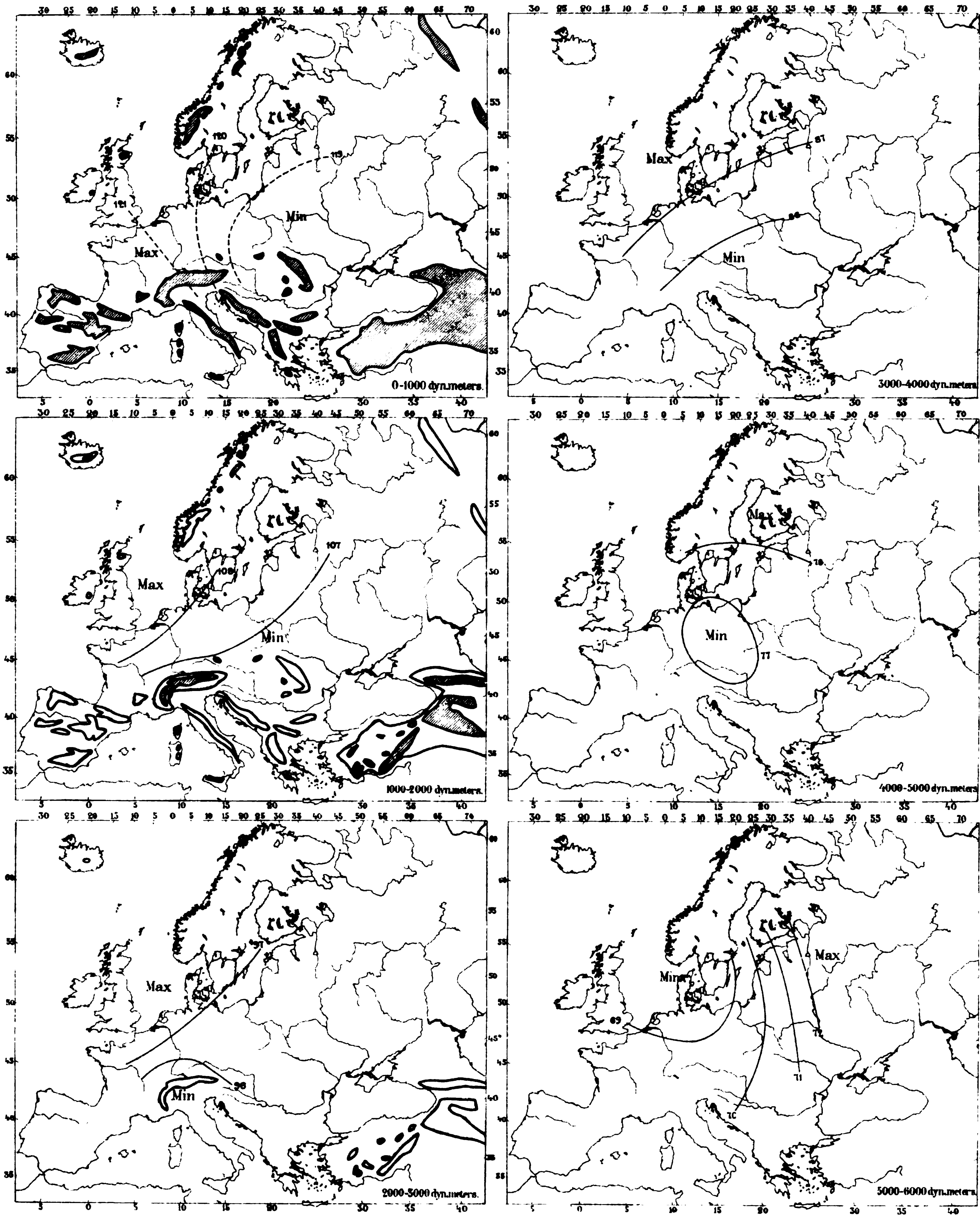


Fig. 23. Field of mass, Nov. 7, 1901, represented by charts giving the average density (10^{-5}ton/m^3) of the air between level surfaces, or the difference of pressure between these surfaces in m-bars.

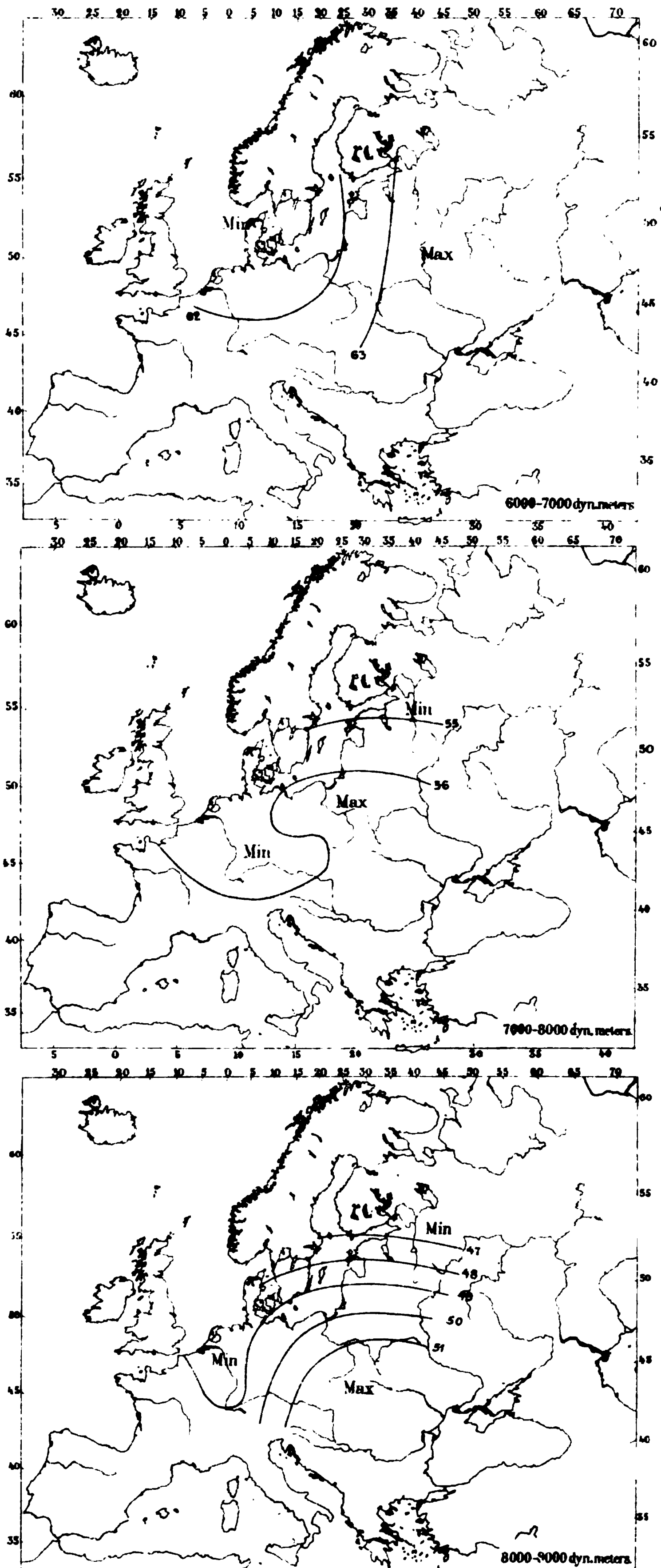


FIG. 23. (Continued). Field of mass, Nov. 7, 1901

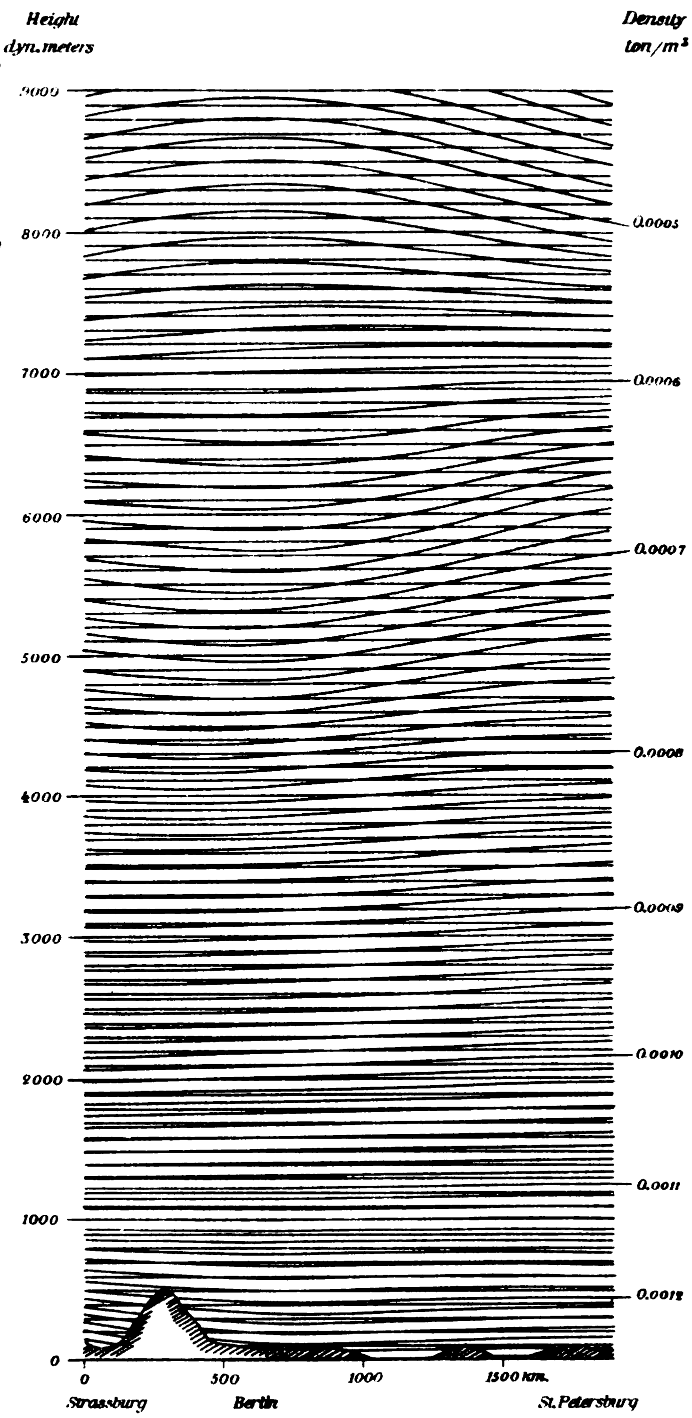


FIG. 24. Profile curves of the level and of the isopycnic surfaces. Each parallelogram represents 0.01 equipotential-isopycnic unit tube.

Table S contains the absolute heights of the lowest standard isobaric surfaces, as well as the heights from surface to surface as obtained by the method of extrapolation from the common meteorological stations.* Among those from which observations have been available 219 have been chosen. Their situation is seen from the last chart of fig. 19. The principle in choosing has been to get as many stations as possible on different levels. The chart therefore contains a great number of stations in mountainous regions, and relatively few in low land.

By means of the figures contained in tables R and S the charts of absolute and of mutual topography (figs. 19 and 20) of the standard isobaric surfaces have been drawn in full accordance with the directions given in sections 66 and 68. The two lowest charts of mutual topography obtained by extrapolation from the 219 common stations and the two corresponding obtained from the ascents from the five aeronautical stations are given side by side in fig. 20, no attempt having been made to mold the corresponding charts into one. (Compare section 68.) The charts of absolute pressure in the four lowest levels (fig. 22) have been derived from the corresponding topographic charts of fig. 19 by the graphic method described in section 67. The charts of pressure differences (fig. 23) are drawn exclusively from the pressure differences contained in table T.

As in the preceding example, we have dotted all isobaric or level curves running below the earth's surface, the lines of intersection of the isobaric or the level surfaces with the earth being marked as thick curves. Thus the half of the 1000 m-bar surface is below the earth, while the 900 m-bar surface passes below the earth only in the mountainous parts of Scandinavia, southern Europe, and adjacent parts of Asia. Above the 800 m-bar surface only the higher parts of the Alps and of the Caucasus rise as small islands.

Figs. 21 and 24 are vertical sections containing profile curves, the first of isobaric and isosteric, the second of equipotential and isopycnic surfaces. These sections are worked out from ascents from Strassburg, Berlin, and St. Petersburg.

Figs. 19 to 24 thus described give the distribution of pressure and mass in a cyclone having its center above Finland. Here the isobaric charts show a minimum of pressure, and the topographic charts deep depressions of the isobaric surfaces. A striking feature of the topography of the isobaric surfaces is their inclination as we proceed upwards. It is characteristic of the other method of representation that the isobaric charts do not show noticeably greater difference of pressure in one level than in another. Thus the topography of the isobaric surfaces is in some sense a more sensitive indicator of the distribution of pressure in higher levels than the isobaric charts for given levels. A most striking feature of the charts of mutual topography is that they indicate decreasing specific volume towards the cyclone center. In the same manner the charts of pressure-differences show an increasing density of the air as we approach this center. Thus both indicate a

*The observations are taken from the meteorological year books published by the different countries. Unfortunately some of them (Italy, Spain) only contain different average values, not original observations, and are therefore of no use for investigations in atmospheric dynamics. As is well known, the simultaneity of the observations from the different countries is not very satisfactory. This will be a great difficulty for dynamical investigations. For our present purpose we may treat them as if they were true simultaneous observations.

concentration of mass in the center of the cyclone. We indicate this only to point out what the charts tell, not to discuss the fact in itself. For we have refrained from discussing the reliability of the observations from which charts have been deduced, our only aim being at present to illustrate our methods, the observations being given and considered as trustworthy.

TABLE T.—*Pressure (m-bars) in standard level surfaces and differences of pressure between them, computed from ascents, Europe, November 7, 1901.*

Station:	Paris.		Strassburg.		Berlin.		Wien.		Petersburg.	
Latitude:	48° 48′		48° 35′		52° 30′		48° 15′		59° 56′	
Longitude:	2° 29′		7° 46′		13° 23′		16° 21′		30° 16′	
Dynamic height:	48		141		48		198		5	
Level (dynamic meters).										
9000	297.0		302.6		290.5		297.5		260.1	
		49.8		48.8		50.4		50.5		46.7
8000	346.8		351.4		340.9		348.0		306.8	
		55.9		55.3		56.2		55.7		54.8
7000	402.7		406.7		397.1		403.7		361.6	
		62.2		62.3		61.7		62.5		63.2
6000	464.9		469.0		458.8		466.2		424.8	
		69.7		69.7		68.7		69.4		71.4
5000	534.6		538.7		527.5		535.6		496.2	
		77.3		77.5		76.8		76.9		78.5
4000	611.9		616.2		604.3		612.5		574.7	
		87.0		86.3		86.3		85.6		87.1
3000	698.9		702.5		690.6		698.1		661.8	
		97.1		95.5		96.7		96.2		96.6
2000	796.0		798.0		787.3		794.3		758.4	
		108.0		106.9		107.3		106.9		107.1
1000	904.0		904.9		894.6		901.2		865.5	
		121.1		121.0		119.3		118.9		119.3
0	1025.1		1025.9		1013.9		1020.1		984.8	

72. Unit-Tubes.—The two sets of curves in figs. 17, 18, 21, and 24 divide the vertical plane into a set of curvilinear parallelograms. These parallelograms are evidently the cross-sections of a set of tubes formed by the intersection of the two sets of surfaces. We may denote them as isobaric-isosteric tubes when they are formed by intersection of the isobaric and the isosteric surfaces (figs. 17 and 21), and as equipotential-isopycnic tubes when formed by intersection of the equipotential and the isopycnic surfaces (figs. 18 and 24). They may further be called unit-tubes if the intersecting surfaces are drawn for unit-differences of the scalar quantities whose fields they represent. The name unit-tubes may be retained also in case of the one set of surfaces being drawn for intervals of a certain multiple of the unit, while the other set is drawn for intervals equal to the corresponding fraction of the unit. In figs. 17 and 21 are isobaric curves drawn for every centibar and isosteric for every 10 m^3/tons. Every parallelogram therefore represents 10 m. t. s. unit isobaric-isosteric tubes. In figs. 18 and 24 a level line is drawn for every 100 dynamic meters, *i. e.*, for every 1000 dynamic decimeters, while the isopycnic curves are drawn for every hundred-thousandth ton/m^3. Every parallelogram in these figures thus represents one-hundredth of a m. t. s. equipotential-isopycnic unit-tube.

In case of true equilibrium there will be no intersection of the surfaces and therefore no tubes. On the other hand, as the angle of intersection increases, the number of unit-tubes will increase. This number can therefore be taken as a

measure for the departure from the state of true equilibrium. For this reason it will be useful to develop some simple relations involving the number of unit-tubes.

Proceeding along an isobaric unit-sheet, we get unit-change of specific volume, and consequently unit-change of thickness of the sheet for every isosteric surface met with. Instead of counting the isosteric surfaces, we may also count the unit-tubes. Introducing the ascendant (section 17) of the specific volume, we see that the projection of this vector on the isobaric surface points in the direction ot increasing thickness of the sheet. We can therefore count algebraically, reckoning a tube positive when the projection of the ascendant points in the direction in which we proceed, otherwise negative. By this mode of counting we get a measure for the increase of thickness of the unit-sheet. From the unit-sheet we may pass to any sheet composed of any number of unit-sheets; the increase of thickness of the sheet from one vertical to another will be equal to the number of isobaric-isosteric unit-tubes contained between them, counted algebraically in the defined manner. The increase of height comes out in dynamic decimeters if the m. t. s. units be used.

Counting in the same way the number of equipotential-isopycnic unit-tubes contained in an equipotential sheet, we find the variations in the difference of pressure between the upper and the lower limiting surface of the sheet. The rule of signs is formally the same as in the preceding case, the projection of the ascendant of the density pointing in the direction where the difference of pressure increases. Thus, in order to find the increase in the difference of pressure between foot and top of two verticals having their end-points in the same two level surfaces, we have simply to count algebraically the number of equipotential-isopycnic unit-tubes contained within the closed curve formed by the two verticals and two level curves joining their end-points.

73. Relation between Sections and Charts. — These rules lead to a new view of the charts representing the mutual topographies or the differences of pressures. The curves of these charts may be considered as the horizontal projections of vertical walls, dividing the sheets into a set of tubes. These tubes with vertical walls are easily seen to have a close relation to the unit-tubes with oblique walls.

To consider first the charts of mutual topography, each vertical wall has a constant dynamic height. Two different walls therefore have a constant difference of dynamic height. From the numerical relation developed in the preceding article we therefore conclude that every tube with vertical walls must contain a constant number ot isobaric-isosteric unit-tubes. If m. t. s. units be used, this number will be equal to the difference of height between the vertical walls, expressed in dynamic decimeters. Thus every section of the tube, it being plane or curved, normal or oblique, contains this constant number of unit-parallelograms. This does not mean that the course of the unit-tubes with their parallelogram-section is exactly the same as that of the tubes with vertical walls. But the latter give the *average* course ot the first. Thus, if a unit-tube passes out through the vertical wall, for instance at its base, a corresponding tube will enter through the *same* wall at its top. We thus arrive at this result: The charts of mutual topography of isobaric surfaces show the average course and the number of the isobaric-isosteric unit-tubes in the sheet between the surfaces.

Passing to the charts of pressure differences, we get this perfectly analogous result: The charts for pressure differences between successive level surfaces show the course and the number of equipotential-isopycnic unit-tubes in the sheet between two levels.

On the charts of mutual topography (figs. 14 and 20) the curves are drawn for differences of height of 10 dynamic meters, *i. e.*, for 100 dynamic decimeters. Thus between the vertical walls represented by the curves there run 100 isobaric-isosteric unit-tubes. On the charts of pressure differences (figs. 16 and 23) the curves are drawn for the intervals of pressure of 1 m-bar, *i. e.*, 0.1 c-bar. Between the vertical walls represented by these curves there will consequently run 0.1 equipotential-isopycnic unit-tubes, if the m. t. s. units be used.

74. Complete Representation of the Fields of Moving Forces and Moved Masses in the Atmosphere. — Our aim has been to arrive at a complete representation of the fields of pressure and of mass. But it is worth while mentioning that in reality we have attained more than this.

For the investigation of atmospheric equilibrium and motion a third field, that of gravitational force, is of fundamental importance. Being invariable, this field need not, like the changing fields of pressure and mass, be specially represented. But it merits attention that in our representations of the variable field of pressure is implied also that of the invariable gravitational field.

The charts giving the dynamic topography of the isobaric surfaces are representations of the gravitational field of force tangentially to these surfaces. Mentioning the charts of dynamic topography of the earth's surface and of the bottom of the sea, we have already developed the idea of these charts as representing two-dimensional fields of force (section 18). Evidently a combination of the two-dimensional fields for the succession of isobaric surfaces will give a complete representation of the three-dimensional field in space.

The other representation of the field of pressure is by isobaric curves drawn on level surfaces. Now, the level or equipotential surfaces give themselves a direct representation of the gravity potential and thus of the gravitational field of force. It is the field ot pressure, which is represented in the more indirect way, as the field ot gravity potential in the preceding case. We have here a perfect parallelism. The isobaric charts in level surfaces represent the two-dimensional fields of the pressure gradient in these surfaces, just as the topographic charts of the isobaric surfaces represented the two-dimensional fields of the potential gradients in these surtaces. The comprehension of these isobaric charts for the successive levels give the representation of the three-dimensional field of the pressure gradient in space.

Whichever of the two methods we choose, we thus get simultaneously a representation of the fields of force due to gravity and to pressure. At the same time, the charts of relative topography or of relative pressure represent the field of mass. We have thus obtained a complete representation of the fields both of the moving forces and of the masses moved.

CHAPTER VIII.

PRACTICAL SOLUTION OF THE HYDROSTATIC PROBLEM FOR THE SEA.

75. Normal Equilibrium Relation and Small Deviations from this Relation. — In order to illustrate the principle of unit-sheets, we have already calculated the depth corresponding to a given pressure (section 36) and the pressure at a given depth (section 39) of the sea having a constant salinity of 35 $^0/_{00}$ and a constant temperature of 0° C. This calculation gave us the fundamental tables 7 H and 15 H of our Hydrographic Tables. In the ideal case of a sea with these constant values of temperature and salinity we have thus fully solved the hydrostatic problem in both its forms.

The treatment of the problem generally is very much simplified by the circumstance that the variations in temperature and salinity only produce minute changes in the equilibrium relation between depth and pressure. We can therefore consider the equilibrium relation represented by tables 7 H or 15 H as the "normal" one. The problem is then reduced to the determination of the small deviations from this relation produced by the variations of temperature and salinity, or, as we may call it, the "anomalies" of the equilibrium relation.

To find the expressions for these anomalies, we have to start with the hydrostatic equation in either of its integral forms, section 40 (*a*) or (*b*). Instead of gravity potential ϕ we introduce the dynamic depth D, measured in dynamic meters and counted positive downwards (section 10). Simultaneously we count the pressure only as sea-pressure (section 27), expressed in decibars. Choosing the lower limit of the integrals in the sea's surface, we then get as expression for the depth D corresponding to a given pressure p,

(*a*) $$D = \int_0^p \alpha dp$$

and as expression for the pressure p at the given depth D,

(*b*) $$p = \int_0^D \rho dD$$

Applying our notations from sections 27 and 29, and introducing

(*c*) $$\alpha = \alpha_{35,0,p} + \delta$$

(*d*) $$\rho = \rho_{35,0,D} + \epsilon$$

we separate the specific volume and the density of the sea-water into their normal values $\alpha_{35,0,p}$, $\rho_{35,0,D}$ and their anomalies δ and ϵ. Substituting this in the equations (*a*) and (*b*), we get D and p separated in two terms,

(*e*) $$D = D_{35,0,p} + \Delta D$$

(*f*) $$p = p_{35,0,D} + \Delta p$$

Where

(g) $$D_{35,0,p} = \int_0^p \alpha_{35,0,p} dp$$

(h) $$p_{35,0,D} = \int_0^D \rho_{35,0,D} dD$$

and

(i) $$\Delta D = \int_0^p \delta dp$$

(j) $$\Delta p = \int_0^D \epsilon dD$$

Here (*g*) represents the normal depth corresponding to a given pressure, *i. e.*, the depth tabulated in table 7 H; (*h*) the normal pressure at a given depth, *i. e.*, the pressure registered in table 15 H. We have therefore henceforth to occupy ourselves only with equations (*i*) and (*j*), the first of which gives the anomaly of depth for a given pressure, while the second gives the anomaly of pressure at a given depth.

76. Fundamental Approximation Rules. — The anomalies of depth or of pressure should be determined in accordance with the observed values of salinity and temperature. Generally the values of these quantities are obtained for known values of depth, measured in meters by means of the sounding-line. In other cases a manometer is used, giving the pressure at the places from whence the samples of water are taken, the temperature and salinity of which are determined.

Between the depths of a certain number of common and the same number of dynamic meters there is a difference of about 2 per cent. Between the depths represented by a sea-pressure of a certain number of decibars and that represented by the same number of dynamic meters there is a variable difference not exceeding 3 per cent in the upper layers and 5 per cent in the greatest depths of the sea, as seen from table 7 H. Between the depth represented by a sea-pressure of a certain number of decibars and that represented by the same number of common meters there will finally be a variable difference not exceeding 1 per cent in the smaller and 3 per cent in the greatest depths of the sea. To these differences (from 1 to 5 per cent) of the total depth there will correspond only very small differences of temperature and salinity. For in the upper sheets, where relatively great differences of temperature and salinity may occur, this difference of depth will be very small, and lower down the variations of temperature and salinity will be exceedingly gradual. Thus, these small differences of temperature and salinity will have no appreciable influence upon the small corrections ΔD and Δp. Suppose, therefore, a sample of water to be taken up from a depth of a certain number of common meters. If it be convenient for the calculation we can, without restricting the accuracy of the final result, consider it as taken from the depth of the same number of dynamic meters, or from the isobaric surface of the same number of decibars. Or, suppose the sample to be taken from a place where the manometer has shown

a sea-pressure of a certain number of decibars. If it be convenient for the calculations we may consider it as taken from the depth expressed by the same number of dynamic meters.

As a consequence of these approximation rules, it remains indifferent whether depths or pressures have been observed. The four forms of the problem met with in the atmosphere (section 49) are, therefore, in the case of the sea, reduced practically to two, the calculation of the depth corresponding to a given pressure and the calculation of the pressure at a given depth, it being immaterial whether the temperature is registered as functions of pressure or of depth.

77. Calculation of the Anomalies of Depth and of Pressure. — These approximation rules being accepted, the calculation of the integrals (a) and (b) can be made immediately. Taking first the anomaly of depth of the isobaric surfaces, we remember (section 27) that we can write for the anomaly δ of the specific volume

$$\delta = \delta_s + \delta_\tau + \delta_{s\tau} + \delta_{sp} + \delta_{\tau p} + \delta_{s\tau p}$$

the quantities δ_s, δ_τ, $\delta_{s\tau}$, δ_{sp}, $\delta_{\tau p}$, $\delta_{s\tau p}$ being tabulated in tables 9 H, 10 H, 11 H, 12 H, 13 H, and 14 H, respectively, for all occurring values of temperature, salinity, and pressure. By means of these tables and the observed temperatures and salinities we find the values of these quantities and by adding them the values of δ corresponding to a set of known pressures. Then the value of the integral (i), section 75, is found by a regular process of integration; *i. e.*, we take the average of the successive values of δ, multiply by the corresponding difference of pressure, and form the sum from the pressure 0 at sea-level down to the pressure p. This sum represents the anomaly ΔD of the dynamic depth of the isobaric surface of pressure p.

We find the anomaly of pressure Δp in the given dynamic depth D in exactly the same way, writing

$$\varepsilon = \varepsilon_s + \varepsilon_\tau + \varepsilon_{s\tau} + \varepsilon_{sD} + \varepsilon_{\tau D} + \varepsilon_{s\tau D}$$

using tables 17 H, 18 H, 19 H, 20 H, 21 H, and 22 H and performing the integration in the same regular way.

The systematic performance of the calculation is easily understood by examination of the examples worked out below.

Adding the anomaly of depth ΔD to the normal value $D_{35,0,p}$ we get the equilibrium relation in form of depth for a given pressure. Adding the anomalies of specific volume δ to the normal values $\alpha_{35,0,p}$ we get the actual specific volumes α for given values of the pressure, *i. e.*, the equilibrium relation between pressure and specific volume.

In the same way the addition of the anomalies of pressure Δp to the normal values $p_{35,0,D}$ gives the equilibrium relation in form of pressures in given dynamic depths, and the addition of the anomalies of density ε to the normal densities $\rho_{35,0,D}$ gives the actual densities ρ at given depths, *i. e.*, the equilibrium relation between density and depth.

Example 1.—Norwegian Sea, Station N. 36, June 7, 1904. 64° 55′ N. lat.; 2° 52′ W. long.; 1830 meters, no bottom.

TABLE U.—*Depth corresponding to a given pressure.*

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Observed depths, meters (in first approximation identical with pressure in decibars).	Observed temperatures, °C.	Observed salinities, ‰.	$10^5 \cdot \delta_s$ found by table 9 H.	$10^5 \cdot \delta_T$ found by table 10 H.	$10^5 \cdot \delta_{sT}$ found by table 11 H.	$10^5 \cdot \delta_{sp}$ found by table 12 H.	$10^5 \cdot \delta_{Tp}$ found by table 13 H.	$10^5 \cdot \delta_{sTp}$ found by table 14 H.	$10^5 \cdot \delta = 10^5(\delta_s + \delta_T + \delta_{sT} + \delta_{sp} + \delta_{Tp} + \delta_{sTp})$ $\delta =$ anomaly of specific volume.	Standard pressures, d-bars.	$10^5 \cdot \delta$ interpolated from column 10.	$10^5 \cdot$ anomalies of thickness of isobaric sheets, obtained by multiplication of successive averages of $10^5 \cdot \delta$ (column 12) by corresponding differences of pressure.	$10^5 \cdot \Delta D$, obtained as sums of anomalies of thickness, column 13. $\Delta D =$ anomaly of depth of isobaric surfaces.	Depth in dynamic meters of isobaric surfaces, $D = D_{35,0,p} + \Delta D$, $D_{35,0,p}$ being found from table 7 H, the corresponding anomaly ΔD from column 14.	Specific volume in m³/ton, $\alpha = \alpha_{35,0,p} + \delta$, $\alpha_{35,0,p}$ being found from table 8 H, corresponding anomaly δ from column 12.
0	7.49	34.93	6	72	0	0	0	0	78	0	78	----	0	0	0.97342
10	7.33	34.88	9	70	0	0	0	0	79	10	79	785	785	9.7340	0.97339
20	6.70	34.90	8	61	0	0	0	0	69	20	69	740	1525	19.4672	0.97324
30	6.11	34.96	3	54	0	0	0	0	57	30	57	630	2155	29.1988	0.97308
50	5.39	34.99	1	45	0	0	1	0	47	40	52	545	2700	38.9291	0.97298
75	3.88	34.88	9	29	0	0	1	0	39	50	47	495	3195	48.6584	0.97289
100	3.10	34.87	10	22	0	0	1	0	33	60	44	455	3650	58.3869	0.97281
150	2.42	34.85	12	16	0	0	1	0	29	70	41	425	4075	68.1146	0.97274
200	1.77	34.88	9	11	0	0	1	0	21	80	38	395	4470	77.8417	0.97266
250	1.94	34.96	3	12	0	0	1	0	16	90	35	365	4835	87.5679	0.97259
300	1.65	34.96	3	10	0	0	2	0	15	100	33	340	5175	97.293	0.97252
400	0.77	34.93	6	4	0	0	1	0	11	200	21	2700	7875	194.517	0.97195
500	0.27	34.91	7	2	0	0	0	0	9	300	15	1800	9675	291.687	0.97144
										400	11	1300	10975	388.806	0.97095
										500	9	1000	11975	485.878	0.97049

TABLE V.—*Pressure at a given depth.*

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Observed depths, meters (in first approximation identical with depths in dynamic meters).	Observed temperatures, °C.	Observed salinities, ‰.	$10^5 \cdot \varepsilon_s$ found by table 17 H.	$10^5 \cdot \varepsilon_\tau$ found by table 18 H.	$10^5 \cdot \varepsilon_{s\tau}$ found by table 19 H.	$10^5 \cdot \varepsilon_{sD}$ found by table 20 H.	$10^5 \cdot \varepsilon_{\tau D}$ found by table 21 H.	$10^5 \cdot \varepsilon_{s\tau D}$ found by table 22 H.	$10^5 \cdot \varepsilon = 10^5(\varepsilon_s + \varepsilon_\tau + \varepsilon_{s\tau} + \varepsilon_{sD} + \varepsilon_{\tau D} + \varepsilon_{s\tau D})$ ε = anomaly of density.	Standard dynamic depths, dynamic meters.	$10^5 \cdot \varepsilon$ interpolated from column 10.	$10^5 \cdot$ anomalies of differences of pressure from level to level, found by multiplication of successive averages of $10^5 \cdot \varepsilon$ (column 10) by corresponding differences of dynamic depth.	$10^5 \cdot \Delta p$ in level surfaces, obtained as sums of the anomalies of differences of pressure, column 13. Δp = anomaly of pressure.	Pressure in d-bars in level surfaces, $p = p_{35,0,D} + \Delta p$, $p_{35,0,D}$ being found from table 15 H, the corresponding anomaly Δp from column 14.	Density in ton/m³, $\rho = \rho_{35,0,D} + \varepsilon$, $\rho_{35,0,D}$ being found from table 16 H, the corresponding anomaly from column 12.
0	7.49	34.93	− 6	−76	0	0	0	0	−82	0	−82	----	0	0	1.02731
												825			
10	7.33	34.88	−10	−73	0	0	0	0	−83	10	−83	----	− 825	10.2733	1.02735
												780			
20	6.70	34.90	− 8	−65	0	0	0	0	−73	20	−73	----	− 1605	20.5475	1.02749
												670			
30	6.11	34.96	− 3	−57	0	0	−1	0	−61	30	−61	----	− 2275	30.8233	1.02766
												580			
50	5.39	34.99	− 1	−48	0	0	−1	0	−50	40	−55	----	− 2855	41.1005	1.02777
												525			
75	3.88	34.88	−10	−31	0	0	−1	0	−42	50	−50	----	− 3380	51.3787	1.02787
												485			
100	3.10	34.87	−11	−23	0	0	−1	0	−35	60	−47	----	− 3865	61.6578	1.02795
												455			
150	2.42	34.85	−12	−17	0	0	−1	0	−30	70	−44	----	− 4320	71.9377	1.02803
												425			
200	1.77	34.88	−10	−12	0	0	−1	0	−23	80	−41	----	− 4745	82.2185	1.02811
												395			
250	1.94	34.96	− 3	−13	0	0	−1	0	−17	90	−38	----	− 5140	92.4999	1.02819
												365			
300	1.65	34.96	− 3	−10	0	0	−2	0	−15	100	−35	----	− 5505	102.782	1.02827
												2900			
400	0.77	34.93	− 6	− 5	0	0	−1	0	−12	200	−23	----	− 8405	205.640	1.02888
												1900			
500	0.27	34.91	− 7	− 2	0	0	0	0	− 9	300	−15	----	−10305	308.555	1.02945
												1350			
										400	−12	----	−11655	411.526	1.02997
												1050			
										500	− 9	----	−12705	514.550	1.03049

Example 2.—Baltic, Station F. 64, May 17, 1904. 60° 12.5′ N. lat.; 19° 7′ E. long.; 277 meters.

TABLE W.—*Depth corresponding to a given pressure*

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Observed depths, meters (in first approximation identical with pressure in decibars).	Observed temperatures °C.	Observed salinities, ‰.	$10^5 \cdot \delta_s$ found by table 9 H.	$10^5 \cdot \delta_T$ found by table 10 H.	$10^5 \cdot \delta_{sT}$ found by table 11 H.	$10^5 \cdot \delta_{sp}$ found by table 12 H.	$10^5 \cdot \delta_{Tp}$ found by table 13 H.	$10^5 \cdot \delta_{sTp}$ found by table 14 H.	$10^5 \cdot \delta = 10^5(\delta_s+\delta_T+\delta_{sT}+\delta_{sp}+\delta_{Tp}+\delta_{sTp})$. δ= anomaly of specific volume.	Standard pressures, d-bars.	$10^5 \cdot \delta$ interpolated from column 10.	$10^5 \cdot$ anomalies of thickness of isobaric sheets, obtained by multiplication of successive averages of $10^5\delta$ (column 12) by corresponding differences of pressure.	$10^5 \cdot \Delta D$ obtained as sums of the anomalies of thickness $\Delta - D$ anomaly of depth of isobaric surfaces.	Depth in dynamic meters of isobaric surfaces $D-D_{35,0,p}+\Delta D$, $D_{35,0,p}$ being found from table 7 H, the corresponding anomaly ΔD from column 14.	Specific volume in m³/ton $a=a_{35,0,p}+\delta$, $a_{35,0,p}$ being found from table 8 H, the corresponding anomaly δ from column 12.
0	3.16	5.81	2275	23	−28	0	0	0	2270	0	2270		0	0	0.99534
10	3.16	5.81	2275	23	−28	−1	0	0	2269	10	2269	22695	22695	9.9531	0.99520
20	2.82	5.97	2262	19	−25	−1	0	0	2255	20	2255	22620	45315	19.9051	0.99510
30	2.78	5.99	2261	19	−25	−2	0	0	2253	30	2253	22540	67855	29.8558	0.99504
40	2.31	5.99	2261	15	−21	−2	0	0	2253	40	2253	22530	90385	39.8059	0.99499
50	1.69	6.08	2254	10	−15	−3	0	0	2246	50	2246	22495	112880	49.7553	0.99488
75	1.47	6.19	2245	9	−13	−4	0	0	2237	60	2242	22440	135320	59.7036	0.99479
100	1.48	6.22	2242	9	−13	−5	0	0	2233	70	2239	22405	157725	69.6511	0.99472
125	1.88	6.29	2237	12	−17	−6	1	0	2227	80	2236	22375	180100	79.5980	0.99464
150	2.37	6.33	2234	16	−21	−7	1	0	2223	90	2235	22355	202455	89.5440	0.99459
200	2.68	6.38	2230	18	−24	−9	1	0	2216	100	2233	22340	224795	99.4896	0.99452
250	2.76	6.44	2225	19	−25	−11	2	0	2210	200	2216	222400	447195	198.910	0.99390
275	2.73	6.44	2225	18	−25	−13	2	0	2207	300	2206	221100	668295	298.273	0.99335

TABLE X.—*Pressure at a given depth.*

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Observed depths, meters (in first approximation identical with depths in dynamic meters).	Observed temperatures °C.	Observed salinities, ‰.	$10^5 \cdot \varepsilon_s$ found by table 17 H.	$10^5 \cdot \varepsilon_\tau$ found by table 18 H.	$10^5 \cdot \varepsilon_{s\tau}$ found by table 19 H.	$10^5 \cdot \varepsilon_{sD}$ found by table 20 H.	$10^5 \cdot \varepsilon_{\tau D}$ found by table 21 H.	$10^5 \cdot \varepsilon_{s\tau D}$ found by table 22 H.	$10^5 \cdot \varepsilon = 10^5(\varepsilon_s + \varepsilon_\tau + \varepsilon_{s\tau} + \varepsilon_{sD} + \varepsilon_{\tau D} + \varepsilon_{s\tau D})$. ε = anomaly of density.	Standard dynamic depths, dynamic meters.	$10^5 \cdot \varepsilon$ interpolated from column 10.	$10^5 \cdot$ anomalies of differences of pressure from level to level, found by multiplication of successive averages of $10^5 \cdot \varepsilon$ (column 12) by corresponding differences of dynamic depth.	$10^5 \cdot \Delta p$ in level surfaces, obtained as sums of the anomalies of differences of pressure, column 13. Δp = anomaly of pressure.	Pressure in d-bars in level surfaces, $p = p_{35,0,D} + \Delta p$, $p_{35,0,D}$ being found from table 15 H, the corresponding anomaly Δp from column 14.	Density in ton/m³, $\rho = \rho_{35,0,D} + \varepsilon$, $\rho_{35,0,D}$ being found from table 16 H, the corresponding anomaly from column 12.
0	3.16	5.81	−2350	−24	30	0	0	0	−2344	0	−2344	-----	0	0	1.00469
10	3.16	5.81	−2350	−24	30	0	0	0	−2344	10	−2344	−23440	− 23440	10.0471	1.00474
20	2.82	5.97	−2337	−20	26	1	0	0	−2330	20	−2330	−23370	− 46810	20.0954	1.00492
30	2.78	5.99	−2336	−20	26	1	0	0	−2329	30	−2329	−23295	− 70105	30.1449	1.00498
40	2.31	5.99	−2336	−16	22	2	0	0	−2328	40	−2328	−23285	− 93390	40.1951	1.00504
50	1.69	6.08	−2329	−11	16	2	0	0	−2322	50	−2322	−23250	−116640	50.2461	1.00515
75	1.47	6.19	−2319	−10	14	3	0	0	−2312	60	−2318	−23200	−139840	60.2980	1.00524
100	1.48	6.22	−2316	−10	14	4	0	0	−2308	70	−2314	−23160	−163000	70.3509	1.00533
125	1.88	6.29	−2311	−13	18	5	−1	0	−2302	80	−2310	−23120	−186120	80.4047	1.00542
150	2.37	6.33	−2308	−17	22	6	−1	0	−2298	90	−2309	−23095	−209215	90.4591	1.00548
200	2.68	6.38	−2304	−19	25	8	−2	0	−2292	100	−2308	−23085	−232300	100.514	1.00554
250	2.76	6.44	−2299	−20	25	10	−2	0	−2286	200	−2292	−230000	−462300	201.101	1.00619
275	2.73	6.44	−2299	−19	25	11	−3	0	−2285	300	−2285	−228850	−691150	301.747	1.00675

78. Example of the Hydrostatic Results of Soundings in the Sea. — On pages 126–129 are given the schemes for the hydrostatic derivation of the results of soundings in the sea. The examples are chosen from soundings executed by the northern European states, one (Norwegian Expedition, May–June, 1904) being chosen from the Norwegian sea, and one (Finnish Expedition, May, 1904) from the inner Baltic.*

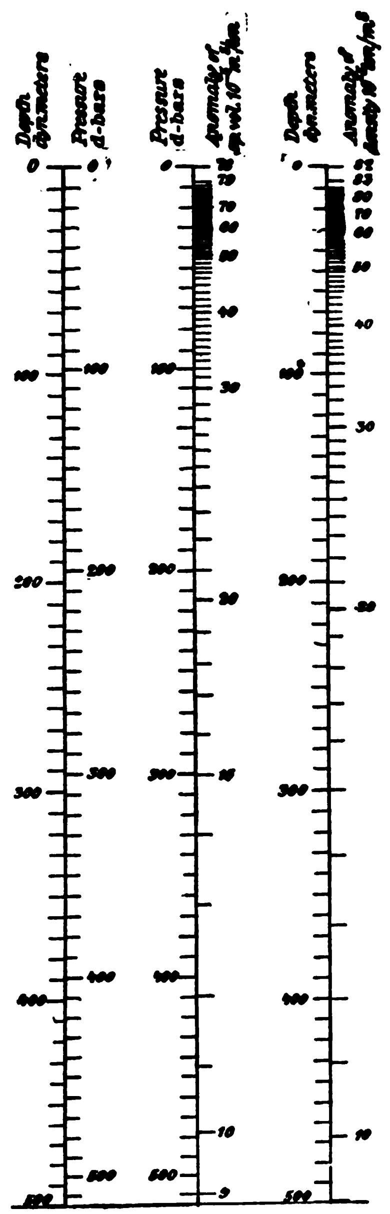


Fig. 25. — State of equilibrium in the Atlantic, 64° 55′ N. lat., 2° 52′ W. long., June 7, 1904.

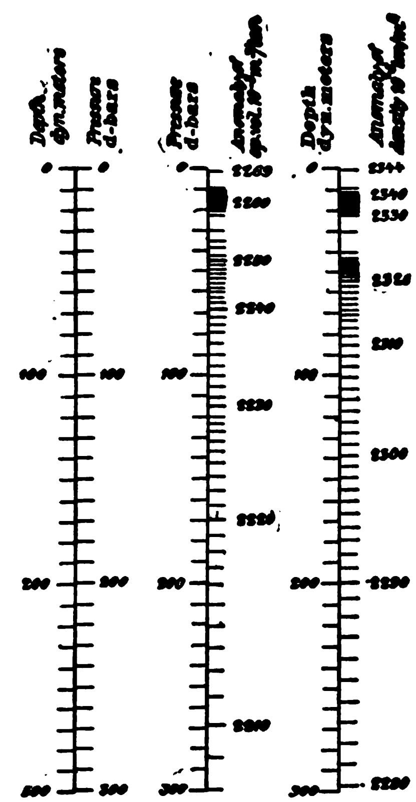


Fig. 26 — State of equilibrium in the Baltic, 60° 12.5′ N. lat., 19° 7′ E. long., May 17, 1904.

* Conseil Permanent International pour l'Exploration de la Mer. Bulletin des Resultats Acquis pendent les Courses Periodiques publié par le Bureau du Conseil avec l'assistance de M. Knudsen. Année 1903–1904. No. 4: Mai, 1904, pp. 96 and 77. Copenhague, 1904.

The soundings are as seen to have been taken with increasing intervals downwards, corresponding to the decreasing variations of temperature and salinity in the greater depths. This inconstancy of intervals, though unavoidable practically, is irrational from a theoretical point of view. Therefore the final results are interpolated for two sets of constant intervals given in column 11 of each scheme. These intervals are 10 dynamic meters or 10 decibars in the upper sheets, and 100 decibars or 100 dynamic meters in the greater depths.

The data for each sounding are treated according to the two different methods, that of depth corresponding to a given pressure (tables U and W), and that of pressure at a given depth (tables V and X).

79. Graphic Representation. — The results worked out in these examples are represented graphically in figs. 25 and 26. The first vertical in each figure gives the depth of the isobaric surfaces exactly as the first of fig. 1 (p. 45) gives these depths for sea-water of 35 $^0/_{00}$ salinity and temperature 0° C. The comparison shows perspicuously a greater depth of the isobaric surfaces in the brackish water of the Baltic than in that of the Atlantic, while the Atlantic vertical would have shown only microscopical differences from that of the normal sea-water (fig. 1), both figures being reduced to the same scale.

On the second vertical of figs. 25 and 26 the first division gives the situation of the isobaric surfaces, transferred from the first vertical. The second division does not, however, give the true specific volume as in fig. 1, but the *anomaly* of the specific volume taken from column 10 of the schemes (pp. 126, 128). In this way the difference from one vertical to another is made much more perspicuous. A vertical would have no anomalous divisions if the water had the "normal" salinity of 35 $^0/_{00}$ and the "normal" temperature of 0° C. The anomalous divisions therefore show the deviation from this normal state. As is seen, these anomalies are of relatively great numerical value in the brackish water of the Baltic, but of much smaller value in the Atlantic. Otherwise the anomaly varies rapidly near the surface and slower as we proceed downward, the variations with the depth being of the same order of magnitude along the vertical in the Baltic as along that in the Atlantic.

The third vertical in figs. 25 and 26 gives in exactly the same way the anomaly of density in different dynamic depths.

CHAPTER IX.

SYNOPTICAL REPRESENTATION OF THE FIELDS OF FORCE AND OF MASS IN THE SEA.

80. Quasi Static State. — The motion of the sea being generally much slower than that of the atmosphere, we may characterize sea-motion with still greater reason than that of the air as slow, and going on near a state of equilibrium. Excepting local phenomena, such as the formation of whirlpools in narrow straits or waves on the surface, we find the conditions of equilibrium apparently fulfilled to a great extent during the motion.

On the other hand we have, still more than was the case in the atmosphere, small distances in a vertical and great in a lateral direction. Consequently the conditions of the quasi static state (section 60) are fulfilled in the sea and with still greater approximation than in the atmosphere. We therefore state this principle, forming the basis of all practical investigations in oceanic dynamics:

The condition of equilibrium is apparently fulfilled along every vertical or quasi vertical line. But as we proceed in a horizontal direction, there is a gradual change in this apparent state of equilibrium from vertical to vertical.

81. Topography of Isobaric Surfaces. — Owing to this principle we can proceed formally as in the case of the atmosphere. Let us suppose first the depth of a given sea-pressure to have been determined by a set of simultaneous soundings. Then on a chart containing the situation of the hydrographic stations, we can note these depths and draw topographic charts, exactly as in the corresponding case of the atmosphere. Fig. 27 below gives examples of such charts.

It is important, however, to get a clear perception of these charts. For an important difference enters between the results attained in the case of the atmosphere and in that of the sea.

First, the surfaces whose topography is represented are surfaces of equal value of the *sea-pressure*, not of the total pressure. Secondly, the topography is given *relatively to the physical sea-level*, from which the measurements are made, and not from the ideal sea-level of the gravity potential zero. To be able to draw charts of absolute topography we want to know the topography of the physical sea-level, and this can not be found from the results of the soundings. This is of great importance to keep in mind. For owing to the motions of the sea and to the varying atmospheric pressure, the distance of physical from ideal sea-level will be of the same order of magnitude as that of isobaric from corresponding level surfaces. Thus, not only theoretically, but also practically, the topographic charts of the isobaric surfaces given in fig. 27 are charts of *relative topography*, *relatively to the unknown topography of physical sea-level.* This is an important restriction on the completeness of the result, making the discussion of sea-motion much less direct than that of atmospheric motions.

It may be useful in this state of indetermination of the results to remark that we can give a slightly changed interpretation to these charts. Let us, instead of *sea*-pressure, consider *total* pressure, obtained by addition of the atmospheric pressure upon the sea's surface. Let us further, to simplify the conditions, imagine the atmosphere to be removed and be replaced by a layer of sea-water of the proper thickness to exert the actual atmospheric pressure. In this case the isobaric surface of absolute pressure 10 decibars will always very nearly coincide with the physical sea-level, passing a little below it in places where the atmospheric pressure upon the physical sea-level is smaller than 10 decibars, and a little above it in the artificially introduced water-layer, where the atmospheric pressure is of smaller value. Now, the working out of the soundings gave the distance from the physical sea-level to a surface of the constant sea-pressure p. But this distance will be essentially the same as the distance from the defined ideal isobaric surface of the total pressure 10 decibars to the isobaric surface where the total pressure is $p + 10$ decibars.

We can thus also interpret the charts of fig. 27 as representing the topography of true isobaric surfaces of a total pressure of $p + 10$ decibars, taken *relatively to the unknown topography of the ideal 10-decibar surface.*

Whichever view we take of the chart representing the distribution of pressure in the sea, the representation remains incomplete. But of the distribution of mass, on the other hand, we are able to give as complete a representation as in the case of the atmosphere.

Forming the differences of depth between two isobaric surfaces of the standard pressures of p and $p + 1$ decibars, we get the numbers representing the specific volume of the water in the standard sheet between the two surfaces. But the thickness of these sheets being only about 1 meter, we would get too many charts by taking every sheet. We have therefore introduced for this purpose dynamic decameters as units of dynamic depth in the upper sheets of the sea, to a depth of 6 dynamic decameters. Simultaneously we use the bar as unit-pressure. For the deeper strata, where the changes with the depth are slower, we have used dynamic hectometers as units of dynamic depth and the decabar as corresponding unit of pressure. The charts representing the specific volume in the corresponding sheets are given in fig. 28.

82. Pressure along Level Surfaces. — Suppose us, on the other hand, to have determined the sea-pressure at a given dynamic depth. We are then able to draw a chart representing the distribution of sea-pressure in this depth. But it must be remembered that this depth is measured from the physical sea-level. The chart thus gives the distribution of the sea-pressure not along a true level surface, but along *a surface of constant dynamic depth below the physical sea-level.*

As in the preceding case, we may take a different view of the chart, giving another definition of the indeterminate element. We then imagine the atmosphere to be replaced by a layer of water having the density of the water at the sea's surface and of the proper thickness to exert the pressure of the atmosphere against the

sea's surface. This layer will then fill out the deepenings where the physical surface of the sea is lower than the ideal sea-level. This being done, we can transport all verticals so as to begin at the ideal sea-level. The charts will then represent *a certain pressure along true level surfaces, namely, that which added to the pressure along the ideal sea-level would give the true pressure.* The indeterminate element, then, will be the pressure existing along the ideal sea-level under the defined conditions.

Whichever interpretation we choose for the isobaric charts, the indetermination due to our ignorance of the true topography of physical sea-level will remain. But quite independently of this, the charts giving the differences of pressure from one surface to another will give a full representation of the distribution of density in the standard layers.

83. Change of Topographic into Isobaric Charts. — An isobaric chart for the depth of a certain number of dynamic meters will be exceedingly like the topographic chart of an isobaric surface of the same number of decibars. In the same manner, the chart of pressure differences between two level surfaces will be exceedingly like the chart of mutual topography of the two corresponding isobaric surfaces. If, therefore, the topographic charts be drawn, we can derive the corresponding isobaric charts from them, no independent calculation of pressures at given depths being required.

To change a chart of mutual topography of isobaric surfaces into one of pressure differences between the corresponding levels, table 23 H, which changes densities into corresponding specific volumes, can be used with satisfactory exactitude. This is evident at once if we remember that the charts of mutual topography represent the average specific volume in the isobaric sheets, and those of pressure differences the average density in the level sheets. If, therefore, the water were under exactly the same pressure in the isobaric and the corresponding level sheets, this table would change with perfect exactitude the required differences of pressure between the level surfaces into the corresponding vertical distances between the isobaric surfaces, or *vice versa*. Now, corresponding isobaric and level sheets are not exactly at the same depth, and, therefore, not exactly under the same pressure. But the difference is too small to produce any visible error on the charts drawn according to the directions appended to the table.

We then consider the problem of changing the topographic chart of an isobaric surface into the isobaric chart in the corresponding level surface. Of course, the method of doing this will be independent of the question whether the given chart represents true topography or only relative topography referred to an initial surface of an unknown topography. But in the latter case the resulting isobaric chart will be one of corresponding incompleteness, as explained above (section 81). For simplicity we make our developments as if always true topographies, true level surfaces, and true pressures were under consideration.

Given the isobaric surface of pressure p, represented topographically by the level curves of depth, $D_1, D_2, \ldots D_n$; further, the level D, in which the cor-

responding isobaric chart should be drawn; and the series of pressures, p_1, p_2, . . . p_n, for which the isobaric curves should be drawn. The problem is to find the situations of these curves from the known situations of the level curves D_1, D_2, . . . That is, we shall find the depth D_x of a point on the isobaric surface p, vertically below which we have the given pressure p_n at the level surface D.

In the first approximation we may consider the water as homogeneous along every vertical, while its density may vary from vertical to vertical. Under this supposition we have simple proportionality along every vertical between dynamic depth and pressure, thus

$$D_x = D\frac{p}{p_n}$$

But now D and p represent corresponding pressures and depths in the sense defined. Thus ***numerically*** we may write D instead of p, and consequently

$$(a) \qquad D_x = \frac{D^2}{p_n}$$

This formula is easily tabulated for all depths D at which we wish to draw isobaric charts, and for all pressures p_n which may occur at these depths.

The numbers thus tabulated will, however, be slightly erroneous, because the water between the isobaric and the corresponding level surface is not homogeneous, there being a slight increase of density downward as a consequence of the compression. The amount of this error is easily found in the case of sea-water of 35 ‰ salinity and 0° C. For in this case we have tabulated both the depths at given pressures and the pressures in given depths (tables 7 H and 15 H). From these we find the true D_x, and thus the error involved in the use of the formula (a) in the case of "normal" sea-water. From this the correction in all other cases is easily found. For evidently the error will be proportional to the distance between the isobaric and the corresponding level surface. This distance is zero for water of unit density, and otherwise proportional to the excess of the density above unity.

Table 24 H of our Hydrographic Tables is calculated in this way by the formula (a), with addition of the always very small corrections obtained in the manner described from tables 7 H and 15 H. The practical use of the tables is easily seen from the appended examples.

Evidently these tables also enable us to solve the inverse problem — to change isobaric charts for given levels into topographic charts for the corresponding isobaric surfaces.

84. Vertical Sections. — As we did in the case with the atmosphere, so we can draw diagrams containing the profile curves either of isobaric and isosteric surfaces or of equipotential and isopycnic surfaces. This will, however, present a practical difficulty. Since the deviations of the different profile-curves from the horizontal course are so minute, extreme exaggeration of the vertical dimensions in comparison with the horizontal would be required to make them visible.

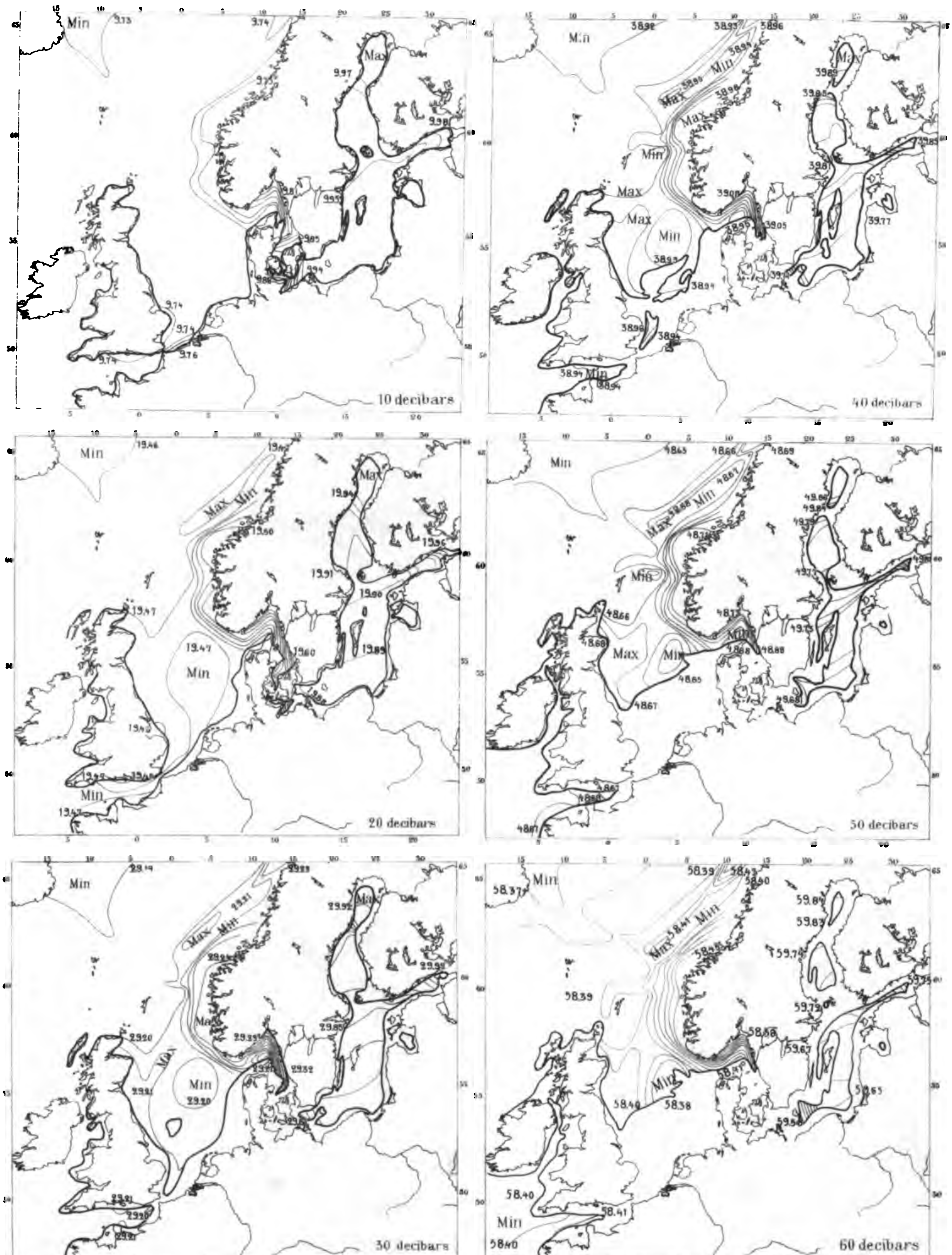


FIG. 27. Field of sea-pressure, May, 1904, represented by charts of relative topography, giving the depth of surfaces of equal sea-pressure below physical sea-level in dyn. meters.

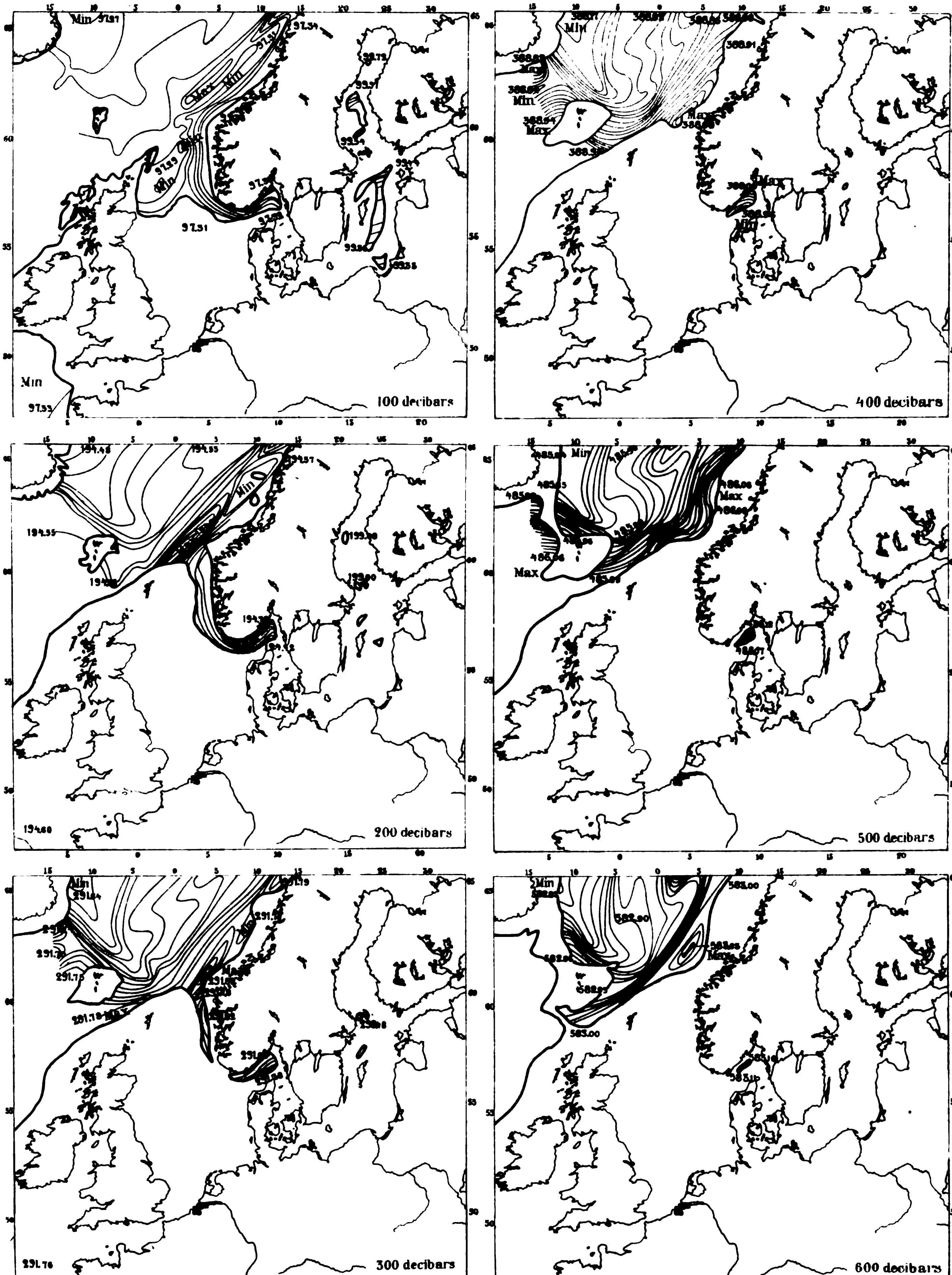


Fig. 27 (Continued). Field of sea-pressure, May, 1904, represented by charts of relative topography, giving the depth of surfaces of equal sea-pressure below physical sea-level in dyn. meters.

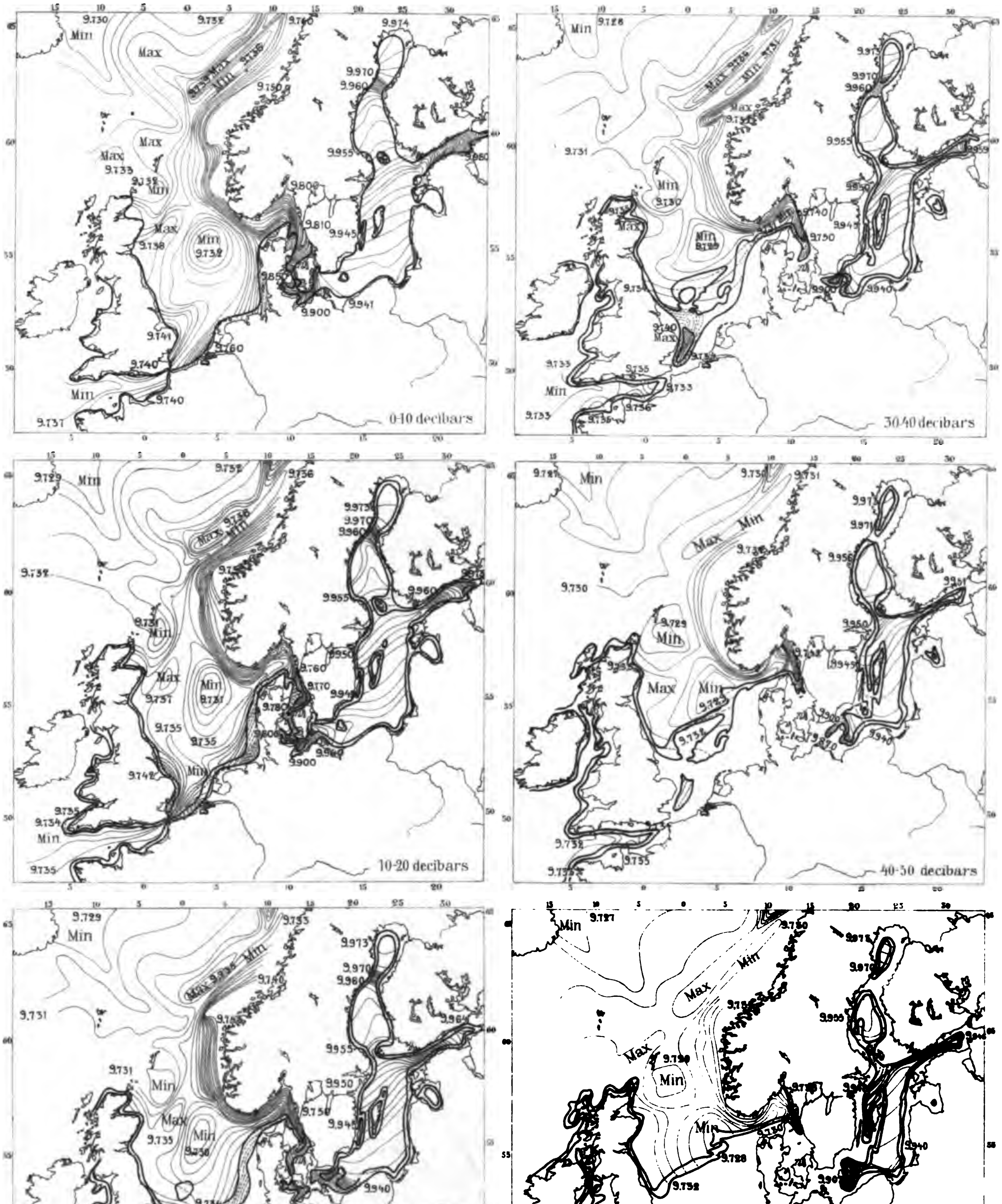


Fig. 28. Field of mass, May 1904, represented by charts giving the average specific volume ($10^{-1}m^3$/ton) of the sea-water in isobaric sheets, or the thickness of these sheets in dyn. meters.

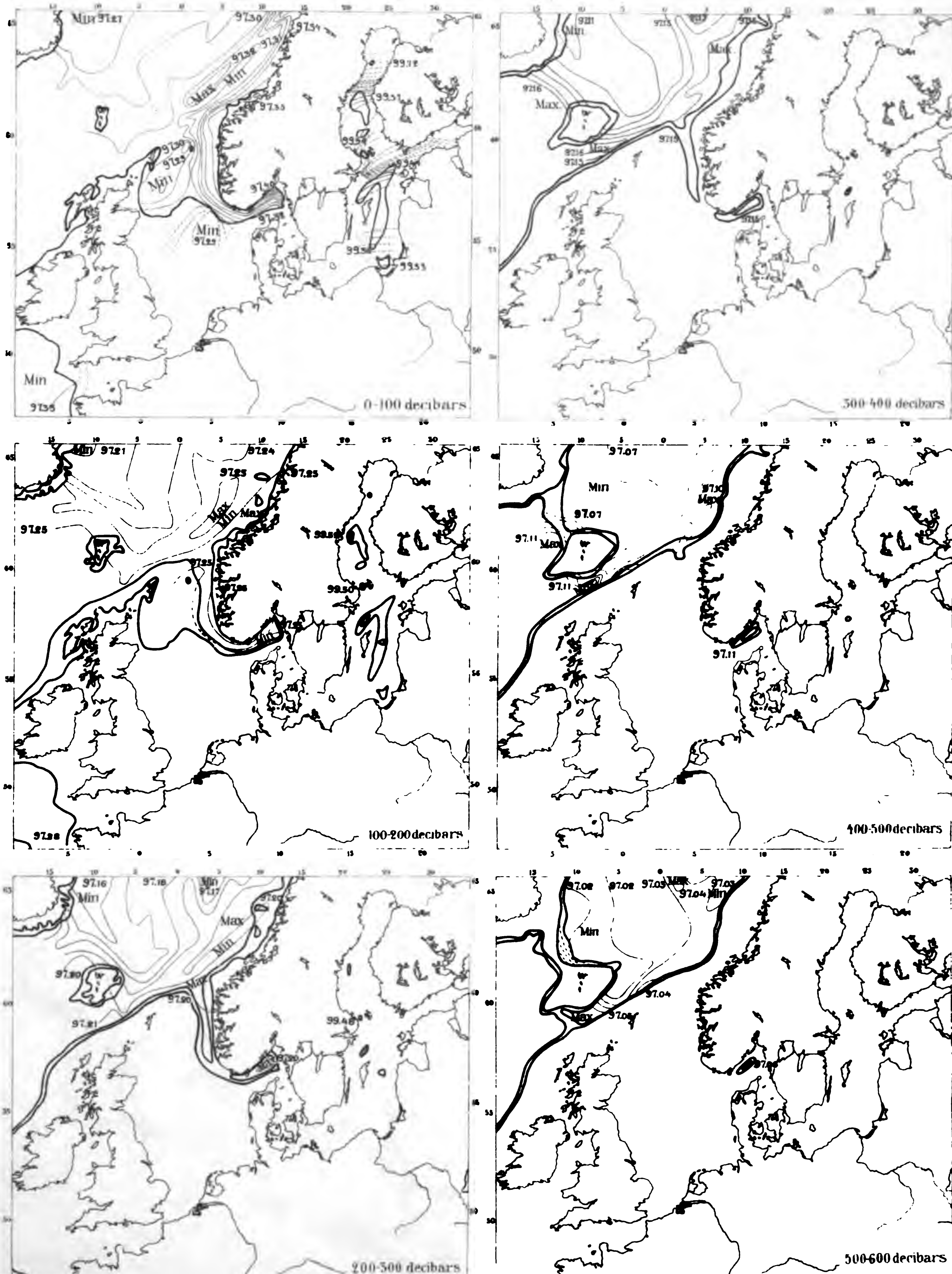


FIG. 28 (Continued). Field of mass, May, 1904, represented by charts giving the average specific volume ($10^{-5}m^3$/ton) of the sea-water in isobaric sheets, or the thickness of these sheets in dyn. meters.

But, then, instead of drawing profile curves of true specific volume, we may draw profile curves for constant values of the anomaly of the specific volume. In order to draw sections of this kind we have to place at proper mutual distances verticals like the second of figs. 25 and 26 and join points of the same value of pressure and points of the same value of the anomaly of the specific volume. In the same way, by taking verticals like the third of figs. 25 and 26, we might draw curves through points of the same potential values and through points of the same value of the density anomaly.

These curves for equal values of the anomaly show on an exaggerated scale the deviation of the isosteric or of the isopycnic curves from the horizontal course.

85. Example:—Northern European Waters, May, 1904.—Since November, 1901, hydrographic expeditions have been sent out iour times a year from most northern European states for the exploration of the northern European waters. The chart (fig. 29) shows the places where soundings were made by the expeditions in May–June, 1904.* The soundings are very far from being simultaneous. But having no data from true simultaneous soundings, and our object here being mainly to exemplify our methods, and not as yet to discuss the actual states of the sea, we have treated the soundings as if they were simultaneous. How great errors consequently may be introduced it is not possible to find out before more is known of oceanic motion.

The charts of fig. 27 show the topography of surfaces of equal sea-pressure relatively to the physical sea-level, or, according to the other interpretation, the topography of the true isobaric surfaces relatively to the ideal initial surface of the pressure of 10 decibars. The curves are drawn ior every dynamic centimeter of depth. The most direct method of obtaining them is to note on the chart the depths calculated from the different soundings and to draw the curves by means of these numbers. But it facilitates the work to note not the absolute depths (column 15 of tables U and W, pp. 126, 128), but the anomalies of depth (column 14 of same tables). For the course and the mutual distance between the curves is determined by the anomalies, while the addition of the constant normal depth of the surface is required only to determine the situation of one of the curves representing an integer value of the total depth. The heavy curves show the intersection of the isobaric surfaces with the bottom of the sea. As the isobaric surtaces are practically level, these limiting curves are obtained at once from a bathymetric chart.

The first six charts of fig. 27 show the topography of six isobaric surfaces with the interval of pressure of 10 decibars, the next six that of six others with the interval of pressure of 100 decibars. A general view of the charts shows that they contain a greater number of lines the more we proceed downward. This does not mean, of course, that the deeper surfaces are necessarily less level than the higher ones. On the contrary, at a certain depth, diftering according to circumstances, the isobaric surfaces will show a minimum of deviation from the absolute level surfaces. The greater depths of the isobaric surfaces below the physical sea-level along the

* Bulletin des Resultats Acquis pendent les Courses Periodiques, Année 1903–1904. No. 4. Mai, 1904, pp. 74–98. Copenhague, 1904.

Norwegian coast or in the Baltic, therefore, tell us rather that the sea's surface is higher here than in the open sea. But to what degree this may be the case can not be decided merely from the hydrostatic treatment of sea-soundings. The topography represented by the charts is otherwise a complicated one, showing maxima and minima of distance from the physical sea-level. As a rule there is an increasing distance between physical sea-level and the different isobaric surfaces as we proceed from east to west in the Norwegian Sea, and more especially so as we

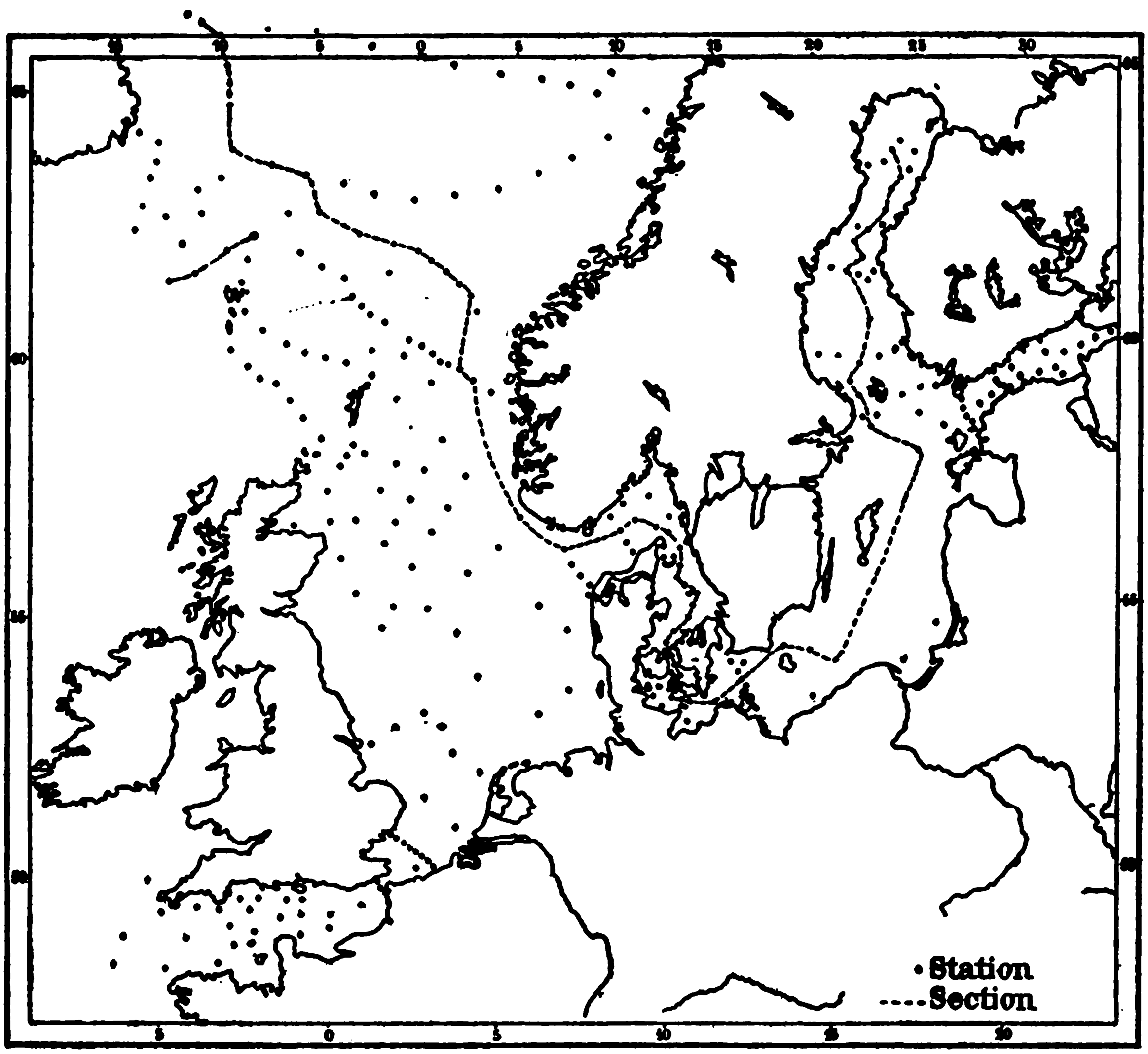


Fig. 29.—Hydrographic stations, May, 1904.

continue into the Baltic. In general, the distances are greater along the coasts than in the open sea, but even there both maxima and minima are found, in some cases side by side in a most striking manner. It is important to observe also that, as we proceed downward, the continuity of the isobaric surfaces is soon broken. Only the 10-decibar surface stretches continuously from the Atlantic into the Baltic. Already the 20-decibar surface is broken in the Belts, and for greater depths the Atlantic and the Baltic belong to different systems hydrostatically. As we proceed

downward the different deep pools are separated from each other. Finally, also, the Norwegian Sea and the open Atlantic are separated by the Shetland-Faroë-Iceland submarine ridge.

The charts of fig. 28 show the mutual topography of the successive isobaric surfaces, the first six for the interval of pressure of 10, and the last six for the interval of pressure of 100 decibars. The curves on these charts are drawn for intervals 1 dynamic millimeter. The curves along which the upper and the lower surface of

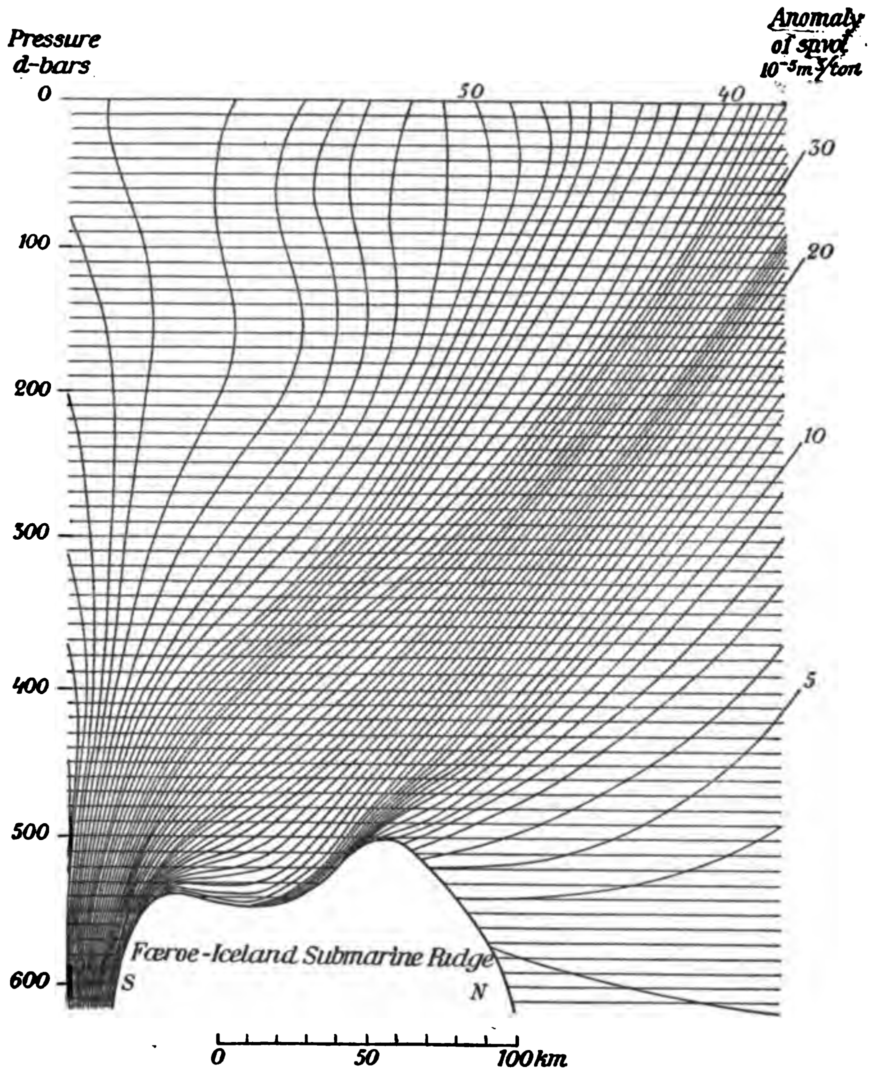


FIG. 30.—Profile curves of isobaric surfaces and surfaces of equal volume anomaly.

the sheets cut the bottom of the sea are drawn heavy. These charts show mainly the same feature as those of fig. 27. In every sheet we find a general increase of thickness as we proceed from Iceland toward Norway, and especially as we get into the Baltic. Otherwise we recognize the same maxima and minima as in the charts of fig. 27. Considered as representing the mass distribution, the charts indicate greater concentration of mass where the sheets have their minimum of

thickness and less concentration of mass where they have their maximum of thickness. On the first six charts representing sheets of 10 decibars the figures added to the curves represent the average specific volumes in the sheets after a division by 10, and in the six charts for the sheets of 100 decibars the average specific volume of the water in the sheet after a division by 100.

Besides charts representing the topography of isobaric surfaces, we might also have drawn charts representing the pressures at level surfaces. But these would have been so like the topographic ones that it would have been of no interest to draw them. A glance at table 24 H shows, for instance, that in the Baltic, where the density of the water is so near unity, we have in the upper sheets only to change the numbers added to the curves, 9.95 into 10.05, 19.94 into 20.06 and so on. Then the charts would at once be the isobaric charts for the depth of 10, 20 . . . dynamic meters below sea-level. In the greater depths also a slight change in the situation of the curves representing the integer values would be required. Outside the belts the change would have been a little greater. In the upper layers the isobaric curves would follow each other with 5.5 per cent smaller intervals than the corresponding level curves drawn in fig. 28. This percentage would increase gradually downward with the increasing density due to the compression reaching 6 at the depth of 600 dynamic meters. As, however, the course of the curves is unchanged, the two kinds of charts would be extremely like each other, the most striking difference being that maxima on the one would have been minima on the

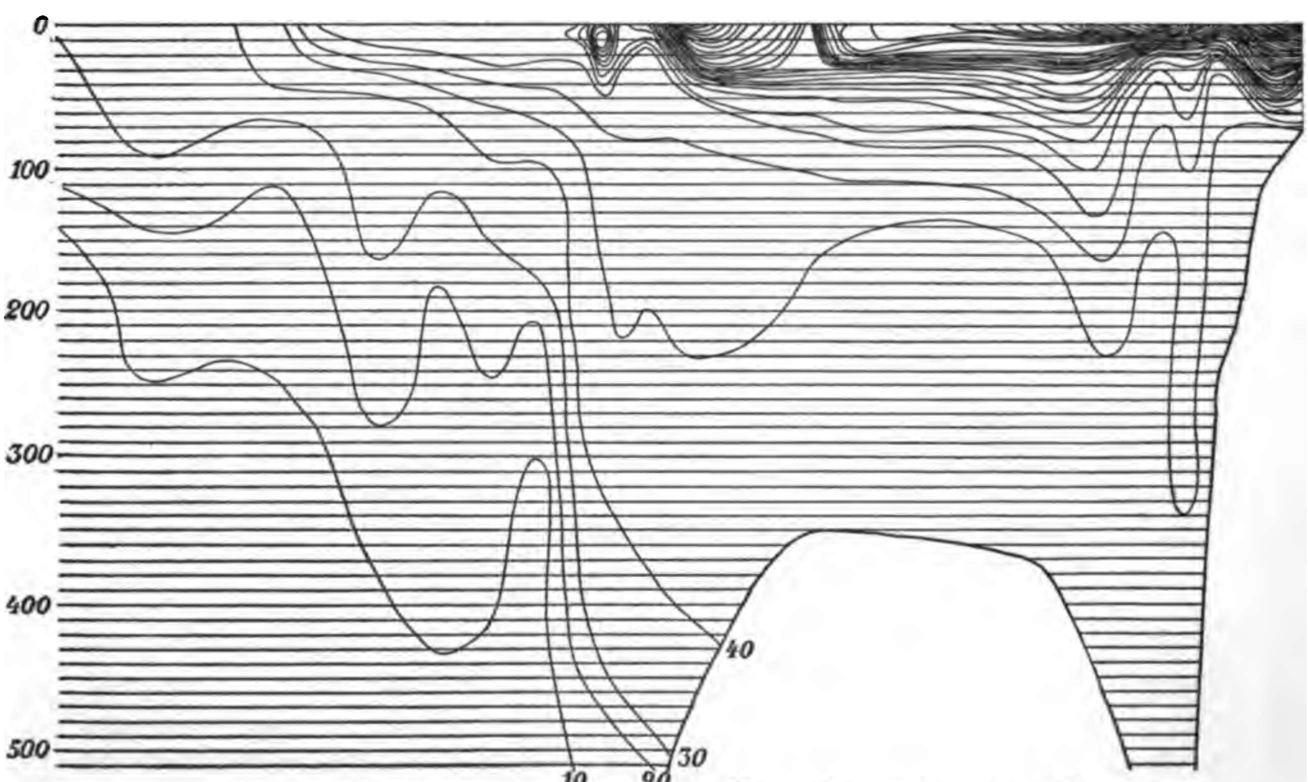
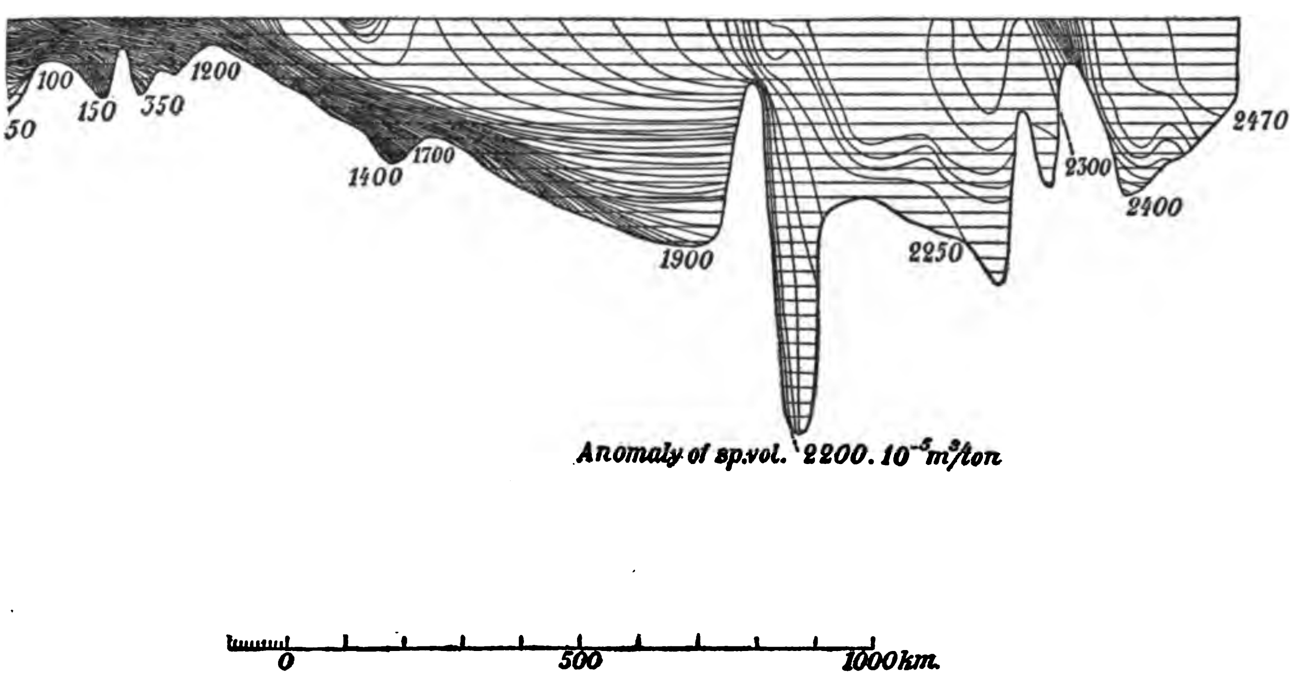


FIG. 31.—Profile curves of isobaric surfaces and surfaces of equal-volume

other, and *vice versa*. If both kinds of charts be drawn, care must be taken that they be not interchanged. The best distinction between them will be this: The figures added to the curves on the topographic charts are all a little below a certain decimal number, thus 9.74, 19.46, 29.20 . . . , while the figures on the isobaric charts are always a little above the same decimal numbers, as 10.26, 20.57, 30.91 . . . .

In figs. 30 and 31 we have, finally, two sections containing the profile curves of the isobaric surfaces drawn simply as horizontal lines, and those of the surfaces of equal anomaly of the specific volume. These give, as we have developed on an exaggerated scale, the elevations and depressions of the true isosteric curves, making the intersection with the isobaric curves more conspicuous. The first is taken across the Faroë Island bank, the second passes from the Baltic through the Belts, along the Norwegian submarine channel across the Norwegian Sea, as shown by the two lines on the station chart (fig. 29). The great density of lines of equal-volume anomaly in the Belts is especially conspicuous. Here we have the change from the brackish Baltic waters to those of the greater salinity of the open sea, and at the same time the greatest deviation from the true equilibrium conditions.

Corresponding sections containing the profile curves of the equipotential surfaces and the surfaces of equal anomaly of density would have had the same appearance, the only difference being that the curves of equal-density anomaly would run a little closer together than those of equal-volume anomaly.



Belts Baltic

anomaly. Every parallelogram re presents 0.0001 isobaric-isosteric unit-tubes.

86. Remark on Unit-Tubes. — In sections 72 and 73 we have developed some properties of the isobaric-isosteric unit-tubes, formed by the intersection of the isobaric and the isosteric surfaces. It is important to remark that these properties are retained by the tubes whose cross-section is seen in figs. 30 and 31, nothing being changed by the fact that surfaces of equal-volume anomaly have been used instead of the true isosteric surfaces to define the tubes. To show this we remark that the "normal" specific volume is constant all along an isobaric sheet, the anomaly only varying. Consequently we get unit-change of specific volume and unit-change of thickness of the sheet for every volume anomaly met with, these surfaces being drawn for unit-differences of the specific volume. Instead of counting the surfaces we can count the tubes. Further, the variation of the total specific volume and the anomaly going always in the same direction, we can use the same rule for the signs of the tubes, based upon the direction of the projection on the isobaric surfaces of the ascendant of the true specific volume or of its anomaly.

We can therefore use the expression isobaric-isosteric unit-tubes irrespectively of their being defined by true isosteric surfaces or surfaces of equal-volume anomaly. In both cases the algebraic counting of the tubes will give the change of thickness from place to place in an isobaric sheet and the horizontal course of the tubes within the sheets will be given by charts like those of fig. 28, giving the topography of the surfaces limiting the sheet relatively to each other.

The isobaric curves in figs. 30 and 31 being drawn for the interval of 1 centibar, and the curves of equal-volume anomaly for intervals of 0.0001 m^3/ton, each parallelogram in the figure will represent 0.0001 unit-tube. The curves on the charts of fig. 28 being drawn for intervals of 1 dynamic millimeter, the interval between the successive curves will represent 0.01 unit-tube.

What we have thus said of the isobaric-isosteric unit-tubes may, the terms being properly changed, be applied to the equipotential-isopycnic tubes, whether they be defined by the true isopycnic surfaces or by surfaces of equal anomaly of density.

DYNAMIC METEOROLOGY AND HYDROGRAPHY

By V. BJERKNES

AND DIFFERENT COLLABORATORS

HYDROGRAPHIC TABLES

METEOROLOGICAL TABLES.

Table 1 M.—*Normal decrease of the acceleration of gravity with the height.*

Height (meters).	0	100	200	300	400	500	600	700	800	900
29000	−0.0895	−0.0898	−0.0901	−0.0904	−0.0907	−0.0910	−0.0913	−0.0917	−0.0920	−0.0923
28000	− .0864	− .0867	− .0870	− .0873	− .0876	− .0880	− .0883	− .0886	− .0889	− .0892
27000	− .0833	− .0836	− .0839	− .0842	− .0846	− .0849	− .0852	− .0855	− .0858	− .0861
26000	− .0802	− .0805	− .0809	− .0812	− .0815	− .0818	− .0821	− .0824	− .0827	− .0830
25000	− .0772	− .0775	− .0778	− .0781	− .0784	− .0787	− .0790	− .0793	− .0796	− .0799
24000	− .0741	− .0744	− .0747	− .0750	− .0753	− .0756	− .0759	− .0762	− .0765	− .0768
23000	− .0710	− .0713	− .0716	− .0719	− .0722	− .0725	− .0728	− .0731	− .0734	− .0738
22000	− .0679	− .0682	− .0685	− .0688	− .0691	− .0694	− .0697	− .0701	− .0704	− .0707
21000	− .0648	− .0651	− .0654	− .0657	− .0660	− .0663	− .0667	− .0670	− .0673	− .0676
20000	− .0617	− .0620	− .0623	− .0626	− .0630	− .0633	− .0636	− .0639	− .0642	− .0645
19000	− .0586	− .0589	− .0593	− .0596	− .0599	− .0602	− .0605	− .0608	− .0611	− .0614
18000	− .0555	− .0559	− .0562	− .0565	− .0568	− .0571	− .0574	− .0577	− .0580	− .0583
17000	− .0525	− .0528	− .0531	− .0534	− .0537	− .0540	− .0543	− .0546	− .0549	− .0552
16000	− .0494	− .0497	− .0500	− .0503	− .0506	− .0509	− .0512	− .0515	− .0518	− .0522
15000	− .0463	− .0466	− .0469	− .0472	− .0475	− .0478	− .0481	− .0485	− .0488	− .0491
14000	− .0432	− .0435	− .0438	− .0441	− .0444	− .0447	− .0451	− .0454	− .0457	− .0460
13000	− .0401	− .0404	− .0407	− .0410	− .0414	− .0417	− .0420	− .0423	− .0426	− .0429
12000	− .0370	− .0373	− .0376	− .0380	− .0383	− .0386	− .0389	− .0392	− .0395	− .0398
11000	− .0339	− .0343	− .0346	− .0349	− .0352	− .0355	− .0358	− .0361	− .0364	− .0367
10000	− .0309	− .0312	− .0315	− .0318	− .0321	− .0324	− .0327	− .0330	− .0333	− .0336
9000	− .0278	− .0281	− .0284	− .0287	− .0290	− .0293	− .0296	− .0299	− .0302	− .0306
8000	− .0247	− .0250	− .0253	− .0256	− .0259	− .0262	− .0265	− .0268	− .0272	− .0275
7000	− .0216	− .0219	− .0222	− .0225	− .0228	− .0231	− .0235	− .0238	− .0241	− .0244
6000	− .0185	− .0188	− .0191	− .0194	− .0198	− .0201	− .0204	− .0207	− .0210	− .0213
5000	− .0154	− .0157	− .0160	− .0164	− .0167	− .0170	− .0173	− .0176	− .0179	− .0182
4000	− .0123	− .0127	− .0130	− .0133	− .0136	− .0139	− .0142	− .0145	− .0148	− .0151
3000	− .0093	− .0096	− .0099	− .0102	− .0105	− .0108	− .0111	− .0114	− .0117	− .0120
2000	− .0062	− .0065	− .0068	− .0071	− .0074	− .0077	− .0080	− .0083	− .0086	− .0089
1000	− .0031	− .0034	− .0037	− .0040	− .0043	− .0046	− .0049	− .0052	− .0056	− .0059
0	− .0000	− .0003	− .0006	− .0009	− .0012	− .0015	− .0019	− .0022	− .0025	− .0028
	0	100	200	300	400	500	600	700	800	900

Table 2 M.—*Normal value of the acceleration of gravity at sea-level.*

Latitude (degrees).	0	1	2	3	4	5	6	7	8	9
80	9.8306	9.8309	9.8312	9.8314	9.8316	9.8318	9.8319	9.8320	9.8321	9.8321
70	9.8261	9.8266	9.8272	9.8277	9.8282	9.8287	9.8291	9.8295	9.8299	9.8303
60	9.8191	9.8199	9.8207	9.8214	9.8222	9.8229	9.8235	9.8242	9.8249	9.8255
50	9.8107	9.8116	9.8124	9.8133	9.8142	9.8150	9.8159	9.8167	9.8176	9.8184
40	9.8017	9.8026	9.8035	9.8044	9.8053	9.8062	9.8071	9.8080	9.8089	9.8098
30	9.7932	9.7940	9.7948	9.7956	9.7965	9.7973	9.7982	9.7990	9.7999	9.8008
20	9.7864	9.7869	9.7876	9.7882	9.7889	9.7895	9.7902	9.7910	9.7917	9.7925
10	9.7819	9.7822	9.7825	9.7829	9.7833	9.7838	9.7842	9.7847	9.7852	9.7858
0	9.7803	9.7803	9.7804	9.7805	9.7806	9.7807	9.7809	9.7811	9.7813	9.7816

Example 1: Latitude 42° 27′, table 2 M gives 9.8039
Height 18429 meters, table 1 M gives .... −0.0569
Gravity at latitude 42° 27′, height 18429 meters.. 9.7470

Example 2: Measured gravity at the earth's surface. 9.7881
Height of station 1470 meters, table 1 M gives (sign reversed) .... +0.0045
Gravity reduced to sea level 9.7926

Table 3 M.—*Heights reduced from meters to dynamic meters, the acceleration of gravity at sea-level being 9.80.*

Height (meters).	0	100	200	300	400	500	600	700	800	900
29000	28290	28387	28484	28582	28679	28776	28873	28970	29067	29164
28000	27319	27416	27513	27610	27708	27805	27902	27999	28096	28193
27000	26347	26445	26542	26639	26736	26833	26930	27028	27125	27222
26000	25376	25473	25570	25667	25764	25862	25959	26056	26153	26250
25000	24404	24501	24598	24695	24792	24890	24987	25084	25181	25279
24000	23431	23528	23626	23723	23820	23917	24015	24112	24209	24306
23000	22458	22556	22653	22750	22847	22945	23042	23139	23237	23334
22000	21485	21583	21680	21777	21875	21972	22069	22166	22264	22361
21000	20512	20609	20707	20804	20901	20999	21096	21193	21291	21388
20000	19538	19636	19733	19830	19928	20025	20122	20220	20317	20415
19000	18564	18662	18759	18856	18954	19051	19149	19246	19344	19441
18000	17590	17687	17785	17882	17980	18077	18175	18272	18369	18467
17000	16615	16713	16810	16908	17005	17103	17200	17298	17395	17493
16000	15640	15738	15835	15933	16030	16128	16225	16323	16420	16518
15000	14665	14763	14860	14958	15055	15153	15250	15348	15446	15543
14000	13690	13787	13885	13982	14080	14178	14275	14373	14470	14568
13000	12714	12811	12909	13007	13104	13202	13299	13397	13495	13592
12000	11738	11835	11933	12031	12128	12226	12323	12421	12519	12616
11000	10761	10859	10957	11054	11152	11250	11347	11445	11543	11640
10000	9785	9882	9980	10078	10175	10273	10371	10468	10566	10664
9000	8807	8905	9003	9101	9198	9296	9394	9492	9589	9687
8000	7830	7928	8026	8123	8221	8319	8417	8514	8612	8710
7000	6852	6950	7048	7146	7244	7341	7439	7537	7635	7732
6000	5874	5972	6070	6168	6266	6363	6461	6559	6657	6755
5000	4896	4994	5092	5190	5287	5385	5483	5581	5679	5777
4000	3918	4015	4113	4211	4309	4407	4505	4603	4700	4798
3000	2939	3037	3134	3232	3330	3428	3526	3624	3722	3820
2000	1959	2057	2155	2253	2351	2449	2547	2645	2743	2841
1000	980	1078	1176	1274	1372	1470	1568	1666	1763	1861
0	0	98	196	294	392	490	588	686	784	882
	0	100	200	300	400	500	600	700	800	900

Proportionality Table.

Meters.	0	1	2	3	4	5	6	7	8	9
90	88	89	90	91	92	93	94	95	96	97
80	78	79	80	81	82	83	84	85	86	87
70	69	70	71	72	73	74	74	75	76	77
60	59	60	61	62	63	64	65	66	67	68
50	49	50	51	52	53	54	55	56	57	58
40	39	40	41	42	43	44	45	46	47	48
30	29	30	31	32	33	34	35	36	37	38
20	20	21	22	23	24	24	25	26	27	28
10	10	11	12	13	14	15	16	17	18	19
0	0	1	2	3	4	5	6	7	8	9
	0	1	2	3	4	5	6	7	8	9

Table 4 M.—*Corrections to table 3 M for values of the acceleration of gravity at sea-level different from 9.80.*

Height (meters).	Acceleration of gravity at sea-level.								
	9.76	9.77	9.78	9.79	9.80	9.81	9.82	9.83	9.84
29000	−116	−87	−58	−29	0	29	58	87	116
28000	−112	−84	−56	−28	0	28	56	84	112
27000	−108	−81	−54	−27	0	27	54	81	108
26000	−104	−78	−52	−26	0	26	52	78	104
25000	−100	−75	−50	−25	0	25	50	75	100
24000	− 96	−72	−48	−24	0	24	48	72	96
23000	− 92	−69	−46	−23	0	23	46	69	92
22000	− 88	−66	−44	−22	0	22	44	66	88
21000	− 84	−63	−42	−21	0	21	42	63	84
20000	− 80	−60	−40	−20	0	20	40	60	80
19000	− 76	−57	−38	−19	0	19	38	57	76
18000	− 72	−54	−36	−18	0	18	36	54	72
17000	− 68	−51	−34	−17	0	17	34	51	68
16000	− 64	−48	−32	−16	0	16	32	48	64
15000	− 60	−45	−30	−15	0	15	30	45	60
14000	− 56	−42	−28	−14	0	14	28	42	56
13000	− 52	−39	−26	−13	0	13	26	39	52
12000	− 48	−36	−24	−12	0	12	24	36	48
11000	− 44	−33	−22	−11	0	11	22	33	44
10000	− 40	−30	−20	−10	0	10	20	30	40
9000	− 36	−27	−18	− 9	0	9	18	27	36
8000	− 32	−24	−16	− 8	0	8	16	24	32
7000	− 28	−21	−14	− 7	0	7	14	21	28
6000	− 24	−18	−12	− 6	0	6	12	18	24
5000	− 20	−15	−10	− 5	0	5	10	15	20
4000	− 16	−12	− 8	− 4	0	4	8	12	16
3000	− 12	− 9	− 6	− 3	0	3	6	9	12
2000	− 8	− 6	− 4	− 2	0	2	4	6	8
1000	− 4	− 3	− 2	− 1	0	1	2	3	4
0	0	0	0	0	0	0	0	0	0
	9.76	9.77	9.78	9.79	9.80	9.81	9.82	9.83	9.84

Example to tables 3 M and 4 M.

1	2	3	4	5
1649	1568	48	− 2	1614
2865	2743	64	− 3	2804
4810	4700	10	− 6	4704
12428	12128	27	−15	12140

Column 1. Heights above sea-level given in meters.
2. Values of table 3 M for the heights 1600, 2800, 4800, 12400.
3. Values of proportionality table for the heights 49, 65, 10, 28.
4. Corrections from table 4 M for $g=9.7873$ at sea-level and for the heights of column 1.
5. Sum of numbers in columns 2, 3 and 4, giving the dynamic heights corresponding to the geometrical heights of column 1.

Table 5 M.—*Heights reduced from dynamic meters to meters, the acceleration of gravity at sea-level being 9.80.*

Height (dynamic meters).	0	100	200	300	400	500	600	700	800	900
29000	29729	29832	29935	30038	30141	30244	30347	30451	30554	30657
28000	28700	28803	28906	29009	29112	29215	29318	29420	29523	29626
27000	27670	27773	27876	27979	28082	28185	28288	28391	28494	28597
26000	26641	26744	26847	26950	27053	27156	27259	27362	27464	27567
25000	25612	25715	25818	25921	26024	26127	26230	26333	26435	26538
24000	24584	24687	24790	24893	24995	25098	25201	25304	25407	25510
23000	23556	23659	23762	23864	23967	24070	24173	24276	24378	24481
22000	22528	22631	22734	22836	22939	23042	23145	23248	23350	23453
21000	21501	21603	21706	21809	21912	22014	22117	22220	22323	22425
20000	20474	20576	20679	20782	20884	20987	21090	21193	21295	21398
19000	19447	19549	19652	19755	19858	19960	20063	20166	20268	20371
18000	18420	18523	18626	18728	18831	18934	19036	19139	19242	19344
17000	17394	17497	17599	17702	17805	17907	18010	18112	18215	18318
16000	16368	16471	16574	16676	16779	16881	16984	17086	17189	17292
15000	15343	15445	15548	15651	15753	15856	15958	16061	16163	16266
14000	14318	14420	14523	14625	14728	14830	14933	15035	15138	15240
13000	13293	13395	13498	13600	13703	13805	13908	14010	14113	14215
12000	12268	12371	12473	12576	12678	12781	12883	12986	13088	13190
11000	11244	11347	11449	11552	11654	11756	11859	11961	12064	12166
10000	10220	10323	10425	10528	10630	10732	10835	10937	11040	11142
9000	9197	9299	9402	9504	9606	9709	9811	9913	10016	10118
8000	8174	8276	8378	8481	8583	8685	8788	8890	8992	9095
7000	7151	7253	7355	7458	7560	7662	7765	7867	7969	8071
6000	6128	6231	6333	6435	6537	6640	6742	6844	6946	7049
5000	5106	5208	5311	5413	5515	5617	5719	5822	5924	6026
4000	4084	4186	4289	4391	4493	4595	4697	4800	4902	5004
3000	3063	3165	3267	3369	3471	3573	3676	3778	3880	3982
2000	2042	2144	2246	2348	2450	2552	2654	2756	2858	2961
1000	1021	1123	1225	1327	1429	1531	1633	1735	1837	1939
0	0	102	204	306	408	510	612	714	816	919
	0	100	200	300	400	500	600	700	800	900

Proportionality Table.

Meters.	0	1	2	3	4	5	6	7	8	9
90	92	93	94	95	96	97	98	99	100	101
80	82	83	84	85	86	87	88	89	90	91
70	71	72	73	74	76	77	78	79	80	81
60	61	62	63	64	65	66	67	68	69	70
50	51	52	53	54	55	56	57	58	59	60
40	41	42	43	44	45	46	47	48	49	50
30	31	32	33	34	35	36	37	38	39	40
20	20	21	22	23	24	26	27	28	29	30
10	10	11	12	13	14	15	16	17	18	19
0	0	1	2	3	4	5	6	7	8	9
	0	1	2	3	4	5	6	7	8	9

Table 6 M.—*Corrections to table 5 M for values of the acceleration of gravity at sea-level different from 9.80.*

Height (dynamic meters).	Acceleration of gravity at sea-level.								
	9.76	9.77	9.78	9.79	9.80	9.81	9.82	9.83	9.84
29000	121	91	60	30	0	−30	−60	−91	−121
28000	117	88	58	29	0	−29	−58	−88	−117
27000	113	84	56	28	0	−28	−56	−84	−113
26000	108	81	54	27	0	−27	−54	−81	−108
25000	104	78	52	26	0	−26	−52	−78	−104
24000	100	75	50	25	0	−25	−50	−75	−100
23000	96	72	48	24	0	−24	−48	−72	− 96
22000	92	69	46	23	0	−23	−46	−69	− 92
21000	87	66	44	22	0	−22	−44	−66	− 87
20000	83	62	42	21	0	−21	−42	−62	− 83
19000	79	59	40	20	0	−20	−40	−59	− 79
18000	75	56	37	19	0	−19	−37	−56	− 75
17000	71	53	35	18	0	−18	−35	−53	− 71
16000	67	50	33	17	0	−17	−33	−50	− 67
15000	62	47	31	16	0	−16	−31	−47	− 62
14000	58	44	29	15	0	−15	−29	−44	− 58
13000	54	41	27	14	0	−14	−27	−41	− 54
12000	50	37	25	13	0	−13	−25	−37	− 50
11000	46	34	23	11	0	−11	−23	−34	− 46
10000	42	31	21	10	0	−10	−21	−31	− 42
9000	37	28	19	9	0	− 9	−19	−28	− 37
8000	33	25	17	8	0	− 8	−17	−25	− 33
7000	29	22	15	7	0	− 7	−15	−22	− 29
6000	25	19	12	6	0	− 6	−12	−19	− 25
5000	21	16	10	5	0	− 5	−10	−16	− 21
4000	17	12	8	4	0	− 4	− 8	−12	− 17
3000	13	9	6	3	0	− 3	− 6	− 9	− 13
2000	8	6	4	2	0	− 2	− 4	− 6	− 8
1000	4	3	2	1	0	− 1	− 2	− 3	− 4
0	0	0	0	0	0	0	0	0	0
	9.76	9.77	9.78	9.79	9.80	9.81	9.82	9.83	9.84

Examples to tables 5 M and 6 M.

1	2	3	4	5
1614	1633	14	+ 2	1649
2804	2858	4	+ 3	2865
4704	4800	4	+ 6	4810
12140	12371	41	+16	12428

Column 1. Heights above sea-level given in dynamic meters.
2. Values of table 5 M for the dynamic heights, 1600, 2800, 4700, 12100.
3. Values of proportionality table for dynamic heights 14, 4, 4, 40.
4. Corrections from table 6 M for $g = 9.7873$ at sea-level and for the heights of column 1.
5. Sum of numbers in columns 2, 3 and 4, giving the geometrical heights corresponding to the dynamic heights of column 1.

Table 7 M.— *Virtual temperature of saturated air for given pressures.*

Pressure (m-bars).	Temperature (° C.).																	
	−50	−40	−30	−20	−15	−10	−5	−2	0	1	2	3	4	5	6	7	8	9
200	0.0	0.1	0.2															
250	0.0	0.0	0.1	0.4														
300	0.0	0.0	0.1	0.3														
350	0.0	0.0	0.1	0.3	0.5													
400	0.0	0.0	0.1	0.2	0.4	0.6												
450	0.0	0.0	0.1	0.2	0.4	0.6	0.9											
500	0.0	0.0	0.1	0.2	0.3	0.5	0.8	1.1	1.3									
550	0.0	0.0	0.1	0.2	0.3	0.5	0.7	1.0	1.1	1.2	1.3	1.4	1.5					
600	0.0	0.0	0.1	0.2	0.3	0.4	0.7	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.9	2.0
650	0.0	0.0	0.1	0.2	0.2	0.4	0.6	0.8	1.0	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.9
700	0.0	0.0	0.1	0.1	0.2	0.4	0.6	0.8	0.9	1.0	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7
750	0.0	0.0	0.0	0.1	0.2	0.3	0.5	0.7	0.8	0.9	1.0	1.0	1.1	1.2	1.3	1.4	1.5	1.6
800	0.0	0.0	0.0	0.1	0.2	0.3	0.5	0.7	0.8	0.8	0.9	1.0	1.1	1.1	1.2	1.3	1.4	1.5
850	0.0	0.0	0.0	0.1	0.2	0.3	0.5	0.6	0.7	0.8	0.9	0.9	1.0	1.1	1.2	1.2	1.3	1.4
900	0.0	0.0	0.0	0.1	0.2	0.3	0.5	0.6	0.7	0.8	0.8	0.9	0.9	1.0	1.1	1.2	1.3	1.4
950	0.0	0.0	0.0	0.1	0.2	0.3	0.4	0.6	0.7	0.7	0.8	0.8	0.9	1.0	1.0	1.1	1.2	1.3
1000	0.0	0.0	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.7	0.8	0.8	0.9	1.0	1.1	1.1	1.2
1050	0.0	0.0	0.0	0.1	0.2	0.2	0.4	0.5	0.6	0.6	0.7	0.7	0.8	0.9	0.9	1.0	1.1	1.2
1100	0.0	0.0	0.0	0.1	0.1	0.2	0.4	0.5	0.6	0.6	0.7	0.7	0.8	0.8	0.9	1.0	1.0	1.1

Pressure (m-bars).	Temperature (° C.).																			
	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
650	2.0	2.2	2.3	2.5	2.7	2.8	3.0	3.2	3.5	3.7										
700	1.9	2.0	2.1	2.3	2.5	2.6	2.8	3.0	3.2	3.4	3.7	3.9	4.2	4.5	4.8					
750	1.7	1.9	2.0	2.1	2.3	2.5	2.6	2.8	3.0	3.2	3.4	3.7	3.9	4.2	4.5	4.8	5.1	5.4	5.7	6.1
800	1.6	1.8	1.9	2.0	2.2	2.3	2.5	2.6	2.8	3.0	3.2	3.4	3.7	3.9	4.2	4.5	4.7	5.0	5.4	5.7
850	1.5	1.6	1.8	1.9	2.0	2.2	2.3	2.5	2.7	2.8	3.0	3.2	3.4	3.7	3.9	4.3	4.5	4.7	5.1	5.4
900	1.5	1.6	1.7	1.8	1.9	2.0	2.2	2.3	2.5	2.7	2.9	3.0	3.3	3.5	3.7	4.0	4.2	4.5	4.8	5.1
950	1.4	1.5	1.6	1.7	1.8	1.9	2.1	2.2	2.4	2.5	2.7	2.9	3.1	3.3	3.5	3.8	4.0	4.2	4.5	4.8
1000	1.3	1.4	1.5	1.6	1.7	1.8	2.0	2.1	2.3	2.4	2.6	2.7	2.9	3.1	3.3	3.6	3.8	4.0	4.3	4.6
1050	1.2	1.3	1.4	1.5	1.6	1.8	1.9	2.0	2.1	2.3	2.4	2.6	2.8	3.0	3.2	3.4	3.6	3.8	4.1	4.3
1100	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.2	2.3	2.5	2.7	2.8	3.0	3.3	3.4	3.7	3.9	4.1

Pressure (m-bars).	Temperature (° C.).																			
	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49
800	6.1	6.5	6.9	7.3	7.8															
850	5.7	6.1	6.5	6.9	7.3	7.8	8.3	8.8	9.3	9.9										
900	5.4	5.7	6.1	6.5	6.9	7.3	7.8	8.3	8.8	9.3	9.9	10.4	11.1	11.7	12.4					
950	5.1	5.4	5.8	6.1	6.5	6.9	7.4	7.8	8.3	8.8	9.3	9.9	10.5	11.1	11.7	12.4	13.1	13.9	14.7	15.6
1000	4.8	5.2	5.5	5.8	6.2	6.6	7.0	7.4	7.9	8.3	8.8	9.4	9.9	10.5	11.1	11.8	12.5	13.2	14.0	14.8
1050	4.6	4.9	5.2	5.5	5.9	6.3	6.7	7.1	7.5	7.9	8.4	8.9	9.4	10.0	10.6	11.2	11.9	12.5	13.3	14.0
1100	4.4	4.7	5.0	5.3	5.6	6.0	6.3	6.7	7.1	7.6	8.0	8.5	9.0	9.5	10.1	10.7	11.3	11.9	12.6	13.4

Example:

Pressure 871 m-bars, temperature......... 14.2°. Table 7 M gives 2.0.

63 per cent. of 2.0 gives...................... 1.3°

Virtual temperature.................. 15.5° for air of 14.2° C. and 63 per cent relative humidity at the pressure of 871 m-bars.

Table 8 M.—*Virtual temperature of saturated air in given heights.*

Height (dynamic meters).	Temperature (° C.).																	
	−50	−40	−30	−20	−15	−10	−5	−2	0	1	2	3	4	5	6	7	8	9
10000	0.0	0.0	0.1	0.4														
9500	0.0	0.0	0.1	0.4														
9000	0.0	0.0	0.1	0.3														
8500	0.0	0.0	0.1	0.3	0.5													
8000	0.0	0.0	0.1	0.3	0.5													
7500	0.0	0.0	0.1	0.3	0.4	0.7												
7000	0.0	0.0	0.1	0.2	0.4	0.6												
6500	0.0	0.0	0.1	0.2	0.4	0.6	1.0											
6000	0.0	0.0	0.1	0.2	0.4	0.6	0.9	1.2										
5500	0.0	0.0	0.1	0.2	0.3	0.5	0.8	1.1	1.3									
5000	0.0	0.0	0.1	0.2	0.3	0.5	0.8	1.0	1.2	1.3	1.4							
4500	0.0	0.0	0.1	0.2	0.3	0.5	0.7	0.9	1.1	1.2	1.3	1.4	1.5	1.6				
4000	0.0	0.0	0.1	0.2	0.3	0.4	0.7	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.8	1.9	2.0
3500	0.0	0.0	0.1	0.2	0.3	0.4	0.6	0.8	1.0	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.8	1.9
3000	0.0	0.0	0.1	0.1	0.2	0.4	0.6	0.8	0.9	1.0	1.1	1.1	1.2	1.3	1.4	1.5	1.6	1.8
2500	0.0	0.0	0.0	0.1	0.2	0.4	0.6	0.7	0.9	0.9	1.0	1.1	1.2	1.2	1.3	1.4	1.5	1.7
2000	0.0	0.0	0.0	0.1	0.2	0.3	0.5	0.7	0.8	0.9	0.9	1.0	1.1	1.2	1.3	1.3	1.4	1.6
1500	0.0	0.0	0.0	0.1	0.2	0.3	0.5	0.6	0.8	0.8	0.9	0.9	1.0	1.1	1.2	1.3	1.4	1.5
1000	0.0	0.0	0.0	0.1	0.2	0.3	0.5	0.6	0.7	0.8	0.8	0.9	1.0	1.0	1.1	1.2	1.3	1.4
500	0.0	0.0	0.0	0.1	0.2	0.3	0.4	0.6	0.7	0.7	0.8	0.8	0.9	1.0	1.0	1.1	1.2	1.3
0	0.0	0.0	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.7	0.8	0.8	0.9	1.0	1.0	1.1	1.2

Height (dynamic meters).	Temperature (° C.).																			
	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
3500	2.0	2.2	2.3	2.5	2.7															
3000	1.9	2.0	2.2	2.3	2.5	2.7	2.9	3.1	3.3	3.5										
2500	1.8	1.9	2.1	2.2	2.4	2.5	2.7	2.9	3.1	3.3	3.5	3.7	4.0	4.3	4.6					
2000	1.7	1.8	1.9	2.1	2.2	2.4	2.5	2.7	2.9	3.1	3.3	3.5	3.8	4.0	4.3	4.6	4.9	5.2	5.5	5.9
1500	1.6	1.7	1.8	1.9	2.1	2.2	2.4	2.5	2.7	2.9	3.1	3.3	3.5	3.8	4.0	4.3	4.6	4.9	5.2	5.5
1000	1.5	1.6	1.7	1.8	2.0	2.1	2.2	2.4	2.6	2.7	2.9	3.1	3.3	3.5	3.8	4.0	4.3	4.6	4.9	5.2
500	1.4	1.5	1.6	1.7	1.8	2.0	2.1	2.2	2.4	2.6	2.7	2.9	3.1	3.3	3.5	3.8	4.0	4.3	4.6	4.9
0	1.3	1.4	1.5	1.6	1.7	1.8	2.0	2.1	2.3	2.4	2.6	2.7	2.9	3.1	3.3	3.6	3.8	4.0	4.3	4.6

Height (dynamic meters).	Temperature (° C.).																			
	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49
1500	5.9	6.2	6.7	7.1	7.5															
1000	5.5	5.9	6.2	6.6	7.1	7.5	7.9	8.4	9.0	9.5										
500	5.2	5.5	5.8	6.2	6.6	7.0	7.4	7.9	8.4	8.9	9.4	10.0	10.6	11.2	11.9	12.6	13.3	14.1	14.9	15.8
0	4.8	5.2	5.5	5.8	6.2	6.6	7.0	7.4	7.9	8.3	8.8	9.4	9.9	10.5	11.1	11.8	12.5	13.2	14.0	14.8

Example:

Dynamic height 2637; temperature......... 10.4°. Table 8 M gives 1.9.
40 per cent. of 1.9 gives................ .... 0.8°
Virtual temperature...................... 11.2° for air of 10.4° C. and 46 per cent. relative humidity in the height of 2637 dynamic meters.

Table 9 M.—*Mutual distances in dynamic meters between standard isobaric surfaces.*

Standard isobaric surface (m-bars)	Average temperature of sheet (° C.).	0	1	2	3	4	5	6	7	8	9	
100												100
	−100	3442	3422	3402	3382	3362	3343	3323	3303	3283	3263	
	− 90	3641	3621	3601	3581	3561	3542	3522	3502	3482	3462	
	− 80	3840	3820	3800	3780	3760	3741	3721	3701	3681	3661	
	− 70	4039	4019	3999	3979	3959	3939	3920	3900	3880	3860	
	− 60	4238	4218	4198	4178	4158	4138	4119	4099	4079	4059	
	− 50	4437	4417	4397	4377	4357	4337	4318	4298	4278	4258	
	− 40	4636	4616	4596	4576	4556	4536	4516	4497	4477	4457	
	− 30	4835	4815	4795	4775	4755	4735	4715	4696	4676	4656	
200												200
	− 90	2130	2118	2107	2095	2083	2072	2060	2048	2037	2025	
	− 80	2246	2235	2223	2211	2200	2188	2176	2165	2153	2142	
	− 70	2363	2351	2339	2328	2316	2304	2293	2281	2270	2258	
	− 60	2479	2467	2456	2444	2432	2421	2409	2398	2386	2374	
	− 50	2595	2584	2572	2561	2549	2537	2526	2514	2502	2491	
	− 40	2712	2700	2689	2677	2665	2654	2642	2630	2619	2607	
	− 30	2828	2817	2805	2793	2782	2770	2758	2747	2735	2723	
	− 20	2945	2933	2921	2910	2898	2886	2875	2863	2851	2840	
300												300
	− 80	1594	1585	1577	1569	1561	1552	1544	1536	1528	1519	
	− 70	1676	1668	1660	1652	1643	1635	1627	1619	1610	1602	
	− 60	1759	1751	1742	1734	1726	1718	1709	1701	1693	1685	
	− 50	1841	1833	1825	1817	1808	1800	1792	1784	1775	1767	
	− 40	1924	1916	1908	1899	1891	1883	1874	1866	1858	1850	
	− 30	2007	1998	1990	1982	1974	1965	1957	1949	1941	1932	
	− 20	2089	2081	2073	2064	2056	2048	2040	2031	2023	2015	
	− 10	2172	2164	2155	2147	2139	2130	2122	2114	2106	2097	
400												400
	− 70	1300	1294	1287	1281	1275	1268	1262	1255	1249	1243	
	− 60	1364	1358	1351	1345	1339	1332	1326	1319	1313	1307	
	− 50	1428	1422	1416	1409	1403	1396	1390	1384	1377	1371	
	− 40	1492	1486	1480	1473	1467	1460	1454	1448	1441	1435	
	− 30	1556	1550	1544	1537	1531	1524	1518	1512	1505	1499	
	− 20	1621	1614	1608	1601	1595	1588	1582	1576	1569	1563	
	− 10	1685	1678	1672	1665	1659	1653	1646	1640	1633	1627	
	− 0	1749	1742	1736	1729	1723	1717	1710	1704	1697	1691	
500												500
	− 60	1115	1109	1104	1099	1094	1089	1083	1078	1073	1068	
	− 50	1167	1162	1157	1151	1146	1141	1136	1130	1125	1120	
	− 40	1219	1214	1209	1204	1198	1193	1188	1183	1178	1172	
	− 30	1272	1266	1261	1256	1251	1246	1240	1235	1230	1225	
	− 20	1324	1319	1314	1308	1303	1298	1293	1287	1282	1277	
	− 10	1376	1371	1366	1361	1355	1350	1345	1340	1335	1329	
	− 0	1429	1423	1418	1413	1408	1403	1397	1392	1387	1382	
	+ 0	1429	1434	1439	1444	1450	1455	1460	1465	1471	1476	
600												600
		0	1	2	3	4	5	6	7	8	9	

Table 9 M (continued).—*Mutual distances in dynamic meters between standard isobaric surfaces.*

Standard isobaric surface (m-bars)	Average temperature of sheet (° C.).	0	1	2	3	4	5	6	7	8	9	
600												600
	− 50	987	982	978	973	969	965	960	956	951	947	
	− 40	1031	1027	1022	1018	1013	1009	1004	1000	996	991	
	− 30	1075	1071	1066	1062	1058	1053	1049	1044	1040	1035	
	− 20	1119	1115	1111	1106	1102	1097	1093	1088	1084	1080	
	− 10	1164	1159	1155	1150	1146	1142	1137	1133	1128	1124	
	− 0	1208	1204	1199	1195	1190	1186	1181	1177	1173	1168	
	+ 0	1208	1212	1217	1221	1226	1230	1235	1239	1243	1248	
	+ 10	1252	1257	1261	1265	1270	1274	1279	1283	1288	1292	
700												700
	− 40	893	889	885	882	878	874	870	866	862	859	
	− 30	931	928	924	920	916	912	908	905	901	897	
	− 20	970	966	962	958	954	951	947	943	939	935	
	− 10	1008	1004	1000	997	993	989	985	981	977	974	
	− 0	1046	1043	1039	1035	1031	1027	1023	1020	1016	1012	
	+ 0	1046	1050	1054	1058	1062	1066	1069	1073	1077	1081	
	+ 10	1085	1089	1092	1096	1100	1104	1108	1112	1115	1119	
	+ 20	1123	1127	1131	1135	1138	1142	1146	1150	1154	1158	
800												800
	− 40	788	784	781	778	774	771	767	764	761	757	
	− 30	822	818	815	811	808	805	801	798	795	791	
	− 20	855	852	849	845	842	838	835	832	828	825	
	− 10	889	886	882	879	876	872	869	866	862	859	
	− 0	923	920	916	913	909	906	903	899	896	893	
	+ 0	923	926	930	933	937	940	943	947	950	953	
	+ 10	957	960	964	967	970	974	977	980	984	987	
	+ 20	991	994	997	1001	1004	1008	1011	1014	1018	1021	
	+ 30	1024	1028	1031	1035	1038	1041	1045	1048	1051	1055	
900												900
	− 40	705	702	699	696	693	690	687	684	680	677	
	− 30	735	732	729	726	723	720	717	714	711	708	
	− 20	765	762	759	756	753	750	747	744	741	738	
	− 10	795	792	789	786	783	780	777	774	771	768	
	− 0	826	823	820	817	814	811	808	804	801	798	
	+ 0	826	829	832	835	838	841	844	847	850	853	
	+ 10	856	859	862	865	868	871	874	877	880	883	
	+ 20	886	889	892	895	898	901	904	907	910	913	
	+ 30	916	919	922	925	928	931	935	938	941	944	
	+ 40	947	950	953	956	959	962	965	968	971	974	
1000												1000
		0	1	2	3	4	5	6	7	8	9	

Example.

1	2	3	4
700			3107
	+ 8.0°	1077	
800			2030
	+13.0	967	
900			1063
	+21.0	889	
1000			174

Column 1. Standard isobaric surfaces.
2. Average virtual temperature of standard sheets, taken from the virtual temperature diagram.
3. Distances between the isobaric surfaces found from table 9 M for the virtual temperatures of column 2.
4. Heights of standard isobaric surfaces calculated by addition of their mutual distances, 174 being the height of the 1000 m-bar standard surface above sea-level. (See example to table 11 M.)

Table 10 M.—*Distances in dynamic meters from a standard isobaric surface to a surface of given pressure, the average virtual temperature of the sheet between the surfaces being 0° C.*

I. Distances from the 100-millibar Surface.

Pressure (m-bars).	0	1	2	3	4	5	6	7	8	9
0	∞	36087	30656	27478	25224	23475	22047	20839	19792	18869
10	18044	17297	16615	15988	15407	14866	14361	13885	13438	13014
20	12612	12230	11865	11517	11183	10863	10556	10260	9975	9700
30	9435	9178	8929	8688	8454	8227	8006	7791	7582	7379
40	7180	6987	6798	6614	6433	6257	6085	5917	5752	5590
50	5432	5277	5124	4975	4829	4685	4544	4405	4269	4135
60	4003	3873	3746	3621	3497	3376	3256	3138	3022	2908
70	2795	2684	2574	2466	2360	2254	2151	2048	1947	1847
80	1749	1651	1555	1460	1366	1274	1182	1091	1002	913
90	826	739	653	569	485	402	320	239	158	79
100 m-bars.										
100	0	−78	−155	−232	−307	−383	−457	−530	−603	−675
110	−747	−818	−888	−958	−1027	−1095	−1163	−1231	−1297	−1363
120	−1429	−1494	−1558	−1623	−1686	−1749	−1811	−1873	−1934	−1996
130	−2056	−2116	−2176	−2235	−2293	−2352	−2410	−2467	−2524	−2581
140	−2637	−2693	−2748	−2803	−2857	−2912	−2965	−3019	−3072	−3125
150	−3177	−3230	−3281	−3333	−3384	−3434	−3485	−3535	−3585	−3634
160	−3683	−3732	−3780	−3829	−3876	−3924	−3972	−4019	−4065	−4112
170	−4158	−4204	−4250	−4295	−4341	−4385	−4430	−4474	−4518	−4562
180	−4606	−4650	−4693	−4736	−4778	−4821	−4863	−4905	−4947	−4988
190	−5030	−5071	−5112	−5153	−5193	−5233	−5273	−5313	−5353	−5393

II. Distances from the 200-millibar Surface.

Pressure (m-bars).	0	1	2	3	4	5	6	7	8	9
100	5432	5353	5276	5200	5124	5049	4975	4901	4829	4756
110	4685	4614	4544	4474	4405	4336	4269	4201	4135	4068
120	4003	3938	3873	3809	3746	3683	3621	3558	3497	3436
130	3376	3315	3256	3197	3138	3080	3022	2965	2908	2851
140	2795	2739	2684	2629	2574	2520	2466	2413	2360	2307
150	2254	2202	2151	2099	2048	1997	1947	1897	1847	1798
160	1749	1700	1651	1603	1555	1507	1460	1413	1366	1320
170	1274	1227	1182	1136	1091	1046	1002	957	913	869
180	826	782	739	696	653	611	569	527	485	443
190	402	361	320	279	239	198	158	118	79	39
200 m-bars.										
200	0	−39	−78	−117	−155	−194	−232	−270	−307	−345
210	−382	−420	−457	−494	−530	−567	−603	−639	−675	−711
220	−747	−782	−818	−853	−888	−923	−958	−992	−1027	−1061
230	−1095	−1129	−1163	−1197	−1230	−1264	−1297	−1330	−1363	−1396
240	−1429	−1461	−1494	−1526	−1558	−1590	−1622	−1654	−1686	−1717
250	−1749	−1780	−1811	−1842	−1873	−1904	−1934	−1965	−1995	−2026
260	−2056	−2086	−2116	−2146	−2176	−2205	−2235	−2264	−2293	−2323
270	−2352	−2381	−2410	−2438	−2467	−2496	−2524	−2552	−2580	−2609
280	−2637	−2665	−2692	−2720	−2748	−2775	−2803	−2830	−2857	−2885
290	−2912	−2939	−2965	−2992	−3019	−3046	−3072	−3099	−3125	−3151

Table 10 M (continued).—*Distances in dynamic meters from a standard isobaric surface to a surface of given pressure, the average virtual temperature of the sheet between the surfaces being 0° C.*

III. Distances from the 300-millibar Surface.

Pressure (m-bars).	0	1	2	3	4	5	6	7	8	9
200	3177	3138	3099	3061	3022	2984	2946	2908	2870	2832
210	2795	2758	2721	2684	2647	2611	2574	2538	2502	2466
220	2430	2395	2360	2324	2289	2254	2220	2185	2151	2116
230	2082	2048	2014	1980	1947	1914	1880	1847	1814	1781
240	1749	1716	1683	1651	1619	1587	1555	1523	1492	1460
250	1429	1397	1366	1335	1304	1273	1243	1212	1182	1152
260	1121	1091	1061	1031	1002	972	943	913	884	855
270	826	797	768	739	710	682	653	625	597	569
280	541	513	485	457	429	402	374	347	320	293
290	266	239	212	185	158	132	105	79	52	26
300 m-bars.										
300	0	− 26	− 52	− 78	− 104	− 130	− 155	− 181	− 206	− 232
310	− 257	− 282	− 307	− 332	− 357	− 382	− 407	− 432	− 457	− 481
320	− 506	− 530	− 555	− 579	− 603	− 627	− 651	− 675	− 699	− 723
330	− 747	− 771	− 794	− 818	− 841	− 865	− 888	− 911	− 935	− 958
340	− 981	−1004	−1027	−1050	−1072	−1095	−1118	−1141	−1163	−1186
350	−1208	−1230	−1253	−1275	−1297	−1319	−1341	−1363	−1385	−1407
360	−1429	−1450	−1472	−1494	−1515	−1537	−1558	−1580	−1601	−1622
370	−1643	−1665	−1685	−1707	−1728	−1749	−1769	−1790	−1811	−1832
380	−1852	−1873	−1893	−1914	−1934	−1955	−1975	−1995	−2016	−2036
390	−2056	−2076	−2096	−2116	−2136	−2156	−2176	−2196	−2215	−2235

IV. Distances from the 400-millibar Surface.

Pressure (m-bars).	0	1	2	3	4	5	6	7	8	9
300	2254	2228	2202	2176	2151	2125	2099	2074	2048	2023
310	1997	1972	1947	1922	1897	1872	1847	1822	1798	1773
320	1749	1724	1700	1676	1651	1627	1603	1579	1555	1531
330	1508	1484	1460	1437	1413	1390	1366	1343	1320	1297
340	1274	1251	1228	1205	1182	1159	1136	1114	1091	1069
350	1046	1024	1002	979	957	935	913	891	869	847
360	826	804	782	761	739	718	696	675	653	632
370	611	590	569	548	527	506	485	464	443	423
380	402	381	361	340	320	300	279	259	239	219
390	198	178	158	138	118	99	79	59	39	20
400 m-bars.										
400	0	− 20	− 39	− 59	− 78	− 97	− 117	− 136	− 155	− 174
410	− 193	− 213	− 232	− 251	− 270	− 289	− 307	− 326	− 345	− 364
420	− 382	− 401	− 419	− 438	− 457	− 475	− 493	− 512	− 530	− 549
430	− 567	− 585	− 603	− 621	− 639	− 657	− 675	− 693	− 711	− 729
440	− 747	− 765	− 782	− 800	− 818	− 835	− 853	− 871	− 888	− 906
450	− 923	− 940	− 958	− 975	− 993	−1010	−1027	−1044	−1061	−1078
460	−1095	−1112	−1129	−1146	−1163	−1180	−1197	−1214	−1230	−1247
470	−1264	−1280	−1297	−1314	−1330	−1347	−1363	−1380	−1396	−1412
480	−1429	−1445	−1461	−1478	−1494	−1510	−1526	−1542	−1558	−1574
490	−1590	−1606	−1622	−1638	−1654	−1670	−1686	−1702	−1717	−1733

Table 10 M (continued).—*Distances in dynamic meters from a standard isobaric surface to a surface of given pressure, the average virtual temperature of the sheet between the surfaces being 0° C.*

V. DISTANCES FROM THE 500-MILLIBAR SURFACE.

Pressure (m-bars).	0	1	2	3	4	5	6	7	8	9
400	1749	1729	1709	1690	1671	1651	1632	1613	1593	1574
410	1555	1536	1517	1498	1479	1460	1441	1422	1404	1385
420	1366	1348	1329	1310	1292	1274	1255	1237	1218	1200
430	1182	1164	1146	1127	1109	1091	1073	1055	1037	1020
440	1002	984	966	949	931	913	896	878	861	843
450	826	808	791	774	756	739	722	705	688	670
460	653	636	619	602	586	569	552	535	518	501
470	485	468	452	435	418	402	385	369	353	336
480	320	304	287	271	255	239	222	206	190	174
490	158	142	126	110	95	79	63	47	31	16
500 m-bars.										
500	0	− 16	− 31	− 47	− 62	− 78	− 93	− 109	− 124	− 140
510	− 155	− 170	− 186	− 201	− 216	− 232	− 247	− 262	− 277	− 292
520	− 307	− 322	− 337	− 352	− 367	− 382	− 397	− 412	− 427	− 442
530	− 457	− 471	− 486	− 501	− 515	− 530	− 545	− 559	− 574	− 588
540	− 603	− 618	− 632	− 646	− 661	− 675	− 690	− 704	− 718	− 733
550	− 747	− 761	− 775	− 789	− 804	− 818	− 832	− 846	− 860	− 874
560	− 888	− 902	− 916	− 930	− 944	− 958	− 972	− 985	− 999	−1013
570	−1027	−1040	−1054	−1068	−1082	−1095	−1109	−1123	−1136	−1150
580	−1163	−1177	−1190	−1204	−1217	−1230	−1244	−1257	−1270	−1284
590	−1297	−1310	−1324	−1337	−1350	−1363	−1376	−1389	−1403	−1416

VI. DISTANCES FROM THE 600-MILLIBAR SURFACE.

Pressure (m-bars).	0	1	2	3	4	5	6	7	8	9
500	1429	1413	1397	1382	1366	1351	1335	1320	1304	1289
510	1274	1258	1243	1228	1212	1197	1182	1167	1152	1136
520	1121	1106	1091	1076	1061	1046	1031	1017	1002	987
530	972	957	943	928	913	899	884	869	855	840
540	826	811	797	782	768	753	739	725	710	696
550	682	668	653	639	625	611	597	583	569	555
560	541	527	513	499	485	471	457	443	430	416
570	402	388	375	361	347	333	320	306	293	279
580	266	252	239	225	212	198	185	172	158	145
590	132	119	105	92	79	66	52	39	26	13
600 m-bars.										
600	0	− 13	− 26	− 39	− 52	− 65	− 78	− 91	− 104	− 117
610	− 130	− 142	− 155	− 168	− 181	− 194	− 206	− 219	− 232	− 244
620	− 257	− 270	− 282	− 295	− 307	− 320	− 332	− 345	− 357	− 370
630	− 382	− 395	− 407	− 419	− 432	− 444	− 457	− 469	− 481	− 493
640	− 506	− 518	− 530	− 542	− 555	− 567	− 579	− 591	− 603	− 615
650	− 627	− 639	− 651	− 663	− 675	− 687	− 699	− 711	− 723	− 735
660	− 747	− 759	− 771	− 782	− 794	− 806	− 818	− 830	− 841	− 853
670	− 865	− 876	− 888	− 900	− 911	− 923	− 935	− 946	− 958	− 969
680	− 981	− 992	−1004	−1015	−1027	−1038	−1050	−1061	−1073	−1084
690	−1095	−1107	−1118	−1129	−1141	−1152	−1163	−1174	−1186	−1197

Table 10 M (continued).—*Distances in dynamic meters from a standard isobaric surface to a surface of given pressure, the average virtual temperature of the sheet between the surfaces being 0° C.*

VII. DISTANCES FROM THE 700-MILLIBAR SURFACE.

Pressure (m-bars).	0	1	2	3	4	5	6	7	8	9
600	1208	1195	1182	1169	1116	1143	1130	1117	1104	1091
610	1078	1066	1053	1040	1027	1014	1002	989	976	964
620	951	938	926	913	901	888	876	863	851	838
630	826	813	801	788	776	764	751	739	727	715
640	702	690	678	666	653	641	629	617	605	593
650	581	569	557	545	533	521	509	497	485	473
660	461	449	438	426	414	402	390	379	367	355
670	343	332	320	308	297	285	273	262	250	239
680	227	216	204	193	181	170	158	147	136	124
690	113	101	90	79	68	56	45	34	23	11
700 m-bars.										700 m-bars.
700	0	− 11	− 22	− 33	− 45	− 56	− 67	− 78	− 89	− 100
710	− 111	− 122	− 133	− 144	− 155	− 166	− 177	− 188	− 199	− 210
720	− 221	− 232	− 242	− 253	− 264	− 275	− 286	− 297	− 307	− 318
730	− 329	− 340	− 350	− 361	− 372	− 382	− 393	− 404	− 414	− 425
740	− 435	− 446	− 457	− 467	− 478	− 488	− 499	− 509	− 520	− 530
750	− 541	− 551	− 561	− 572	− 582	− 593	− 603	− 613	− 624	− 634
760	− 644	− 655	− 665	− 675	− 685	− 696	− 706	− 716	− 726	− 737
770	− 747	− 757	− 767	− 777	− 787	− 798	− 808	− 818	− 828	− 838
780	− 848	− 858	− 868	− 878	− 888	− 898	− 908	− 918	− 928	− 938
790	− 948	− 958	− 968	− 978	− 987	− 997	−1007	−1017	−1027	− 1037

VIII. DISTANCES FROM THE 800-MILLIBAR SURFACE.

Pressure (m-bars).	0	1	2	3	4	5	6	7	8	9
700	1046	1035	1024	1013	1002	991	980	968	957	946
710	935	924	913	902	891	880	869	858	847	837
720	826	815	804	793	782	771	761	750	739	728
730	718	707	696	685	675	664	653	643	632	622
740	611	600	590	579	569	558	548	537	527	516
750	506	495	485	474	464	454	443	433	423	412
760	402	392	381	371	361	351	340	330	320	310
770	300	289	279	269	259	249	239	229	219	209
780	198	188	178	168	158	148	138	128	118	108
790	99	89	79	69	59	49	39	29	20	10
800 m-bars.										800 m-bars.
800	0	− 10	− 20	− 29	− 39	− 49	− 59	− 68	− 78	− 88
810	− 97	− 107	− 117	− 126	− 136	− 146	− 155	− 165	− 174	− 184
820	− 193	− 203	− 213	− 222	− 232	− 241	− 251	− 260	− 270	− 279
830	− 289	− 298	− 307	− 317	− 326	− 336	− 345	− 354	− 364	− 373
840	− 382	− 392	− 401	− 410	− 420	− 429	− 438	− 447	− 457	− 466
850	− 475	− 484	− 494	− 503	− 512	− 521	− 530	− 539	− 549	− 558
860	− 567	− 576	− 585	− 594	− 603	− 612	− 621	− 630	− 639	− 648
870	− 657	− 666	− 675	− 684	− 693	− 702	− 711	− 720	− 729	− 738
880	− 747	− 756	− 765	− 774	− 782	− 791	− 800	− 809	− 818	− 827
890	− 835	− 844	− 853	− 862	− 871	− 879	− 888	− 897	− 906	− 914

Table 10 M (continued).—*Distances in dynamic meters from a standard isobaric surface to a surface of given pressure, the average virtual temperature of the sheet between the surfaces being 0° C.*

IX. DISTANCES FROM THE 900-MILLIBAR SURFACE.

Pressure (m-bars).	0	1	2	3	4	5	6	7	8	9
800	923	913	903	894	884	874	864	855	845	835
810	826	816	806	797	787	777	768	758	749	739
820	729	720	710	701	691	682	672	663	653	644
830	634	625	616	606	597	587	578	569	559	550
840	541	531	522	513	503	494	485	476	466	457
850	448	439	429	420	411	402	393	384	374	365
860	356	347	338	329	320	311	302	293	284	275
870	266	257	248	239	230	221	212	203	194	185
880	176	167	158	149	141	132	123	114	105	96
890	88	79	70	61	52	44	35	26	17	9
900 m-bars.										
900	0	− 9	− 17	− 26	− 35	− 43	− 52	− 61	− 69	− 78
910	− 87	− 95	− 104	− 112	− 121	− 130	− 138	− 147	− 155	− 164
920	− 172	− 181	− 189	− 198	− 206	− 215	− 223	− 232	− 240	− 249
930	− 257	− 265	− 274	− 282	− 291	− 299	− 307	− 316	− 324	− 333
940	− 341	− 349	− 357	− 366	− 374	− 382	− 391	− 399	− 407	− 415
950	− 424	− 432	− 440	− 448	− 457	− 465	− 473	− 481	− 489	− 498
960	− 506	− 514	− 522	− 530	− 538	− 547	− 555	− 563	− 571	− 579
970	− 587	− 595	− 603	− 611	− 619	− 627	− 635	− 643	− 651	− 659
980	− 667	− 675	− 683	− 691	− 699	− 707	− 715	− 723	− 731	− 739
990	− 747	− 755	− 763	− 771	− 778	− 786	− 794	− 802	− 810	− 818

X. DISTANCES FROM THE 1000-MILLIBAR SURFACE.

Pressure (m-bars)	0	1	2	3	4	5	6	7	8	9
900	826	817	808	800	791	782	774	765	756	748
910	739	730	722	713	705	696	688	679	670	662
920	653	645	636	628	619	611	603	594	586	577
930	569	560	552	543	535	527	518	510	502	493
940	485	477	468	460	452	443	435	427	419	410
950	402	394	386	377	369	361	353	345	336	328
960	320	312	304	295	287	279	271	263	255	247
970	239	231	223	215	207	198	190	182	174	166
980	158	150	142	134	126	118	110	103	95	87
990	79	71	63	55	47	39	31	24	16	8
1000 m-bars.										
1000	0	− 8	− 16	− 23	− 31	− 39	− 47	− 55	− 62	− 70
1010	− 78	− 86	− 93	− 101	− 109	− 117	− 124	− 132	− 140	− 147
1020	− 155	− 163	− 171	− 178	− 186	− 193	− 201	− 209	− 216	− 224
1030	− 232	− 239	− 247	− 254	− 262	− 270	− 277	− 285	− 292	− 300
1040	− 307	− 315	− 322	− 330	− 337	− 345	− 352	− 360	− 367	− 375
1050	− 382	− 390	− 397	− 405	− 412	− 420	− 427	− 434	− 442	− 449
1060	− 457	− 464	− 471	− 479	− 486	− 493	− 501	− 508	− 516	− 523
1070	− 530	− 538	− 545	− 552	− 559	− 567	− 574	− 581	− 589	− 596
1080	− 603	− 610	− 618	− 625	− 632	− 639	− 647	− 654	− 661	− 668
1090	− 675	− 683	− 690	− 697	− 704	− 711	− 718	− 725	− 733	− 740

Example:

	Dynamic meters.
Height of standard surface 300 m-bars found as shown in example to table 9 M	9365
Table 10, III, gives for pressure 257.3 m-bars	+1203
Virtual-temperature diagram giving for the sheet between 300 and 257.3 m-bars the average virtual temperature —40°, table 12 gives for this temperature and the height 1203	— 177
Height of isobaric surface 257.3 m-bars	10391

Table 11 M.—*Distances in dynamic meters from the earth's surface to the nearest standard isobaric surfaces, the average virtual temperature of the sheet being 0° C.*

Pressure at station (m-bars).	Standard surfaces (m-bars). 700	600	500	Δ
600	−1208	0	1429	13
601	−1195	13	1442	13
602	−1182	26	1455	13
603	−1169	39	1468	13
604	−1156	52	1481	13
605	−1143	65	1494	13
606	−1130	78	1507	13
607	−1117	91	1520	13
608	−1104	104	1533	13
609	−1091	117	1546	13
610	−1078	130	1559	12
611	−1066	142	1571	13
612	−1053	155	1584	13
613	−1040	168	1597	13
614	−1027	181	1610	13
615	−1014	194	1623	12
616	−1002	206	1635	13
617	− 989	219	1648	13
618	− 976	232	1661	12
619	− 964	244	1673	13
620	− 951	257	1686	13
621	− 938	270	1699	12
622	− 926	282	1711	13
623	− 913	295	1724	12
624	− 901	307	1736	13
625	− 888	320	1749	12
626	− 876	332	1761	13
627	− 863	345	1774	12
628	− 851	357	1786	13
629	− 838	370	1799	12
630	− 826	382	1811	13
631	− 813	395	1824	12
632	− 801	407	1836	12
633	− 788	419	1848	13
634	− 776	432	1861	12
635	− 764	444	1873	13
636	− 751	457	1886	12
637	− 739	469	1898	12
638	− 727	481	1910	12
639	− 715	493	1922	13
640	− 702	506	1935	12
641	− 690	518	1947	12
642	− 678	530	1959	12
643	− 666	542	1971	13
644	− 653	555	1984	12
645	− 641	567	1996	12
646	− 629	579	2008	12
647	− 617	591	2020	12
648	− 605	603	2032	12
649	− 593	615	2044	12
650	− 581	627	2056	12
651	− 569	639	2068	12
652	− 557	651	2080	12
653	− 545	663	2092	12
654	− 533	675	2104	12
655	− 521	687	2116	12
656	− 509	699	2128	12
657	− 497	711	2140	12
658	− 485	723	2152	12
659	− 473	735	2164	12

Pressure at station (m-bars).	Standard surfaces (m-bars). 800	700	600	500	Δ
660		−461	747	2176	12
661		−449	759	2188	12
662		−438	771	2200	11
663		−426	782	2211	12
664		−414	794	2223	12
665		−402	806	2235	12
666		−390	818	2247	12
667		−379	830	2259	11
668		−367	841	2270	12
669		−355	853	2282	12
670		−343	865	2294	11
671		−332	876	2305	12
672		−320	888	2317	12
673		−308	900	2329	11
674		−297	911	2340	12
675		−285	923	2352	12
676		−273	935	2364	11
677		−262	946	2375	12
678		−250	958	2387	11
679		−239	969	2398	12
680		−227	981	2410	11
681		−216	992	2421	12
682		−204	1004	2433	11
683		−193	1015	2444	12
684		−181	1027	2456	11
685		−170	1038	2467	12
686		−158	1050	2479	11
687		−147	1061	2490	12
688		−136	1073	2502	11
689		−124	1084	2513	11
690		−113	1095	2524	12
691		−101	1107	2536	11
692		− 90	1118	2547	11
693		− 79	1129	2558	12
694		− 68	1141	2570	11
695		− 56	1152	2581	11
696		− 45	1163	2592	11
697		− 34	1174	2603	12
698		− 23	1186	2615	11
699		− 11	1197	2626	11
700	−1046	0	1208	2637	11
701	−1035	11	1219	2648	11
702	−1024	22	1230	2659	11
703	−1013	33	1241	2670	12
704	−1002	45	1253	2682	11
705	− 991	56	1264	2693	11
706	− 980	67	1275	2704	11
707	− 968	78	1286	2715	11
708	− 957	89	1297	2726	11
709	− 946	100	1308	2737	11
710	− 935	111	1319	2748	11
711	− 924	122	1320	2759	11
712	− 913	133	1341	2770	11
713	− 902	144	1352	2781	11
714	− 891	155	1363	2792	11
715	− 880	166	1374		11
716	− 869	177	1385		11
717	− 858	188	1396		11
718	− 847	199	1407		11
719	− 837	210	1418		11

Pressure at station (m-bars).	Standard surfaces (m-bars). 800	700	600	Δ
720	−826	221	1429	11
721	−815	232	1440	10
722	−804	242	1450	11
723	−793	253	1461	11
724	−782	264	1472	11
725	−771	275	1483	11
726	−761	286	1494	11
727	−750	297	1505	10
728	−739	307	1515	11
729	−728	318	1526	11
730	−718	329	1537	11
731	−707	340	1548	10
732	−696	350	1558	11
733	−685	361	1569	11
734	−675	372	1580	10
735	−664	382	1590	11
736	−653	393	1601	11
737	−643	404	1612	10
738	−632	414	1622	11
739	−622	425	1633	10
740	−611	435	1643	11
741	−600	446	1654	11
742	−590	457	1665	10
743	−579	467	1675	11
744	−569	478	1686	10
745	−558	488	1696	11
746	−548	499	1707	10
747	−537	509	1717	11
748	−527	520	1728	10
749	−516	530	1738	11
750	−506	541	1749	10
751	−495	551	1759	10
752	−485	561	1769	11
753	−474	572	1780	10
754	−464	582	1790	11
755	−454	593	1801	10
756	−444	603	1811	10
757	−433	613	1821	11
758	−423	624	1832	10
759	−412	634	1842	10
760	−402	644	1852	11
761	−392	655	1863	10
762	−381	665	1873	10
763	−371	675	1883	10
764	−361	685	1893	11
765	−351	696	1904	10
766	−340	706	1914	10
767	−330	716	1924	10
768	−320	726	1934	11
769	−310	737	1945	10
770	−300	747	1955	10
771	−289	757	1965	10
772	−279	767	1975	10
773	−269	777	1985	10
774	−259	787	1995	11
775	−249	798	2006	10
776	−239	808	2016	10
777	−229	818	2026	10
778	−219	828	2036	10
779	−209	838	2046	10

Table 11 M (continued).—*Distances in dynamic meters from the earth's surface to the nearest standard isobaric surfaces, the average virtual temperature of the sheet being 0° C.*

Pressure at station (m-bars.)	Standard surfaces (m-bars). 900	800	700	600	Δ
780		−198	848	2056	10
781		−188	858	2066	10
782		−178	868	2076	10
783		−168	878	2086	10
784		−158	888	2096	10
785		−148	898	2106	10
786		−138	908	2116	10
787		−128	918	2126	10
788		−118	928	2136	10
789		−108	938	2146	10
790		− 99	948	2156	10
791		− 89	958	2166	10
792		− 79	968	2176	10
793		− 69	978	2186	9
794		− 59	987	2195	10
795		− 49	997	2205	10
796		− 39	1007	2215	10
797		− 29	1017	2225	10
798		− 20	1027	2235	10
799		− 10	1037	2245	9
800	−923	0	1046	2254	10
801	−913	10	1056	2264	10
802	−903	20	1066	2274	9
803	−894	29	1075	2283	10
804	−884	39	1085	2293	10
805	−874	49	1095	2303	10
806	−864	59	1105	2313	9
807	−855	68	1114	2322	10
808	−845	78	1124	2332	10
809	−835	88	1134	2342	9
810	−826	97	1143	2351	10
811	−816	107	1153	2361	10
812	−806	117	1163	2371	9
813	−797	126	1172	2380	10
814	−787	136	1182	2390	10
815	−777	146	1192	2400	9
816	−768	155	1201	2409	10
817	−758	165	1211	2419	9
818	−749	174	1220	2428	10
819	−739	184	1230	2438	9
820	−729	193	1239	2447	10
821	−720	203	1249	2457	10
822	−710	213	1259	2467	9
823	−701	222	1268	2476	10
824	−691	232	1278	2486	9
825	−682	241	1287	2495	10
826	−672	251	1297	2505	9
827	−663	260	1306	2514	10
828	−653	270	1316	2524	9
829	−644	279	1325	2533	10
830	−634	289	1335	2543	9
831	−625	298	1344	2552	9
832	−616	307	1353	2561	10
833	−606	317	1363	2571	9
834	−597	326	1372	2580	10
835	−587	336	1382	2590	9
836	−578	345	1391	2599	9
837	−569	354	1400	2608	10
838	−559	364	1410	2618	9
839	−550	373	1419	2627	9

Pressure at station (m-bars).	Standard surfaces (m-bars). 900	800	700	Δ
840	−541	382	1428	10
841	−531	392	1438	9
842	−522	401	1447	9
843	−513	410	1456	10
844	−503	420	1466	9
845	−494	429	1475	9
846	−485	438	1484	9
847	−476	447	1493	10
848	−466	457	1503	9
849	−457	466	1512	9
850	−448	475	1521	9
851	−439	484	1530	10
852	−429	494	1540	9
853	−420	503	1549	9
854	−411	512	1558	9
855	−402	521	1567	9
856	−393	530	1576	9
857	−384	539	1585	10
858	−374	549	1595	9
859	−365	558	1604	9
860	−356	567	1613	9
861	−347	576	1622	9
862	−338	585	1631	9
863	−329	594	1640	9
864	−320	603	1649	9
865	−311	612	1658	9
866	−302	621	1667	9
867	−293	630	1676	9
868	−284	639	1685	9
869	−275	648	1694	9
870	−266	657	1703	9
871	−257	666	1712	9
872	−248	675	1721	9
873	−239	684	1730	9
874	−230	693	1739	9
875	−221	702	1748	9
876	−212	711	1757	9
877	−203	720	1766	9
878	−194	729	1775	9
879	−185	738	1784	9
880	−176	747	1793	9
881	−167	756	1802	9
882	−158	765	1811	9
883	−149	774	1820	8
884	−141	782	1828	9
885	−132	791	1837	9
886	−123	800	1846	9
887	−114	809	1855	9
888	−105	818	1864	9
889	− 96	827	1873	8
890	− 88	835	1881	9
891	− 79	844	1890	9
892	− 70	853	1899	9
893	− 61	862	1908	9
894	− 52	871	1917	8
895	− 44	879	1925	9
896	− 35	888	1934	9
897	− 26	897	1943	9
898	− 17	906	1952	8
899	− 9	914	1960	9

Pressure at station (m-bars).	Standard surfaces (m-bars). 1000	900	800	700	Δ
900	−826	0	923	1969	9
901	−817	9	932	1978	8
902	−808	17	940	1986	9
903	−800	26	949	1995	9
904	−791	35	958	2004	8
905	−782	43	966	2012	9
906	−774	52	975	2021	9
907	−765	61	984	2030	8
908	−756	69	992	2038	9
909	−748	78	1001	2047	9
910	−739	87	1010	2056	8
911	−730	95	1018	2064	9
912	−722	104	1027	2073	8
913	−713	112	1035	2081	9
914	−705	121	1044	2090	9
915	−696	130	1053	2099	8
916	−688	138	1061	2107	9
917	−679	147	1070	2116	8
918	−670	155	1078	2124	9
919	−662	164	1087	2133	8
920	−653	172	1095	2141	9
921	−645	181	1104	2150	8
922	−636	189	1112	2158	9
923	−628	198	1121	2167	8
924	−619	206	1129	2175	9
925	−611	215	1138	2184	8
926	−603	223	1146	2192	9
927	−594	232	1155	2201	8
928	−586	240	1163	2209	9
929	−577	249	1172	2218	8
930	−569	257	1180	2226	8
931	−560	265	1188	2234	9
932	−552	274	1197	2243	8
933	−543	282	1205	2251	9
934	−535	291	1214	2260	8
935	−527	299	1222	2268	8
936	−518	307	1230	2276	9
937	−510	316	1239	2285	8
938	−502	324	1247	2293	9
939	−493	333	1256	2302	8
940	−485	341	1264	2310	8
941	−477	349	1272	2318	8
942	−468	357	1280	2326	9
943	−460	366	1289	2335	8
944	−452	374	1297	2343	8
945	−443	382	1305	2351	9
946	−435	391	1314	2360	8
947	−427	399	1322	2368	8
948	−419	407	1330	2376	8
949	−410	415	1338	2384	9
950	−402	424	1347	2393	8
951	−394	432	1355	2401	8
952	−386	440	1363	2409	8
953	−377	448	1371	2417	9
954	−369	457	1380	2426	8
955	−361	465	1388	2434	8
956	−353	473	1396	2442	8
957	−345	481	1404	2450	8
958	−336	489	1412	2458	9
959	−328	498	1421	2467	8

Table 11 M (continued).—*Distances in dynamic meters from the earth's surface to the nearest standard isobaric surface, the average virtual temperature of the sheet being 0° C.*

Pressure at station (m-bars).	Standard surfaces (m-bars).				
	1000	900	800	700	Δ
960	−320	506	1429	2475	8
961	−312	514	1437	2483	8
962	−304	522	1445		8
963	−295	530	1453		8
964	−287	538	1461		9
965	−279	547	1470		8
966	−271	555	1478		8
967	−263	563	1486		8
968	−255	571	1494		8
969	−247	579	1502		8
970	−239	587	1510		8
971	−231	595	1518		8
972	−223	603	1526		8
973	−215	611	1534		8
974	−207	619	1542		8
975	−198	627	1550		8
976	−190	635	1558		8
977	−182	643	1566		8
978	−174	651	1574		8
979	−166	659	1582		8
980	−158	667	1590		8
981	−150	675	1598		8
982	−142	683	1606		8
983	−134	691	1614		8
984	−126	699	1622		8
985	−118	707	1630		8
986	−110	715	1638		8
987	−103	723	1646		8
988	− 95	731	1654		8
989	− 87	739	1662		8
990	− 79	747	1670		8
991	− 71	755	1678		8
992	− 63	763	1686		8
993	− 55	771	1694		7
994	− 47	778	1701		8
995	− 39	786	1709		8
996	− 31	794	1717		8
997	− 24	802	1725		8
998	− 16	810	1733		8
999	− 8	818	1741		8
1000	0	826	1749		8
1001	8	834	1757		8
1002	16	842	1765		7
1003	23	849	1772		8
1004	31	857	1780		8
1005	39	865	1788		8
1006	47	873	1796		8
1007	55	881	1804		7
1008	62	888	1811		8
1009	70	896	1819		8

Pressure at station (m-bars).	Standard surfaces (m-bars).			
	1000	900	800	Δ
1010	78	904	1827	8
1011	86	912	1835	7
1012	93	919	1842	8
1013	101	927	1850	8
1014	109	935	1858	8
1015	117	943	1866	7
1016	124	950	1873	8
1017	132	958	1881	8
1018	140	966	1889	7
1019	147	973	1896	8
1020	155	981	1904	8
1021	163	989	1912	8
1022	171	997	1920	7
1023	178	1004	1927	8
1024	186	1012	1935	7
1025	193	1019	1042	8
1026	201	1027	1950	8
1027	209	1035	1958	7
1028	216	1042	1965	8
1029	224	1050	1973	8
1030	232	1058	1981	7
1031	239	1065	1988	8
1032	247	1073	1996	7
1033	254	1080	2003	8
1034	262	1088	2011	8
1035	270	1096	2019	7
1036	277	1103	2026	8
1037	285	1111	2034	7
1038	292	1118	2041	8
1039	300	1126	2049	7
1040	307	1133	2056	8
1041	315	1141	2064	7
1042	322	1148	2071	8
1043	330	1156	2079	7
1044	337	1163	2086	8
1045	345	1171	2094	7
1046	352	1178	2101	8
1047	360	1186	2109	7
1048	367	1193	2116	8
1049	375	1201	2124	7
1050	382	1208	2131	8
1051	390	1216	2139	7
1052	397	1223	2146	8
1053	405	1231	2154	7
1054	412	1238	2161	8

Pressure at station (m-bars).	Standard surfaces (m-bars).			
	1000	900	800	Δ
1055	420	1246	2169	7
1056	427	1253	2176	7
1057	434	1260	2183	8
1058	442	1268	2191	7
1059	449	1275	2198	8
1060	457	1283	2206	7
1061	464	1290	2213	7
1062	471	1297	2220	8
1063	479	1305	2228	7
1064	486	1312	2235	7
1065	493	1319	2242	8
1066	501	1327	2250	7
1067	508	1334	2257	8
1068	516	1342	2265	7
1069	523	1349	2272	7
1070	530	1356	2279	8
1071	538	1364	2287	7
1072	545	1371	2294	7
1073	552	1378	2301	7
1074	559	1385	2308	8
1075	567	1393	2316	7
1076	574	1400	2323	7
1077	581	1407	2330	8
1078	589	1415	2338	7
1079	596	1422	2345	7
1080	603	1429	2352	7
1081	610	1436	2359	8
1082	618	1444	2367	7
1083	625	1451	2374	7
1084	632	1458	2381	7
1085	639	1465	2388	8
1086	647	1473	2396	7
1087	654	1480	2403	7
1088	661	1487	2410	7
1089	668	1494	2417	7
1090	675	1501	2424	8
1091	683	1509	2432	7
1092	690	1516	2439	7
1093	697	1523	2446	7
1094	704	1530	2453	7
1095	711	1537	2460	7
1096	718	1544	2467	7
1097	725	1551	2474	8
1098	733	1559	2482	7
1099	740	1566	2489	7

Example:

	Dynamic meters.
Height of station above sea level	39
Table 11 M gives for the 1000 m-bar surface and the pressure of 1015.9 at station	+123
Virtual-temperature diagram giving for the sheet between the station and the 1000 m-bar surface the average virtual temperature +25°, table 12 M gives for this temperature and the height 123 the correction	+ 11
Height of standard surface 1000 m-bars above sea level	173

Table 12 M.—*Corrections to tables 10 M and 11 M for temperature.*

Height (dynamic meters).	Temperature (°C.). 0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
20	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2
30	0	0	0	0	0	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	3	3	3	3	3	3	3	3	3	4	4	4
40	0	0	0	0	1	1	1	1	1	1	1	2	2	2	2	2	2	2	3	3	3	3	3	3	4	4	4	4	4	4	4	5	5	5	5
50	0	0	0	1	1	1	1	1	1	2	2	2	2	2	3	3	3	3	3	3	4	4	4	4	4	5	5	5	5	5	5	6	6	6	6
60	0	0	0	1	1	1	1	2	2	2	2	2	3	3	3	3	4	4	4	4	4	5	5	5	5	5	6	6	6	6	7	7	7	7	7
70	0	0	1	1	1	1	2	2	2	2	3	3	3	3	4	4	4	4	5	5	5	5	6	6	6	6	7	7	7	7	8	8	8	8	9
80	0	0	1	1	1	1	2	2	2	3	3	3	4	4	4	4	5	5	5	6	6	6	6	7	7	7	8	8	8	8	9	9	9	10	10
90	0	0	1	1	1	2	2	2	3	3	3	4	4	4	5	5	5	6	6	6	7	7	7	8	8	8	9	9	9	10	10	10	11	11	11
100	0	0	1	1	1	2	2	3	3	3	4	4	4	5	5	5	6	6	7	7	7	8	8	8	9	9	10	10	10	11	11	11	12	12	12
110	0	0	1	1	2	2	2	3	3	4	4	4	5	5	6	6	6	7	7	8	8	8	9	9	10	10	10	11	11	12	12	12	13	13	14
120	0	0	1	1	2	2	3	3	4	4	4	5	5	6	6	7	7	7	8	8	9	9	10	10	11	11	11	12	12	13	13	14	14	15	15
130	0	0	1	1	2	2	3	3	4	4	5	5	6	6	7	7	8	8	9	9	10	10	10	11	11	12	12	13	13	14	14	15	15	16	16
140	0	1	1	2	2	3	3	4	4	5	5	6	6	7	7	8	8	9	9	10	10	11	11	12	12	13	13	14	14	15	15	16	16	17	17
150	0	1	1	2	2	3	3	4	4	5	5	6	7	7	8	8	9	9	10	10	11	12	12	13	13	14	14	15	15	16	16	17	18	18	19
160	0	1	1	2	2	3	4	4	5	5	6	6	7	8	8	9	9	10	11	11	12	12	13	13	14	15	15	16	16	17	18	18	19	19	20
170	0	1	1	2	2	3	4	4	5	6	6	7	7	8	9	9	10	11	11	12	12	13	14	14	15	16	16	17	17	18	19	19	20	21	21
180	0	1	1	2	3	3	4	5	5	6	7	7	8	9	9	10	11	11	12	13	13	14	15	15	16	16	17	18	18	19	20	20	21	22	22
190	0	1	1	2	3	3	4	5	6	6	7	8	8	9	10	10	11	12	13	13	14	15	15	16	17	17	18	19	19	20	21	22	22	23	24
200	0	1	1	2	3	4	4	5	6	7	7	8	9	10	10	11	12	12	13	14	15	15	16	17	18	18	19	20	21	21	22	23	23	24	25
210	0	1	2	2	3	4	5	5	6	7	8	8	9	10	11	12	12	13	14	15	15	16	17	18	18	19	20	21	22	22	23	24	25	25	26
220	0	1	2	2	3	4	5	6	6	7	8	9	10	10	11	12	13	14	15	15	16	17	18	19	19	20	21	22	23	23	24	25	26	27	27
230	0	1	2	3	3	4	5	6	7	8	8	9	10	11	12	13	13	14	15	16	17	18	19	19	20	21	22	23	24	24	25	26	27	28	29
240	0	1	2	3	4	4	5	6	7	8	9	10	11	11	12	13	14	15	16	17	18	18	19	20	21	22	23	24	25	25	26	27	28	29	30
250	0	1	2	3	4	5	5	6	7	8	9	10	11	12	13	14	15	16	16	17	18	19	20	21	22	23	24	25	26	27	27	28	29	30	31
260	0	1	2	3	4	5	6	7	8	9	10	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	30	31	32
270	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
280	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
290	0	1	2	3	4	5	6	7	8	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	27	28	29	30	31	32	33	34	35	36
300	0	1	2	3	4	5	7	8	9	10	11	12	13	14	15	16	18	19	20	21	22	23	24	25	26	27	29	30	31	32	33	34	35	36	37
310	0	1	2	3	5	6	7	8	9	10	11	12	14	15	16	17	18	19	20	22	23	24	25	26	27	28	30	31	32	33	34	35	36	37	39
320	0	1	2	4	5	6	7	8	9	11	12	13	14	15	16	18	19	20	21	22	23	25	26	27	28	29	30	32	33	34	35	36	38	39	40
330	0	1	2	4	5	6	7	8	10	11	12	13	15	16	17	18	19	21	22	23	24	25	27	28	29	30	31	33	34	35	36	37	39	40	41
340	0	1	2	4	5	6	7	9	10	11	12	14	15	16	17	19	20	21	22	24	25	26	27	29	30	31	32	34	35	36	37	39	40	41	42
350	0	1	3	4	5	6	8	9	10	12	13	14	15	17	18	19	21	22	23	24	26	27	28	29	31	32	33	35	36	37	38	40	41	42	44
360	0	1	3	4	5	7	8	9	11	12	13	15	16	17	18	20	21	22	24	25	26	28	29	30	32	33	34	36	37	38	40	41	42	44	45
370	0	1	3	4	5	7	8	9	11	12	14	15	16	18	19	20	22	23	24	26	27	28	30	31	33	34	35	37	38	39	41	42	43	45	46
380	0	1	3	4	6	7	8	10	11	13	14	15	17	18	19	21	22	24	25	26	28	29	31	32	33	35	36	38	39	40	42	43	45	46	47
390	0	1	3	4	6	7	9	10	11	13	14	16	17	19	20	21	23	24	26	27	29	30	31	33	34	36	37	39	40	41	43	44	46	47	49

Example 1:

	Dynamic meters.
Given the average virtual temperature of 25.0° C. and the height of.........	123
Table 12 M gives for 123 meters and 25.0° C. the correction.....................	11
123 meters at 0° C. reduced to 25.0° C. gives the height of	134

Table 12 M (continued).—*Corrections to tables 10 M and 11 M for temperature.*

Height (dynamic meters).	Temperature (°C.)																																	
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
400	0	1	3	4	6	7	9	10	12	13	15	16	18	19	21	22	23	25	26	28	29	31	32	34	35	37	38	40	41	42	44	45	47	48	50
410	0	2	3	5	6	8	9	11	12	14	15	17	18	20	21	23	24	26	27	29	30	32	33	35	36	38	39	41	42	44	45	47	48	50	51
420	0	2	3	5	6	8	9	11	12	14	15	17	18	20	22	23	25	26	28	29	31	32	34	35	37	38	40	42	43	45	46	48	49	51	52
430	0	2	3	5	6	8	9	11	13	14	16	17	19	20	22	24	25	27	28	30	32	33	35	36	38	39	41	43	44	46	47	49	50	52	54
440	0	2	3	5	6	8	10	11	13	15	16	18	19	21	23	24	26	27	29	31	32	34	35	37	39	40	42	44	45	47	48	50	52	53	55
450	0	2	3	5	7	8	10	12	13	15	16	18	20	21	23	25	26	28	30	31	33	35	36	38	40	41	43	45	46	48	49	51	53	54	56
460	0	2	3	5	7	8	10	12	13	15	17	19	20	22	24	25	27	29	30	32	34	35	37	39	40	42	44	45	47	49	51	52	54	56	57
470	0	2	3	5	7	9	10	12	14	15	17	19	21	22	24	26	28	29	31	33	34	36	38	40	41	43	45	46	48	50	52	53	55	57	59
480	0	2	4	5	7	9	11	12	14	16	18	19	21	23	25	26	28	30	32	33	35	37	39	40	42	44	46	47	49	51	53	55	56	58	60
490	0	2	4	5	7	9	11	13	14	16	18	20	22	23	25	27	29	31	32	34	36	38	39	41	43	45	47	48	50	52	54	56	57	59	61
500	0	2	4	5	7	9	11	13	15	16	18	20	22	24	26	27	29	31	33	35	37	38	40	42	44	46	48	49	51	53	55	57	59	60	62
510	0	2	4	6	7	9	11	13	15	17	19	21	22	24	26	28	30	32	34	35	37	39	41	43	45	47	49	50	52	54	56	58	60	62	64
520	0	2	4	6	8	10	11	13	15	17	19	21	23	25	27	29	30	32	34	36	38	40	42	44	46	48	50	51	53	55	57	59	61	63	65
530	0	2	4	6	8	10	12	14	16	17	19	21	23	25	27	29	31	33	35	37	39	41	43	45	47	49	50	52	54	56	58	60	62	64	66
540	0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	45	47	49	51	53	55	57	59	61	63	65	67
550	0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50	52	54	56	58	60	62	64	66	68
560	0	2	4	6	8	10	12	14	16	18	21	23	25	27	29	31	33	35	37	39	41	43	45	47	49	51	53	55	57	59	62	64	66	68	70
570	0	2	4	6	8	10	13	15	17	19	21	23	25	27	29	31	33	35	38	40	42	44	46	48	50	52	54	56	58	61	63	65	67	69	71
580	0	2	4	6	8	11	13	15	17	19	21	23	25	28	30	32	34	36	38	40	42	45	47	49	51	53	55	57	59	62	64	66	68	70	72
590	0	2	4	6	9	11	13	15	17	19	22	24	26	28	30	32	35	37	39	41	43	45	48	50	52	54	56	58	61	63	65	67	69	71	73
600	0	2	4	7	9	11	13	15	18	20	22	24	26	29	31	33	35	37	40	42	44	46	48	51	53	55	57	59	62	64	66	68	70	73	75
610	0	2	4	7	9	11	13	16	18	20	22	25	27	29	31	34	36	38	40	42	45	47	49	51	54	56	58	60	63	65	67	69	72	74	76
620	0	2	5	7	9	11	14	16	18	20	23	25	27	30	32	34	36	39	41	43	45	48	50	52	55	57	59	61	64	66	68	70	73	75	77
630	0	2	5	7	9	12	14	16	18	21	23	25	28	30	32	35	37	39	42	44	46	48	51	53	55	58	60	62	65	67	69	72	74	76	78
640	0	2	5	7	9	12	14	16	19	21	23	26	28	30	33	35	38	40	42	45	47	49	52	54	56	59	61	63	66	68	70	73	75	77	80
650	0	2	5	7	10	12	14	17	19	21	24	26	29	31	33	36	38	40	43	45	48	50	52	55	57	60	62	64	67	69	71	74	76	79	81
660	0	2	5	7	10	12	15	17	19	22	24	27	29	31	34	36	39	41	44	46	48	51	53	56	58	60	63	65	68	70	73	75	77	80	82
670	0	2	5	7	10	12	15	17	20	22	25	27	29	32	34	37	39	42	44	47	49	52	54	56	59	61	64	66	69	71	74	76	79	81	83
680	0	2	5	7	10	12	15	17	20	22	25	27	30	32	35	37	40	42	45	47	50	52	55	57	60	62	65	67	70	72	75	77	80	82	85
690	0	3	5	8	10	13	15	18	20	23	25	28	30	33	35	38	40	43	45	48	51	53	56	58	61	63	66	68	71	73	76	78	81	83	86
700	0	3	5	8	10	13	15	18	21	23	26	28	31	33	36	38	41	44	46	49	51	54	56	59	62	64	67	69	72	74	77	79	82	85	87
710	0	3	5	8	10	13	16	18	21	23	26	29	31	34	36	39	42	44	47	49	52	55	57	60	62	65	68	70	73	75	78	81	83	86	88
720	0	3	5	8	11	13	16	18	21	24	26	29	32	34	37	40	42	45	47	50	53	55	58	61	63	66	69	71	74	76	79	82	84	87	90
730	0	3	5	8	11	13	16	19	21	24	27	29	32	35	37	40	43	45	48	51	53	56	59	62	64	67	70	72	75	78	80	83	86	88	91
740	0	3	5	8	11	14	16	19	22	24	27	30	33	35	38	41	43	46	49	52	54	57	60	62	65	68	70	73	76	79	81	84	87	89	92
750	0	3	5	8	11	14	16	19	22	25	27	30	33	36	38	41	44	47	49	52	55	58	60	63	66	69	71	74	77	80	82	85	88	91	93
760	0	3	6	8	11	14	17	19	22	25	28	31	33	36	39	42	45	47	50	53	56	58	61	64	67	70	72	75	78	81	84	86	89	92	95
770	0	3	6	8	11	14	17	20	23	25	28	31	34	37	39	42	45	48	51	54	56	59	62	65	68	71	73	76	79	82	85	87	90	93	96
780	0	3	6	9	11	14	17	20	23	26	29	31	34	37	40	43	46	49	51	54	57	60	63	66	69	71	74	77	80	83	86	89	91	94	97
790	0	3	6	9	12	14	17	20	23	26	29	32	35	38	41	43	46	49	52	55	58	61	64	67	69	72	75	78	81	84	87	90	93	95	98

Example 2:

	Dynamic meters.
Given the temperature 24° C. and the height of........................	1462
Table 12 M gives for 1000 meters and 24° C. the correction.........	88
Table 12 M gives for 462 meters and 24° C. the correction.........	40
1462 meters at 0° C. reduced to 24° C. gives	1590

Table 12 M (continued).—*Corrections to tables 10 M and 11 M for temperature.*

Height (dynamic meters).	Temperature (° C.). 0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
800	0	3	6	9	12	15	18	21	23	26	29	32	35	38	41	44	47	50
810	0	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	47	50
820	0	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51
830	0	3	6	9	12	15	18	21	24	27	30	33	36	40	43	46	49	52
840	0	3	6	9	12	15	18	22	25	28	31	34	37	40	43	46	49	52
850	0	3	6	9	12	16	19	22	25	28	31	34	37	40	44	47	50	53
860	0	3	6	9	13	16	19	22	25	28	32	35	38	41	44	47	50	54
870	0	3	6	10	13	16	19	22	25	29	32	35	38	41	45	48	51	54
880	0	3	6	10	13	16	19	23	26	29	32	35	39	42	45	48	52	55
890	0	3	7	10	13	16	20	23	26	29	33	36	39	42	46	49	52	55
900	0	3	7	10	13	16	20	23	26	30	33	36	40	43	46	49	53	56
910	0	3	7	10	13	17	20	23	27	30	33	37	40	43	47	50	53	57
920	0	3	7	10	13	17	20	24	27	30	34	37	40	44	47	51	54	57
930	0	3	7	10	14	17	20	24	27	31	34	37	41	44	48	51	55	58
940	0	3	7	10	14	17	21	24	28	31	34	38	41	45	48	52	55	59
950	0	3	7	10	14	17	21	24	28	31	35	38	42	45	49	52	56	59
960	0	4	7	11	14	18	21	25	28	32	35	39	42	46	49	53	56	60
970	0	4	7	11	14	18	21	25	28	32	36	39	43	46	50	53	57	60
980	0	4	7	11	14	18	22	25	29	32	36	39	43	47	50	54	57	61
990	0	4	7	11	15	18	22	25	29	33	36	40	44	47	51	54	58	62
1000	0	4	7	11	15	18	22	26	29	33	37	40	44	48	51	55	59	62
1010	0	4	7	11	15	18	22	26	30	33	37	41	44	48	52	55	59	63
1020	0	4	7	11	15	19	22	26	30	34	37	41	45	49	52	56	60	64
1030	0	4	8	11	15	19	23	26	30	34	38	42	45	49	53	57	60	64
1040	0	4	8	11	15	19	23	27	30	34	38	42	46	50	53	57	61	65
1050	0	4	8	12	15	19	23	27	31	35	38	42	46	50	54	58	62	65
1060	0	4	8	12	16	19	23	27	31	35	39	43	47	50	54	58	62	66
1070	0	4	8	12	16	20	24	27	31	35	39	43	47	51	55	59	63	67
1080	0	4	8	12	16	20	24	28	32	36	40	44	47	51	55	59	63	67
1090	0	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68
1000	0	4	7	11	15	18	22	26	29	33	37	40	44	48	51	55	59	62
2000	0	7	15	22	29	37	44	51	59	66	73	81	88	95	103	110	117	125
3000	0	11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187
4000	0	15	29	44	59	73	88	103	117	132	147	161	176	190	205	220	234	249
5000	0	18	37	55	73	92	110	128	147	165	183	201	220	238	256	275	293	311
6000	0	22	44	66	88	110	132	154	176	198	220	242	264	286	308	330	352	374
7000	0	26	51	77	103	128	154	179	205	231	256	282	308	333	359	385	410	436
8000	0	29	59	88	117	147	176	205	234	264	293	322	352	381	410	440	469	498
9000	0	33	66	99	132	165	198	231	264	297	330	363	396	429	462	495	527	560
10000	0	37	73	110	147	183	220	256	293	330	366	403	440	476	513	549	586	623

Height (dynamic meters).	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
800	53	56	59	62	64	67	70	73	76	79	82	85	88	91	94	97	100
810	53	56	59	62	65	68	71	74	77	80	83	86	89	92	95	98	101
820	54	57	60	63	66	69	72	75	78	81	84	87	90	93	96	99	102
830	55	58	61	64	67	70	73	76	79	82	85	88	91	94	97	100	103
840	55	58	62	65	68	71	74	77	80	83	86	89	92	95	98	102	105
850	56	59	62	65	68	72	75	78	81	84	87	90	93	97	100	103	106
860	57	60	63	66	69	72	76	79	82	85	88	91	95	98	101	104	107
870	57	61	64	67	70	73	76	80	83	86	89	92	96	99	102	105	108
880	58	61	64	68	71	74	77	81	84	87	90	93	97	100	103	106	110
890	59	62	65	68	72	75	78	82	85	88	91	95	98	101	104	108	111
900	59	63	66	69	73	76	79	82	86	89	92	96	99	102	105	109	112
910	60	63	67	70	73	77	80	83	87	90	93	97	100	103	107	110	113
920	61	64	67	71	74	78	81	84	88	91	94	98	101	104	108	111	115
930	61	65	68	72	75	78	82	85	89	92	95	99	102	106	109	112	116
940	62	65	69	72	76	79	83	86	90	93	96	100	103	107	110	114	117
950	63	66	70	73	77	80	84	87	90	94	97	101	104	108	111	115	118
960	63	67	70	74	77	81	84	88	91	95	98	102	105	109	113	116	120
970	64	68	71	75	78	82	85	89	92	96	99	103	107	110	114	117	121
980	65	68	72	75	79	83	86	90	93	97	101	104	108	111	115	118	122
990	65	69	73	76	80	83	87	91	94	98	102	105	109	112	116	120	123
1000	66	70	73	77	81	84	88	92	95	99	103	106	110	114	117	121	125
1010	67	70	74	78	81	85	89	92	96	100	104	107	111	115	118	122	126
1020	67	71	75	78	82	86	90	93	97	101	105	108	112	116	120	123	127
1030	68	72	75	79	83	87	91	94	98	102	106	109	113	117	121	125	128
1040	69	72	76	80	84	88	91	95	99	103	107	110	114	118	122	126	130
1050	69	73	77	81	85	88	92	96	100	104	108	112	115	119	123	127	131
1060	70	74	78	82	85	89	93	97	101	105	109	113	116	120	124	128	132
1070	71	74	78	82	86	90	94	98	102	106	110	114	118	122	125	129	133
1080	71	75	79	83	87	91	95	99	103	107	111	115	119	123	127	131	135
1090	72	76	80	84	88	92	96	100	104	108	112	116	120	124	128	132	136
1000	66	70	73	77	81	84	88	92	95	99	103	106	110	114	117	121	125
2000	132	139	147	154	161	168	176	183	190	190	205	212	220	227	234	242	249
3000	198	209	220	231	242	253	264	275	286	297	308	319	330	341	352	363	374
4000	264	278	293	308	322	337	352	366	381	396	410	425	440	454	469	484	498
5000	330	348	366	385	403	421	440	458	476	495	513	531	549	568	586	604	623
6000	396	418	440	462	484	505	527	549	571	593	615	637	659	681	703	725	747
7000	462	487	513	538	564	590	615	641	667	692	718	744	769	795	821	846	872
8000	527	557	586	615	645	674	703	733	762	791	821	850	879	908	938	967	996
9000	593	626	659	692	725	758	791	824	857	890	923	956	989	1022	1055	1088	1121
10000	659	696	733	769	806	842	879	916	952	989	1026	1062	1099	1136	1172	1209	1245

Example 3:

	Dynamic meters.
Given the temperature —52° C. and the height of................	769
Table 12 M gives for 769 meters and —32° C. the correction.........	—90
Table 12 M gives for 769 meters and —20° C. the correction.........	—56
769 meters at 0° C. reduced to —52° C. gives....................	623

Table 12 M (continued).—*Corrections to tables 10 M and 11 M for temperature.*

Height (dynamic meters).	Temperature (° C.).							
	30	40	50	60	70	80	90	100
0	0	0	0	0	0	0	0	0
10	1	1	2	2	3	3	3	4
20	2	3	4	4	5	6	7	7
30	3	4	5	7	8	9	10	11
40	4	6	7	9	10	12	13	15
50	5	7	9	11	13	15	16	18
60	7	9	11	13	15	18	20	22
70	8	10	13	15	18	21	23	26
80	9	12	15	18	21	23	26	29
90	10	13	16	20	23	26	30	33
100	11	15	18	22	26	29	33	37
110	12	16	20	24	28	32	36	40
120	13	18	22	26	31	35	40	44
130	14	19	24	29	33	38	43	48
140	15	21	26	31	36	41	46	51
150	16	22	27	33	38	44	49	55
160	18	23	29	35	41	47	53	59
170	19	25	31	37	44	50	56	62
180	20	26	33	40	46	53	59	66
190	21	28	35	42	49	56	63	70
200	22	29	37	44	51	59	66	73
210	23	31	38	46	54	62	69	77
220	24	32	40	48	56	64	73	81
230	25	34	42	51	59	67	76	84
240	26	35	44	53	62	70	79	88
250	27	37	46	55	64	73	82	92
260	29	38	48	57	67	76	86	95
270	30	40	49	59	69	79	89	99
280	31	41	51	62	72	82	92	103
290	32	42	53	64	74	85	96	106
300	33	44	55	66	77	88	99	110
310	34	45	57	68	79	91	102	114
320	35	47	59	70	82	94	105	117
330	36	48	60	73	85	97	109	121
340	37	50	62	75	87	100	112	125
350	38	51	64	77	90	103	115	128
360	40	53	66	79	92	105	119	132
370	41	54	68	81	95	108	122	136
380	42	56	70	84	97	111	125	139
390	43	57	71	86	100	114	129	143

Height (dynamic meters).	Temperature (° C.).							
	30	40	50	60	70	80	90	100
400	44	59	73	88	103	117	132	147
410	45	60	75	90	105	120	135	150
420	46	62	77	92	108	123	138	154
430	47	63	79	95	110	126	142	158
440	48	64	81	97	113	129	145	161
450	49	66	82	99	115	132	148	165
460	51	67	84	101	118	135	152	168
470	52	69	86	103	121	138	155	172
480	53	70	88	105	123	141	158	176
490	54	72	90	108	126	144	162	179
500	55	73	92	110	128	147	165	183
510	56	75	93	112	131	149	168	187
520	57	76	95	114	133	152	171	190
530	58	78	97	116	136	155	175	194
540	59	79	99	119	138	158	178	198
550	60	81	101	121	141	161	181	201
560	62	82	103	123	144	164	185	205
570	63	84	104	125	146	167	188	209
580	64	85	106	127	149	170	191	212
590	65	86	108	130	151	173	195	216
600	66	88	110	132	154	176	198	220
610	67	89	112	134	156	179	201	223
620	68	91	114	136	159	182	204	227
630	69	92	115	138	162	185	208	231
640	70	94	117	141	164	188	211	234
650	71	95	119	143	167	190	214	238
660	73	97	121	145	169	193	218	242
670	74	98	123	147	172	196	221	245
680	75	100	125	149	174	199	224	249
690	76	101	126	152	177	202	227	253
700	77	103	128	154	179	205	231	256
710	78	104	130	156	182	208	234	260
720	79	105	132	158	185	211	237	264
730	80	107	134	160	187	214	241	267
740	81	108	136	163	190	217	244	271
750	82	110	137	165	192	220	247	275
760	84	111	139	167	195	223	251	278
770	85	113	141	169	197	226	254	282
780	86	114	143	171	200	229	257	286
790	87	116	145	174	203	232	260	289

Height (dynamic meters).	Temperature (° C.).							
	30	40	50	60	70	80	90	100
800	88	117	147	176	205	234	264	293
810	89	119	148	178	208	237	267	297
820	90	120	150	180	210	240	270	300
830	91	122	152	182	213	243	274	304
840	92	123	154	185	215	246	277	308
850	93	125	156	187	218	249	280	311
860	95	126	158	189	221	252	284	315
870	96	127	159	191	223	255	287	319
880	97	129	161	193	226	258	290	322
890	98	130	163	196	228	261	293	326
900	99	132	165	198	231	264	297	330
910	100	133	167	200	233	267	300	333
920	101	135	168	202	236	270	303	337
930	102	136	170	204	238	273	307	341
940	103	138	172	207	241	275	310	344
950	104	139	174	209	244	278	313	348
960	105	141	176	211	246	281	316	352
970	107	142	178	213	249	284	320	355
980	108	144	179	215	251	287	323	359
990	109	145	181	218	254	290	326	363
1000	110	147	183	220	256	293	330	366
1010	111	148	185	222	259	296	333	370
1020	112	149	187	224	262	299	336	374
1030	113	151	189	226	264	302	340	377
1040	114	152	190	229	267	305	343	381
1050	115	154	192	231	269	308	346	385
1060	116	155	194	233	272	311	349	388
1070	118	157	196	235	274	314	353	392
1080	119	158	198	237	277	316	356	396
1090	120	160	200	240	270	319	359	399
1000	110	147	183	220	256	293	330	366
2000	220	293	366	440	513	586	659	733
3000	330	440	549	659	769	879	989	1099
4000	440	586	733	879	1026	1172	1319	1465
5000	549	733	916	1099	1282	1465	1648	1832
6000	659	879	1099	1319	1538	1758	1978	2198
7000	769	1026	1282	1538	1795	2051	2308	2564
8000	879	1172	1465	1758	2051	2344	2637	2930
9000	989	1319	1648	1978	2308	2637	2967	3297
10000	1099	1465	1832	2198	2564	2930	3297	3663

Example 4:

	Dynamic meters.
Given temperature —69° C. and the height of	4785
Table 12 M gives for 4000 meters and —60° C. the correction	—879
Table 12 M gives for 785 meters and —60° C. the correction	—172
Table 12 M gives for 4000 meters and —9° C. the correction	—132
Table 12 M gives for 785 meters and —9° C. the correction	— 26
4785 meters at 0° C. reduced to —69° C. gives	3576

Table 13 M.—*Artificial temperature to be used in table 12 M for calculating pressures in given heights.*

Temperature (°C.).	0	1	2	3	4	5	6	7	8	9
−90	134.3	136.5	138.8	141.1	143.4	145.7	148.1	150.5	152.9	155.3
−80	113.2	115.2	117.2	119.3	121.3	123.4	125.6	127.7	129.9	132.1
−70	94.1	96.0	97.8	99.7	101.5	103.4	105.3	107.2	109.2	111.2
−60	76.9	78.5	80.2	81.9	83.6	85.3	87.0	88.8	90.6	92.3
−50	61.2	62.7	64.2	65.8	67.3	68.9	70.5	72.0	73.7	75.3
−40	46.9	48.2	49.6	51.0	52.5	53.9	55.3	56.8	58.2	59.7
−30	33.7	35.0	36.2	37.5	38.8	40.1	41.5	42.8	44.2	45.5
−20	21.6	22.8	23.9	25.1	26.3	27.5	28.7	30.0	31.2	32.4
−10	10.4	11.5	12.6	13.7	14.8	15.9	17.0	18.1	19.3	20.4
−0	0	1.0	2.0	3.0	4.1	5.1	6.1	7.2	8.2	9.3
0	0	− 1.0	− 2.0	− 3.0	− 3.9	− 4.9	− 5.9	− 6.8	− 7.8	− 8.7
10	− 9.6	−10.6	−11.5	−12.4	−13.3	−14.2	−15.1	−16.0	−16.9	−17.8
20	−18.6	−19.5	−20.4	−21.2	−22.1	−22.9	−23.7	−24.6	−25.4	−26.2
30	−27.0	−27.8	−28.6	−29.4	−30.2	−31.0	−31.8	−32.6	−33.4	−34.1
40	−34.9	−35.6	−36.4	−37.2	−37.9	−38.6	−39.4	−40.1	−40.8	−41.5

Example:

	Dynamic meters.
To calculate the pressure at the height of	5000
Height of nearest standard surface, 600 m-bars, found as in example to table 9 M	4323
Difference	+677
Table 10 M (VI) gives for +677 the pressure 550.4 m-bars; the virtual-temperature diagram giving for the sheet between the surfaces 600 and 550.4 the average virtual temperature −5°; table 13 M gives the artificial temperature + 5.1; table 12 M gives for temperature 5.1 and height 677 the correction	+ 12
Sum (artificial height)	689

Table 10 M (VI) gives for +689 the pressure 549.5, which is thus the pressure in the given height of 5000 dynamic meters.

Table 14 M.—*Specific volume of the air at the standard pressures.*

Pressure (m-bars).	Virtual temperature (centigrade).															
	−100	−90	−80	−70	−60	−50	−40	−30	−20	−10	0	10	20	30	40	50
0	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
100	4966	5253	5540	5827	6114	6401	6688	6975	7262	7549	7836	8123	8410	8697	8984	9271
200	2483	2626	2770	2913	3057	3201	3344	3488	3631	3775	3918	4062	4205	4349	4492	4636
300	1655	1751	1847	1942	2038	2134	2229	2325	2421	2516	2612	2708	2803	2899	2995	3091
400	1241	1313	1385	1457	1529	1600	1672	1744	1816	1887	1959	2031	2103	2174	2246	2318
500	993	1051	1108	1165	1223	1280	1338	1395	1452	1510	1567	1625	1682	1739	1797	1854
600	828	875	923	971	1019	1067	1115	1163	1210	1258	1306	1354	1402	1450	1497	1545
700	709	750	791	832	873	914	955	996	1037	1078	1119	1160	1201	1242	1283	1324
800	621	657	692	728	764	800	836	872	908	944	980	1015	1051	1087	1123	1159
900	552	584	616	647	679	711	743	775	807	839	871	903	934	966	998	1030
1000	497	525	554	583	611	640	669	698	726	755	784	812	841	870	898	927
1100	451	478	504	530	556	582	608	634	660	686	712	738	765	791	817	843

PROPORTIONALITY TABLE.

Pressure (m-bars).	Vitrual temperature (centigrade).										
	0	1	2	3	4	5	6	7	8	9	10
0	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
100	0	29	57	86	115	144	172	201	230	258	287
200	0	14	29	43	57	72	86	100	115	129	144
300	0	10	19	29	38	48	57	67	77	86	96
400	0	7	14	22	29	36	43	50	57	65	72
500	0	6	11	17	13	29	34	40	46	52	57
600	0	5	10	14	19	24	29	33	38	43	48
700	0	4	8	12	16	21	25	29	33	37	41
800	0	4	7	11	14	18	22	25	29	32	36
900	0	3	6	10	13	16	19	22	26	29	32
1000	0	3	6	9	11	14	17	20	23	26	29
1100	0	3	5	8	10	13	16	18	21	23	26

Example.

1	2	3	4	5
700	+ 5.8	1119	24	1143
800	+10.1	1015	0	1015
900	+16.5	903	20	923
1000	+24.7	841	13	854

Column 1. Standard isobaric surfaces.
2. Virtual temperature at standard surfaces, taken from the virtual temperature diagram.
3. Numbers found from table 14 M for pressures of column 1 and temperatures, respectively 0, +10, +10, +20.
4. Numbers found from proportionality table for pressures of column 1, and temperatures respectively 5.8, 0.1, 6.5, 4.7.
5. Sum of numbers in columns 3 and 4, giving the specific volume of the air in the standard surfaces of column 1.

Table 15 M.—*Temperature correction to be added to the virtual temperature at the earth's surface in order to give the most probable average virtual temperature in the sheet between the earth's surface and the nearest standard isobaric surfaces (based upon statistics).*

Height of standard surfaces above station (dynamic meters).	Temperature Correction.											
	Winter.			Spring.			Summer.			Autumn.		
	High.	Mean.	Low.	High.	Mean.	Low.	High.	Mean.	Low.	High.	Mean.	Low.
	° C.	° C.	° C.	° C.	° C.	° C.	° C.	° C.	° C.	° C.	° C.	° C.
2400	+2.6	+0.9	−1.6	−3.2	−5.6	−5.6	−4.0	−6.3	−6.2	−1.1	−4.6	−6.4
2300	+2.7	+1.0	−1.5	−3.0	−5.4	−5.4	−3.8	−6.1	−5.9	−0.9	−4.5	−6.1
2200	+2.7	+1.2	−1.3	−2.8	−5.1	−5.1	−3.6	−5.8	−5.6	−0.8	−4.2	−5.8
2100	+2.8	+1.3	−1.2	−2.6	−4.9	−4.9	−3.4	−5.6	−5.4	−0.6	−4.1	−5.6
2000	+2.8	+1.4	−1.1	−2.4	−4.6	−4.7	−3.2	−5.4	−5.1	−0.5	−3.8	−5.3
1900	+2.8	+1.5	−1.0	−2.2	−4.4	−4.4	−3.0	−5.2	−4.8	−0.3	−3.7	−5.0
1800	+2.8	+1.6	−0.9	−2.0	−4.2	−4.2	−2.8	−5.0	−4.5	−0.2	−3.4	−4.8
1700	+2.8	+1.6	−0.8	−1.8	−3.9	−3.9	−2.6	−4.7	−4.2	−0.1	−3.3	−4.5
1600	+2.8	+1.7	−0.7	−1.6	−3.7	−3.7	−2.4	−4.5	−4.0	+0.1	−3.1	−4.3
1500	+2.7	+1.7	−0.6	−1.4	−3.5	−3.5	−2.2	−4.3	−3.7	+0.2	−2.9	−4.0
1400	+2.6	+1.7	−0.5	−1.2	−3.2	−3.3	−2.0	−4.1	−3.4	+0.3	−2.7	−3.8
1300	+2.5	+1.7	−0.5	−1.0	−3.0	−3.0	−1.8	−3.8	−3.2	+0.4	−2.5	−3.5
1200	+2.3	+1.6	−0.4	−0.9	−2.8	−2.8	−1.6	−3.6	−2.9	+0.5	−2.3	−3.2
1100	+2.2	+1.6	−0.4	−0.7	−2.6	−2.6	−1.4	−3.4	−2.6	+0.5	−2.1	−3.0
1000	+2.1	+1.5	−0.3	−0.6	−2.3	−2.4	−1.2	−3.1	−2.4	+0.6	−1.9	−2.7
900	+2.0	+1.4	−0.3	−0.5	−2.1	−2.1	−1.0	−2.9	−2.2	+0.6	−1.7	−2.5
800	+1.8	+1.3	−0.2	−0.4	−1.9	−1.9	−0.9	−2.6	−1.9	+0.6	−1.5	−2.2
700	+1.6	+1.1	−0.2	−0.3	−1.6	−1.7	−0.7	−2.3	−1.7	+0.5	−1.4	−1.9
600	+1.4	+1.0	−0.1	−0.2	−1.4	−1.4	−0.6	−2.0	−1.4	+0.5	−1.1	−1.7
500	+1.2	+0.8	−0.1	−0.2	−1.2	−1.2	−0.4	−1.7	−1.2	+0.4	−1.0	−1.4
400	+1.0	+0.7	−0.1	−0.1	−1.0	−1.0	−0.3	−1.4	−0.9	+0.4	−0.8	−1.1
300	+0.7	+0.5	0	−0.1	−0.7	−0.7	−0.2	−1.2	−0.7	+0.3	−0.6	−0.9
200	+0.5	+0.3	0	−0.1	−0.5	−0.5	−0.2	−0.8	−0.4	+0.2	−0.4	−0.6
100	+0.2	+0.2	0	0	−0.2	−0.2	−0.1	−0.4	−0.2	+0.1	−0.2	−0.3
0	0	0	0	0	0	0	0	0	0	0	0	0

Extrapolation below the earth's surface common for all pressures and seasons.

Dynamic meters.	Temperature correction (° C.).	Dynamic meters.	Temperature correction (° C.).	Dynamic meters.	Temperature correction (° C.).
− 0	0	−500	+1.2	−1000	+2.5
−100	+0.2	−600	+1.5	−1100	+2.8
−200	+0.5	−700	+1.8	−1200	+3.0
−300	+0.8	−800	+2.0		
−400	+1.0	−900	+2.2		

Example.—Low pressure, autumn, at the station pressure 984.3 m-bars, virtual temperature 9.4°.

1	2	3	4	5	6
800	1624	−4.3	+5.1	+29	1653
900	701	−1.9	+7.5	+19	720
1000	−124	+0.3	+9.7	− 4	−128

Column 1. Standard surfaces.

2. Approximate height of these surfaces, found from table 11 M for the pressure of 984.3 m-bars at the station.
3. Temperature corrections according to table 15 M for low pressure, autumn, and for the heights of column 2.
4. Most probable average virtual temperature of the sheets between the earth and the standard surfaces of column 1, found by addition of the corrections of column 3 to the virtual temperature at the station 9.4°.
5. Corrections to the heights of column 2, found from table 12 M for the heights of column 2 and the average virtual temperatures of column 4.
6. Heights of the standard surfaces above the station, found by addition of the approximate heights of column 2 and the corrections of column 5.

Table 16 M.—*Temperature corrections for the extrapolation of virtual-temperature diagrams (based upon statistics).*

Height (dynamic meters).	Winter.			Spring.			Summer.			Autumn.		
	High.	Mean.	Low.	High.	Mean.	Low.	High.	Mean.	Low.	High.	Mean.	Low.
	°C.	°C.	°C.	°C.	°C.	°C.	°C.	°C.	°C.	°C.	°C.	°C.
2400	+0.6	−2.7	−4.9	−8.0	−11.6	−11.7	−8.9	−11.8	−13.3	−5.2	−9.6	−12.8
2300	+1.0	−2.1	−4.5	−7.6	−11.0	−11.1	−8.4	−11.3	−12.6	−4.7	−9.1	−12.2
2200	+1.4	−1.6	−4.2	−7.2	−10.5	−10.5	−8.0	−10.8	−12.0	−4.2	−8.7	−11.7
2100	+1.8	−1.1	−3.8	−6.8	− 9.9	− 9.9	−7.6	−10.3	−11.4	−3.8	−8.2	−11.1
2000	+2.2	−0.7	−3.4	−6.4	− 9.4	− 9.4	−7.2	− 9.8	−10.8	−3.4	−7.8	−10.6
1900	+2.6	−0.2	−3.1	−6.0	− 8.9	− 8.9	−6.9	− 9.4	−10.1	−3.0	−7.4	−10.0
1800	+3.0	+0.3	−2.8	−5.6	− 8.4	− 8.4	−6.5	− 8.9	− 9.5	−2.6	−7.0	− 9.5
1700	+3.3	+0.7	−2.4	−5.2	− 7.9	− 7.9	−6.1	− 8.5	− 8.9	−2.3	−6.6	− 8.9
1600	+3.6	+1.1	−2.1	−4.8	− 7.4	− 7.4	−5.7	− 8.1	− 8.3	−2.0	−6.2	− 8.4
1500	+3.9	+1.4	−1.8	−4.4	− 6.9	− 6.9	−5.3	− 7.7	− 7.7	−1.6	−5.8	− 7.9
1400	+4.2	+1.8	−1.5	−3.8	− 6.4	− 6.4	−4.9	− 7.2	− 7.1	−1.1	−5.4	− 7.4
1300	+4.2	+2.0	−1.3	−3.2	− 6.0	− 5.9	−4.5	− 6.8	− 6.5	−0.7	−5.0	− 6.8
1200	+4.3	+2.3	−1.1	−2.7	− 5.5	− 5.5	−4.0	− 6.4	− 6.0	−0.3	−4.6	− 6.3
1100	+3.6	+2.4	−0.9	−2.2	− 5.0	− 5.0	−3.5	− 6.0	− 5.4	0.0	−4.2	− 5.8
1000	+3.0	+2.5	−0.7	−1.8	− 4.6	− 4.6	−3.1	− 5.6	− 4.9	+0.3	−3.8	− 5.3
900	+3.2	+2.4	−0.6	−1.4	− 4.1	− 4.2	−2.7	− 5.2	− 4.4	+0.6	−3.4	− 4.8
800	+3.4	+2.3	−0.5	−1.2	− 3.7	− 3.8	−2.2	− 4.8	− 3.9	+0.8	−3.0	− 4.3
700	+3.0	+2.1	−0.5	−0.9	− 3.2	− 3.3	−1.8	− 4.4	− 3.4	+0.8	−2.6	− 3.8
600	+2.7	+1.9	−0.5	−0.6	− 2.8	− 2.9	−1.3	− 3.9	− 2.9	+0.8	−2.2	− 3.3
500	+2.3	+1.6	−0.3	−0.4	− 2.3	− 2.4	−1.0	− 3.2	− 2.4	+0.7	−1.8	− 2.7
400	+1.9	+1.4	−0.2	−0.3	− 1.9	− 1.9	−0.7	− 2.4	− 1.9	+0.6	−1.5	− 2.2
300	+1.4	+1.0	−0.1	−0.2	− 1.4	− 1.4	−0.5	− 2.0	− 1.4	+0.5	−1.2	− 1.7
200	+1.0	+0.7	0	−0.1	− 1.0	− 1.0	−0.3	− 1.6	− 0.9	+0.4	−0.8	− 1.2
100	+0.5	+0.3	0	−0.1	− 0.5	− 0.5	−0.2	− 0.8	− 0.4	+0.2	−0.4	− 0.6
0	0	0	0	0	0	0	0	0	0	0	0	0

Below the earth's surface.

Height (dynamic meters).	Temperature correction (°C.).	Height (dynamic meters).	Temperature correction (°C.).	Height (dynamic meters).	Temperature correction (°C.).
0	0	−400	+2	− 800	+4
−100	+0.5	−500	+2.5	− 900	+4.5
−200	+1	−600	+3	−1000	+5
−300	+1.5	−700	+3.5		

Examples.

(1) Spring, mean pressure. Required the virtual temperature 1761 dynamic meters above the earth's surface:

Given virtual temperature at the earth's surface............	+12.8
Table 16 gives for spring, mean pressure, 1761 dynamic meters..........	− 8.2
Required virtual temperature..............	+ 4.6

(2) Summer, high pressure. Height of station 391 dynamic meters above sea-level. Virtual temperature at the station +7.1. Required the virtual temperatures in standard heights. These are found as shown in the annexed scheme.

Standard heights (dynamic meters).	Corresponding heights above station (dynamic meters).	Corrections from table 16 to be added to virtual temperature +7.1 at station (°C.).	Required virtual temperature in standard heights (°C.).
2500	2109	−7.6	−0.5
2000	1609	−5.7	+1.4
1500	1109	−3.5	+3.6
1000	609	−1.3	+5.8
500	109	−0.2	+6.9
0	−391	+2	+9.1

Table 17 M. — *Change of topographic charts of isobaric surfaces into isobaric charts in level surfaces.*

I. Height of the 1000 m-bars Surface (Dynamic Meters).

Pressure at sea-level (m-bars).	Virtual temperature (° C.) in isobaric surface (1000 m-bars).										
	−50	−40	−30	−20	−10	0	10	20	30	40	50
950	−330	−344	−359	−374	−389	−403	−418	−433	−448	−463	−477
955	−296	−309	−322	−336	−349	−362	−375	−389	−402	−415	−428
960	−262	−274	−286	−297	−309	−321	−333	−344	−356	−368	−380
965	−229	−239	−249	−259	−270	−280	−290	−300	−311	−321	−331
970	−195	−204	−213	−222	−230	−239	−248	−257	−265	−274	−283
975	−162	−170	−177	−184	−192	−199	−206	−213	−221	−228	−235
980	−129	−135	−141	−147	−153	−158	−164	−170	−176	−182	−187
985	− 97	−101	−105	−110	−114	−118	−123	−127	−132	−136	−140
990	− 64	− 67	− 70	− 73	− 76	− 79	− 82	− 85	− 87	− 90	− 93
995	− 32	− 34	− 35	− 36	− 38	− 39	− 41	− 42	− 44	− 45	− 46
1000	0	0	0	0	0	0	0	0	0	0	0
1005	32	33	35	36	38	39	41	42	44	45	46
1010	64	67	69	72	75	78	81	84	87	89	92
1015	95	100	104	108	112	117	121	125	130	134	138
1020	127	132	138	144	149	155	161	166	172	178	183
1025	158	165	172	179	186	193	200	207	214	221	228
1030	189	197	206	214	223	231	240	248	257	265	274
1035	220	230	239	249	259	269	279	289	299	308	318
1040	250	261	273	284	295	306	318	329	340	351	363
1045	281	293	306	319	331	344	356	369	382	394	407
1050	311	325	339	353	367	381	395	409	423	437	451

II. Height of the 900 m-bars Surface (Dynamic Meters).

Pressure in 1000 dynamic meters height (m-bars).	Virtual temperature (° C.) in isobaric surface (900 m-bars).									
	−50	−40	−30	−20	−10	0	10	20	30	40
830	479	455	432	409	385	362	338	315	292	268
835	518	496	474	453	431	409	388	366	345	323
840	556	536	516	496	477	457	437	417	397	377
845	595	576	558	540	522	504	486	467	449	431
850	633	616	600	583	567	550	534	517	501	484
855	671	656	641	626	611	597	582	567	552	538
860	708	695	682	669	656	643	630	616	603	590
865	745	734	723	711	700	688	677	666	654	643
870	782	773	763	753	743	734	724	714	704	695
875	819	811	803	795	787	779	771	763	755	746
880	856	849	843	837	830	824	817	811	804	798
885	892	887	883	878	873	868	863	858	854	849
890	928	925	922	919	916	912	909	906	903	900
895	964	963	961	960	958	956	955	953	952	950
900	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
905	1036	1037	1039	1040	1042	1043	1045	1047	1048	1050
910	1071	1074	1077	1080	1083	1087	1090	1093	1096	1099
915	1106	1110	1115	1120	1125	1129	1134	1139	1144	1148
920	1141	1147	1153	1159	1166	1172	1178	1185	1181	1197
925	1175	1183	1191	1199	1206	1214	1222	1230	1238	1246
930	1209	1219	1228	1238	1247	1256	1266	1275	1285	1294

Example 1.—Given the topographic and isothermic charts of the 1000 m-bar surface. Required the course of the 1025 m-bar curve in sea-level.

To the pressure 1025 m-bars in table 17 M (1) corresponds height 186 dynamic meters for temperature —10° C.; height 193 dynamic meters for temperature 0° C.; height 200 dynamic meters for temperature +10° C.; and so on. The required curve runs through the points of intersection of the isothermic curve —10° C. with the level curve 186 dynamic meters; of the isothermic curve 0° C. with the level curve 193 dynamic meters; of the isothermic curve +10° C. with the level curve 200 dynamic meters; and so on.

Table 17 M (continued).—*Change of topographic charts of isobaric surfaces into isobaric charts in level surfaces.*

III. Height of the 800 m-bars Surface (Dynamic Meters).

Pressure in 2000 dynamic meters height (m-bars).	Virtual temperature (° C.) in isobaric surface (800 m-bars).								
	−50	−40	−30	−20	−10	0	10	20	30
735	1454	1430	1405	1381	1356	1332.	1307	1283	1259
740	1498	1476	1453	1431	1408	1386	1363	1341	1318
745	1542	1521	1501	1480	1460	1439	1418	1398	1377
750	1585	1566	1548	1529	1510	1492	1473	1455	1436
755	1628	1611	1595	1578	1561	1544	1528	1511	1494
760	1670	1656	1641	1626	1611	1596	1582	1567	1552
765	1713	1700	1687	1674	1661	1648	1635	1622	1610
770	1755	1744	1733	1722	1711	1700	1689	1678	1667
775	1796	1787	1778	1769	1760	1751	1741	1732	1723
780	1838	1830	1823	1816	1808	1801	1794	1787	1779
785	1879	1873	1868	1862	1857	1851	1846	1841	1835
790	1919	1916	1912	1909	1905	1901	1898	1894	1891
795	1960	1958	1956	1954	1953	1951	1949	1947	1945
800	2000	2000	2000	2000	2000	2000	2000	2000	2000
805	2040	2042	2044	2045	2047	2049	2051	2052	2054
810	2080	2083	2087	2090	2094	2097	2101	2104	2108
815	2119	2124	2129	2135	2140	2145	2151	2156	2161
820	2158	2165	2172	2179	2186	2193	2200	2207	2214
825	2196	2205	2214	2223	2232	2241	2249	2258	2267
830	2235	2246	2256	2267	2277	2288	2298	2309	2319
835	2273	2286	2298	2310	2322	2335	2347	2359	2371

IV. Height of the 700 m-bars Surface (Dynamic Meters).

Pressure in 3000 dynamic meters height (m-bars).	Virtual temperature (° C.) in isobaric surface (700 m-bars).								
	−60	−50	−40	−30	−20	−10	0	10	20
630	2351	2320	2290	2259	2229	2198	2168	2138	2107
635	2400	2372	2344	2315	2287	2259	2231	2203	2174
640	2449	2423	2397	2371	2345	2319	2293	2267	2241
645	2497	2473	2449	2426	2402	2379	2355	2331	2308
650	2544	2523	2502	2480	2459	2437	2416	2395	2373
655	2592	2573	2553	2534	2515	2496	2477	2458	2438
660	2639	2622	2605	2588	2571	2554	2537	2520	2503
665	2685	2670	2656	2641	2626	2611	2597	2582	2567
670	2731	2719	2706	2693	2681	2668	2656	2643	2630
675	2777	2766	2756	2746	2735	2725	2714	2704	2693
680	2822	2814	2806	2797	2789	2781	2772	2764	2756
685	2867	2861	2855	2849	2842	2836	2830	2824	2817
690	2912	2908	2904	2899	2895	2891	2887	2883	2879
695	2956	2954	2952	2950	2948	2946	2944	2942	2939
700	3000	3000	3000	3000	3000	3000	3000	3000	3000
705	3044	3046	3048	3050	3052	3054	3056	3058	3060
710	3087	3091	3095	3099	3103	3107	3111	3115	3119
715	3129	3136	3142	3148	3154	3160	3166	3172	3178
720	3172	3180	3188	3196	3204	3212	3220	3228	3236
725	3214	3224	3234	3244	3254	3264	3274	3284	3294
730	3256	3268	3280	3292	3304	3316	3328	3340	3352

Example 2.—Given the topographic and the isothermic chart of the 700 m-bars surface. Required the isobaric chart in level 3000 dynamic meters.

Table 17 M (IV) shows that—

The isobaric curve 700 m-bars is identical with the level curve 3000 dynamic meters.

The isobaric curve 695 m-bars goes through the points of intersection of the isothermic curve —60° C. with the level curve 2956 dynamic meters; of the isothermic curve —50° C. with the level curve 2954 dynamic meters; and so on.

The isobaric curve 690 m-bars goes through the points of intersection of the isothermic curve —60° C. with the level curve 2912 dynamic meters; of the isothermic curve —50° C. with the level curve 2908 dynamic meters; and so on.

Table 17 M (continued).—*Change of topographic charts of isobaric surfaces into isobaric charts in level surfaces.*

V. Height of the 600 m-bars Surface (Dynamic Meters).

Pressure in 4000 dynamic meters height (m-bars).	Virtual temperature (° C.) at isobaric surface (600 m-bars).									
	−70	−60	−50	−40	−30	−20	−10	0	10	20
550	3490	3465	3439	3414	3389	3364	3339	3314	3289	3264
555	3543	3521	3498	3476	3453	3431	3408	3386	3363	3341
560	3596	3576	3556	3536	3516	3496	3477	3457	3437	3417
565	3648	3631	3614	3596	3579	3562	3544	3527	3510	3492
570	3700	3685	3670	3656	3641	3626	3611	3596	3582	3567
575	3751	3739	3727	3715	3702	3690	3678	3666	3653	3641
380	3802	3792	3783	3773	3763	3753	3743	3734	3724	3714
585	3852	3845	3838	3830	3823	3816	3809	3801	3794	3787
590	3902	3897	3892	3887	3883	3878	3873	3868	3863	3858
595	3951	3949	3946	3944	3942	3939	3937	3934	3932	3930
600	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000
605	4048	4051	4053	4056	4058	4060	4063	4065	4067	4070
610	4096	4101	4106	4110	4115	4120	4125	4129	4134	4139
615	4144	4151	4158	4165	4172	4179	4186	4193	4200	4207
620	4191	4200	4209	4219	4228	4238	4247	4256	4266	4275
625	4237	4249	4261	4272	4284	4296	4307	4319	4331	4342
630	4283	4297	4311	4325	4339	4353	4367	4381	4395	4409
635	4329	4345	4361	4378	4394	4410	4426	4442	4459	4475
640	4374	4393	4411	4430	4448	4467	4485	4503	4522	4540
645	4419	4440	4461	4481	4502	4522	4543	4564	4584	4605
650	4464	4487	4509	4532	4555	4578	4601	4624	4646	4669

VI. Height of the 500 m-bars Surface (Dynamic Meters).

Pressure in 5000 dynamic meters height (m-bars).	Virtual temperature (° C.) at isobaric surface (500 m-bars).								
	−70	−60	−50	−40	−30	−20	−10	0	10
465	4575	4554	4533	4512	4491	4470	4449	4428	4407
470	4638	4620	4602	4584	4567	4549	4531	4513	4495
475	4700	4685	4670	4656	4641	4626	4611	4597	4582
480	4761	4750	4738	4726	4714	4703	4691	4679	4667
485	4822	4813	4805	4796	4787	4778	4770	4761	4752
490	4882	4876	4871	4865	4859	4853	4847	4842	4836
495	4941	4939	4936	4933	4930	4927	4924	4921	4918
500	5000	5000	5000	5000	5000	5000	5000	5000	5000
505	5058	5061	5064	5066	5069	5072	5075	5078	5081
510	5115	5121	5127	5132	5138	5144	5149	5155	5161
515	5172	5180	5189	5197	5206	5214	5223	5231	5240
520	5228	5239	5250	5262	5273	5284	5295	5306	5318
525	5283	5297	5311	5325	5339	5353	5367	5381	5395
530	5338	5355	5371	5388	5405	5421	5438	5455	5471
535	5392	5412	5431	5450	5470	5489	5508	5528	5547
540	5446	5468	5490	5512	5534	5556	5578	5600	5622
545	5499	5524	5548	5573	5597	5622	5647	5671	5696
550	5551	5579	5606	5633	5660	5687	5714	5742	5769
555	5603	5633	5663	5693	5722	5752	5782	5812	5841
560	5655	5687	5719	5752	5784	5816	5849	5881	5913
565	5706	5741	5775	5810	5845	5880	5914	5949	5984

Table 17 M (continued).—*Change of topographic charts of isobaric surfaces into isobaric charts in level surfaces.*

VII. HEIGHT OF THE 400 M-BARS SURFACE (DYNAMIC METERS).

Pressure in 7000 dynamic meters height (m-bars).	Virtual temperature (° C.) at isobaric surface (400 m-bars).								
	−80	−70	−60	−50	−40	−30	−20	−10	0
350	6253	6214	6176	6137	6098	6060	6021	5982	5944
355	6333	6299	6264	6229	6195	6160	6126	6091	6057
360	6412	6381	6351	6320	6290	6259	6229	6198	6168
365	6489	6463	6436	6410	6383	6357	6331	6304	6278
370	6566	6543	6521	6498	6476	6453	6431	6408	6386
375	6641	6622	6604	6585	6566	6548	6529	6510	6492
380	6715	6700	6685	6670	6656	6641	6626	6611	6596
385	6788	6777	6766	6755	6744	6733	6722	6711	6700
390	6859	6852	6845	6838	6830	6823	6816	6808	6801
395	6930	6927	6923	6919	6916	6912	6909	6905	6901
400	7000	7000	7000	7000	7000	7000	7000	7000	7000
405	7069	7072	7076	7080	7083	7087	7090	7094	7097
410	7137	7144	7151	7158	7165	7172	7179	7186	7193
415	7203	7214	7225	7235	7246	7256	7267	7277	7288
420	7269	7283	7297	7311	7325	7339	7353	7367	7381
425	7334	7352	7369	7386	7404	7421	7438	7456	7473
430	7399	7419	7440	7461	7481	7502	7522	7543	7564
435	7462	7486	7510	7534	7558	7582	7605	7629	7653
440	7524	7551	7579	7606	7633	7660	7687	7715	7742
445	7586	7616	7647	7677	7707	7738	7768	7799	7829
450	7646	7680	7713	7747	7780	7814	7847	7881	7914

VIII. HEIGHT OF THE 300 M-BARS SURFACE (DYNAMIC METERS).

Pressure in 9000 dynamic meters height (m-bars).	Virtual temperature (° C.) at isobaric surface (300 m-bars).								
	−90	−80	−70	−60	−50	−40	−30	−20	−10
250	8030	7977	7924	7871	7818	7765	7711	7658	7605
255	8136	8089	8042	7995	7947	7900	7853	7806	7759
260	8240	8199	8157	8116	8074	8033	7991	7950	7908
265	8343	8307	8271	8235	8199	8163	8127	8091	8055
270	8442	8412	8381	8351	8320	8290	8259	8229	8198
275	8540	8515	8490	8465	8440	8414	8389	8364	8339
280	8636	8616	8596	8576	8556	8536	8516	8497	8477
285	8730	8715	8700	8685	8670	8656	8641	8626	8611
290	8822	8812	8802	8792	8783	8773	8763	8753	8744
295	8912	8907	8902	8897	8892	8887	8883	8878	8873
300	9000	9000	9000	9000	9000	9000	9000	9000	9000
305	9087	9091	9096	9101	9106	9110	9115	9120	9125
310	9172	9181	9191	9200	9209	9219	9228	9238	9247
315	9255	9269	9283	9297	9311	9325	9339	9353	9367
320	9337	9356	9374	9393	9411	9430	9448	9467	9485
325	9418	9441	9464	9487	9509	9532	9555	9578	9601
330	9497	9524	9551	9579	9606	9633	9660	9687	9714
335	9575	9606	9638	9669	9701	9732	9763	9795	9826
340	9652	9687	9723	9758	9794	9830	9865	9901	9936
345	9727	9766	9806	9846	9886	9925	9965	10005	10044
350	9801	9844	9888	9932	9976	10020	10063	10107	10151

DYNAMIC METEOROLOGY AND HYDROGRAPHY

BY V. BJERKNES
AND DIFFERENT COLLABORATORS

APPENDIX

TO

METEOROLOGICAL AND HYDROGRAPHIC TABLES

CONTAINING TABLES TO BE USED WHEN THE OBSERVATIONS ARE GIVEN IN UNITS NOT BELONGING TO THE C.G.S. SYSTEM

Table 1 A. — *Heights reduced from feet to meters.*

Feet.	0	1000	2000	3000	4000	5000	6000	7000	8000	9000
90000	27432	27736	28041	28346	28651	28955	29260	29565	29870	30175
80000	24384	24688	24993	25298	25603	25908	26212	26517	26822	27127
70000	21336	21640	21945	22250	22555	22860	23164	23469	23774	24079
60000	18288	18592	18897	19202	19507	19812	20116	20421	20726	21031
50000	15240	15545	15849	16154	16459	16764	17068	17373	17678	17983
40000	12192	12497	12801	13106	13411	13716	14021	14325	14630	14935
30000	9144	9449	9753	10058	10363	10668	10973	11277	11582	11887
20000	6096	6401	6705	7010	7315	7620	7925	8229	8534	8839
10000	3048	3353	3658	3962	4267	4572	4877	5182	5486	5791
0	0	305	610	914	1219	1524	1829	2134	2438	2743
Feet.	**0**	**10**	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**
900	274	277	280	283	287	290	293	296	299	302
800	244	247	250	253	256	259	262	265	268	271
700	213	216	219	223	226	229	232	235	238	241
600	183	186	189	192	195	198	201	204	207	210
500	152	155	158	162	165	168	171	174	177	180
400	122	125	128	131	134	137	140	143	146	149
300	91	94	98	101	104	107	110	113	116	119
200	61	64	67	70	73	76	79	82	85	88
100	30	34	37	40	43	46	49	52	55	58
0	0	3	6	9	12	15	18	21	24	27

Example: Given 33562 feet. **Meters.**
First section of the table gives for 33000 feet.........10058
Second section of the table gives for 562 feet......... 172
10230

Table 2 A.—*Heights reduced from feet to dynamic meters, the acceleration of gravity at sea-level being 9.80.*

Height (feet).	0	1000	2000	3000	4000	5000	6000	7000	8000	9000
90000	26767	27063	27359	27655	27951	28247	28543	28839	29135	29431
80000	23804	24101	24397	24693	24990	25286	25582	25878	26175	26471
70000	20839	21135	21432	21729	22025	22322	22618	22915	23211	23508
60000	17870	18167	18464	18761	19058	19355	19652	19948	20245	20542
50000	14899	15196	15494	15791	16088	16385	16682	16979	17276	17573
40000	11925	12223	12520	12818	13115	13412	13710	14007	14304	14602
30000	8948	9246	9544	9841	10139	10437	10735	11032	11330	11627
20000	5968	6266	6564	6862	7161	7459	7757	8054	8352	8650
10000	2986	3284	3582	3881	4179	4477	4775	5074	5372	5670
0	0	299	597	896	1195	1493	1792	2090	2389	2687

PROPORTIONALITY TABLE.

Feet.	0	10	20	30	40	50	60	70	80	90
900	269	272	275	278	281	284	287	290	293	296
800	239	242	245	248	251	254	357	260	263	266
700	209	212	215	218	221	224	227	230	233	236
600	179	182	185	188	191	194	197	200	203	206
500	149	152	155	158	161	164	167	170	173	176
400	119	122	125	128	131	134	137	140	143	146
300	90	93	96	99	102	105	108	111	114	116
200	60	63	66	69	72	75	78	81	84	87
100	30	33	36	39	42	45	48	51	54	57
0	0	3	6	9	12	15	18	21	24	27

Table 3 A.—*Corrections to table 2 A for values of the acceleration of gravity at sea-level different from 9.80.*

Height (feet).	Acceleration of gravity.								
	9.76	9.77	9.78	9.79	9.80	9.81	9.82	9.83	9.84
90000	−110	−82	−55	−27	0	27	55	82	110
80000	− 98	−73	−49	−24	0	24	49	73	98
70000	− 85	−64	−43	−21	0	21	43	64	85
60000	− 73	−55	−37	−18	0	18	37	55	73
50000	− 61	−46	−30	−15	0	15	30	46	61
40000	− 49	−37	−24	−12	0	12	24	37	49
30000	− 37	−27	−18	− 9	0	9	18	27	37
20000	− 24	−18	−12	− 6	0	6	12	18	24
10000	− 12	− 9	− 6	− 3	0	3	6	9	12
0	0	0	0	0	0	0	0	0	0

Example, tables 2 A and 3 A.

(1)	(2)	(3)	(4)	(5)
4871	1195	260	+ 3	1458
11492	3284	147	+ 7	3438
19601	5670	179	+12	5861
32416	9544	124	+19	9687

Column 1. Heights above sea-level.
2. Values of table 2 A for the heights 4000, 11000, 19000, 32000.
3. Values of proportionality table for the heights 871, 492, 601, 416.
4. Corrections from table 3 A for $g = 9.8197$ at sea-level and for the heights of column 1.
5. Sum of numbers in columns 2, 3, and 4, giving the dynamic heights corresponding to the geometrical heights of column 1.

Table 4 A.—*Depths reduced from fathoms to meters.*

Fathoms.	0	100	200	300	400	500	600	700	800	900
0	0	183	366	549	732	914	1097	1280	1463	1646
1000	1829	2012	2195	2377	2560	2743	2926	3109	3292	3475
2000	3658	3840	4023	4206	4389	4572	4755	4938	5121	5303
3000	5486	5669	5852	6035	6218	6401	6584	6766	6949	7132
4000	7315	7498	7681	7864	8047	8229	8412	8595	8778	8961
5000	9144	9327	9510	9692	9875	10058	10241	10424	10607	10790
Fathoms.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
0	0	2	4	5	7	9	11	13	15	16
10	18	20	22	24	26	27	29	31	33	35
20	37	38	40	42	44	46	48	49	51	53
30	55	57	59	60	62	64	66	68	69	71
40	73	75	77	79	80	82	84	86	88	90
50	91	93	95	97	99	101	102	104	106	108
60	110	112	113	115	117	119	121	123	124	126
70	128	130	132	133	135	137	139	141	143	144
80	146	148	150	152	154	155	157	159	161	163
90	165	166	168	170	172	174	176	177	179	181

Example: Given 3678 fathoms. Meters.
First section of the table gives for 3600 fathoms......... 6584
Second section of the table gives for 78 fathoms......... 143
6727

Table 5 A.—*Depths reduced from fathoms to dynamic meters, the acceleration of gravity at sea-level being 9.80.*

Depth (fathoms).	0	100	200	300	400	500	600	700	800	900
0	0	179	358	538	717	896	1075	1255	1434	1613
1000	1793	1972	2151	2330	2510	2689	2868	3048	3227	3407
2000	3586	3765	3945	4124	4303	4483	4662	4842	5021	5200
3000	5380	5559	5739	5918	6098	6277	6457	6636	6816	6995
4000	7175	7354	7534	7713	7893	8072	8252	8431	8611	8791
5000	8970	9150	9329	9509	9689	9868	10048	10227	10407	10587

Proportionality Table.

Fathoms.	0	1	2	3	4	5	6	7	8	9
0	0	2	4	5	7	9	11	13	14	16
10	18	20	21	23	25	27	29	30	32	34
20	36	38	39	41	43	45	47	48	50	52
30	54	56	57	59	61	63	65	66	68	70
40	72	73	75	77	79	81	82	84	86	88
50	90	91	93	95	97	99	100	102	104	106
60	108	109	111	113	115	116	118	120	122	124
70	125	127	129	131	133	134	136	138	140	142
80	143	145	147	149	151	152	154	156	158	159
90	161	163	165	167	168	170	172	174	176	177

Table 6 A.—*Corrections to table 5 A for values of the acceleration of gravity different from 9.80.*

Depth (fathoms).	Acceleration of gravity at sea-level.						
	9.78	9.79	9.80	9.81	9.82	9.83	9.84
0	0	0	0	0	0	0	0
1000	−4	−2	0	2	4	5	7
2000	−7	−4	0	4	7	11	15
3000	−11	−5	0	5	11	16	22
4000	−15	−7	0	7	15	22	29
5000	−18	−9	0	9	18	27	37

Example: Depth 2769 fathoms.
Value of table 5 A for the depth of 2700 fathoms 4842
Value of proportionality table for the depth of 69 fathoms........ 124
Correction from table 6 A for gravity 9.8197 at sea-level and for the depth 2769 fathoms......... +10
Depth in dynamic meters........ 4976

Table 7 A.—*Pressure reduced from millimeters of mercury to millibars.*

Milli-meters of mercury.	0	1	2	3	4	5	6	7	8	9
0	0	1.3	2.7	4.0	5.3	6.7	8.0	9.3	10.7	12.0
10	13.3	14.7	16.0	17.3	18.7	20.0	21.3	22.7	24.0	25.3
20	26.7	28.0	29.3	30.7	32.0	33.3	34.7	36.0	37.3	38.7
30	40.0	41.3	42.7	44.0	45.3	46.7	48.0	49.3	50.7	52.0
40	53.3	54.7	56.0	57.3	58.7	60.0	61.3	62.7	64.0	65.3
50	66.7	68.0	69.3	70.7	72.0	73.3	74.7	76.0	77.3	78.7
60	80.0	81.3	82.7	84.0	85.3	86.7	88.0	89.3	90.7	92.0
70	93.3	94.7	96.0	97.3	98.7	100.0	101.3	102.7	104.0	105.3
80	106.7	108.0	109.3	110.7	112.0	113.3	114.7	116.0	117.3	118.7
90	120.0	121.3	122.7	124.0	125.3	126.7	128.0	129.3	130.7	132.0
100	133.3	134.7	136.0	137.3	138.7	140.0	141.3	142.7	144.0	145.3
110	146.7	148.0	149.3	150.7	152.0	153.3	154.7	156.0	157.3	158.6
120	160.0	161.3	162.6	164.0	165.3	166.6	168.0	169.3	170.6	172.0
130	173.3	174.6	176.0	177.3	178.6	180.0	181.3	182.6	184.0	185.3
140	186.6	188.0	189.3	190.6	192.0	193.3	194.6	196.0	197.3	198.6
150	200.0	201.3	202.6	204.0	205.3	206.6	208.0	209.3	210.6	212.0
160	213.3	214.6	216.0	217.3	218.6	220.0	221.3	222.6	224.0	225.3
170	226.6	228.0	229.3	230.6	232.0	233.3	234.6	236.0	237.3	238.6
180	240.0	241.3	242.6	244.0	245.3	246.6	248.0	249.3	250.6	252.0
190	253.3	254.6	256.0	257.3	258.6	260.0	261.3	262.6	264.0	265.3
200	266.6	268.0	269.3	270.6	272.0	273.3	274.6	276.0	277.3	278.6
210	280.0	281.3	282.6	284.0	285.3	286.6	288.0	289.3	290.6	292.0
220	293.3	294.6	296.0	297.3	298.6	300.0	301.3	302.6	304.0	305.3
230	306.6	308.0	309.3	310.6	312.0	313.3	314.6	316.0	317.3	318.6
240	320.0	321.3	322.6	324.0	325.3	326.6	328.0	329.3	330.6	332.0
250	333.3	334.6	336.0	337.3	338.6	340.0	341.3	342.6	344.0	345.3
260	346.6	348.0	349.3	350.6	352.0	353.3	354.6	356.0	357.3	358.6
270	360.0	361.3	362.6	364.0	365.3	366.6	368.0	369.3	370.6	372.0
280	373.3	374.6	376.0	377.3	378.6	380.0	381.3	382.6	384.0	385.3
290	386.6	388.0	389.3	390.6	392.0	393.3	394.6	396.0	397.3	398.6
300	400.0	401.3	402.6	404.0	405.3	406.6	408.0	409.3	410.6	412.0
310	413.3	414.6	416.0	417.3	418.6	420.0	421.3	422.6	424.0	425.3
320	426.6	428.0	429.3	430.6	432.0	433.3	434.6	436.0	437.3	438.6
330	440.0	441.3	442.6	444.0	445.3	446.6	448.0	449.3	450.6	452.0
340	453.3	454.6	456.0	457.3	458.6	460.0	461.3	462.6	464.0	465.3
350	466.6	468.0	469.3	470.6	472.0	473.3	474.6	475.9	477.3	478.6
360	479.9	481.3	482.6	483.9	485.3	486.6	487.9	489.3	490.6	491.9
370	493.3	494.6	495.9	497.3	498.6	499.9	501.3	502.6	503.9	505.3
380	506.6	507.9	509.3	510.6	511.9	513.3	514.6	515.9	517.3	518.6
390	519.9	521.3	522.6	523.9	525.3	526.6	527.9	529.3	530.6	531.9
400	533.3	534.6	535.9	537.3	538.6	539.9	541.3	542.6	543.9	545.3
410	546.6	547.9	549.3	550.6	551.9	553.3	554.6	555.9	557.3	558.6
420	559.9	561.3	562.6	563.9	565.3	566.6	567.9	569.3	570.6	571.9
430	573.3	574.6	575.9	577.3	578.6	579.9	581.3	582.6	583.9	585.3
440	586.6	587.9	589.3	590.6	591.9	593.3	594.6	595.9	597.3	598.6
450	599.9	601.3	602.6	603.9	605.3	606.6	607.9	609.3	610.6	611.9
460	613.3	614.6	615.9	617.3	618.6	619.9	621.3	622.6	623.9	625.3
470	626.6	627.9	629.3	630.6	631.9	633.3	634.6	635.9	637.3	638.6
480	639.9	641.3	642.6	643.9	645.3	646.6	647.9	649.3	650.6	651.9
490	653.3	654.6	655.9	657.3	658.6	659.9	661.3	662.6	663.9	665.3

Table 7 A (continued).—*Pressure reduced from millimeters of mercury to millibars.*

Millimeters of mercury.	0	1	2	3	4	5	6	7	8	9
500	666.6	667.9	669.3	670.6	671.9	673.3	674.6	675.9	677.3	678.6
510	679.9	681.3	682.6	683.9	685.3	686.6	687.9	689.3	690.6	691.9
520	693.3	694.6	695.9	697.3	698.6	699.9	701.3	702.6	703.9	705.3
530	706.6	707.9	709.3	710.6	711.9	713.3	714.6	715.9	717.3	718.6
540	719.9	721.3	722.6	723.9	725.3	726.6	727.9	729.3	730.6	731.9
550	733.3	734.6	735.9	737.3	738.6	739.9	741.3	742.6	743.9	745.3
560	746.6	747.9	749.3	750.6	751.9	753.3	754.6	755.9	757.3	758.6
570	759.9	761.3	762.6	763.9	765.3	766.6	767.9	769.3	770.6	771.9
580	773.3	774.6	775.9	777.3	778.6	779.9	781.3	782.6	783.9	785.3
590	786.6	787.9	789.3	790.6	791.9	793.2	794.6	795.9	797.2	798.6
600	799.9	801.2	802.6	803.9	805.2	806.6	807.9	809.2	810.6	811.9
610	813.2	814.6	815.9	817.2	818.6	819.9	821.2	822.6	823.9	825.2
620	826.6	827.9	829.2	830.6	831.9	833.2	834.6	835.9	837.2	838.6
630	839.9	841.2	842.6	843.9	845.2	846.6	847.9	849.2	850.6	851.9
640	853.2	854.6	855.9	857.2	858.6	859.9	861.2	862.6	863.9	865.2
650	866.6	867.9	869.2	870.6	871.9	873.2	874.6	875.9	877.2	878.6
660	879.9	881.2	882.6	883.9	885.2	886.6	887.9	889.2	890.6	891.9
670	893.2	894.6	895.9	897.2	898.6	899.9	901.2	902.6	903.9	905.2
680	906.6	907.9	909.2	910.6	911.9	913.2	914.6	915.9	917.2	918.6
690	919.9	921.2	922.6	923.9	925.2	926.6	927.9	929.2	930.6	931.9
700	933.2	934.6	935.9	937.2	938.6	939.9	941.2	942.6	943.9	945.2
710	946.6	947.9	949.2	950.6	951.9	953.2	954.6	955.9	957.2	958.6
720	959.9	961.2	962.6	963.9	965.2	966.6	967.9	969.2	970.6	971.9
730	973.2	974.6	975.9	977.2	978.6	979.9	981.2	982.6	983.9	985.2
740	986.6	987.9	989.2	990.6	991.9	993.2	994.6	995.9	997.2	998.6
750	999.9	1001.2	1002.6	1003.9	1005.2	1006.6	1007.9	1009.2	1010.6	1011.9
760	1013.2	1014.6	1015.9	1017.2	1018.6	1019.9	1021.2	1022.6	1023.9	1025.2
770	1026.6	1027.9	1029.2	1030.6	1031.9	1033.2	1034.6	1035.9	1037.2	1038.6
780	1039.9	1041.2	1042.6	1043.9	1045.2	1046.6	1047.9	1049.2	1050.6	1051.9
790	1053.2	1054.6	1055.9	1057.2	1058.6	1059.9	1061.2	1062.6	1063.9	1065.2
800	1066.6	1067.9	1069.2	1070.6	1071.9	1073.2	1074.6	1075.9	1077.2	1078.6
810	1079.9	1081.2	1082.6	1083.9	1085.2	1086.6	1087.9	1089.2	1090.6	1091.9

Example:
Pressure .... 622.2 mm. of mercury.
Table 7 A gives......... 829.5 m-bar.

Table 8 A.— *Pressure reduced from inches of mercury to millibars.*

Inches of mercury.	0	1	2	3	4	5	6	7	8	9
0	0	3.4	6.8	10.2	13.5	16.9	20.3	23.7	27.1	30.5
1	33.9	37.2	40.6	44.0	47.4	50.8	54.2	57.6	61.0	64.3
2	67.7	71.1	74.5	77.9	81.3	84.7	88.0	91.4	94.8	98.2
3	101.6	105.0	108.4	111.7	115.1	118.5	121.9	125.3	128.7	132.1
4	135.5	138.8	142.2	145.6	149.0	152.4	155.8	159.2	162.5	165.9
5	169 3	172.7	176.1	179.5	182.9	186.2	189.6	193.0	196.4	199.8
6	203.2	206.6	209.9	213.3	216.7	220.1	223.5	226.9	230.3	233.7
7	237.0	240.4	243.8	247.2	250.6	254.0	257.4	260.7	264.1	267.5
8	270.9	274.3	277.7	281.1	284.4	287.8	291.2	294.6	298.0	301.4
9	304.8	308.1	311.5	314.9	318.3	321.7	325.1	328.5	331.9	335.2
10	338.6	342.0	345.4	348.8	352.2	355.6	358.9	362.3	365.7	369.1
11	372.5	375.9	379.3	382.6	386.0	389.4	392.8	396.2	399.6	403.0
12	406.3	409.7	413.1	416.5	419.9	423.3	426.7	430.1	433.4	436.8
13	440.2	443.6	447.0	450.4	453.8	457.1	460.5	463.9	467.3	470.7
14	474.1	477.5	480.8	484.2	487.6	491.0	494.4	497.8	501.2	504.6
15	507.9	511.3	514.7	518.1	521.5	524.9	528.3	531.6	535.0	538.4
16	541.8	545.2	548.6	552.0	555.3	558.7	562.1	565.5	568.9	572.3
17	575.7	579.0	582.4	585.8	589.2	592.6	596.0	599.4	602.8	606.1
18	609.5	612.9	616.3	619.7	623.1	626.5	629.8	633.2	636.6	640.0
19	643.4	646.8	650.2	653.5	656.9	660.3	663.7	667.1	670.5	673.9
20	677.2	680.6	684.0	687.4	690.8	694.2	697.6	701.0	704.3	707.7
21	711.1	714.5	717.9	721.3	724.7	728.0	731.4	734.8	738.2	741.6
22	745.0	748.4	751.7	755.1	758.5	761.9	765.3	768.7	772.1	775.5
23	778.8	782.2	785.6	789.0	792.4	795.8	799.2	802.5	805.9	809.3
24	812.7	816.1	819.5	822.9	826.2	829.6	833.0	836.4	839.8	843.2
25.0	846.6	846.9	847.2	847.6	847.9	848.3	848.6	848.9	849.3	849.6
25.1	849.9	850.3	850.6	851.0	851.3	851.6	852.0	852.3	852.7	853.0
25.2	853.3	853.7	854.0	854.4	854.7	855.0	855.4	855.7	856.0	856.4
25.3	856.7	857.1	857.4	857.7	858.1	858.4	858.8	859.1	859.4	859.8
25.4	860.1	860.4	860.8	861.1	861.5	861.8	862.1	862.5	862.8	863.2
25.5	863.5	863.8	864.2	864.5	864.8	865.2	865.5	865.9	866.2	866.5
25.6	866.9	867.2	867.6	867.9	868.2	868.6	868.9	869.2	869.6	869.9
25.7	870.3	870.6	870.9	871.3	871.6	872.0	872.3	872.6	873.0	873.3
25.8	873.7	874.0	874.3	874.7	875.0	875.3	875.7	876.0	876.4	876.7
25.9	877.0	877.4	877.7	878.1	878.4	878.7	879.1	879.4	879.7	880.1
26.0	880.4	880.8	881.1	881.4	881.8	882.1	882.5	882.8	883.1	883.5
26.1	883.8	884.1	884.5	884.8	885.2	885.5	885.8	886.2	886.5	886.9
26.2	887.2	887.5	887.9	888.2	888.6	888.9	889.2	889.6	889.9	890.2
26.3	890.6	890.9	891.3	891.6	891.9	892.3	892.6	893.0	893.3	893.6
26.4	894.0	894.3	894.6	895.0	895.3	895.7	896.0	896.3	896.7	897.0
26.5	897.4	897.7	898.0	898.4	898.7	899.0	899.4	899.7	900.1	900.4
26.6	900.7	901.1	901.4	901.8	902.1	902.4	902.8	903.1	903.5	903.8
26.7	904.1	904.5	904.8	905.1	905.5	905.8	906.2	906.5	906.8	907.2
26.8	907.5	907.9	908.2	908.5	908.9	909.2	909.5	909.9	910.2	910.6
26.9	910.9	911.2	911.6	911.9	912.3	912.6	912.9	913.3	913.6	913.9
27.0	914.3	914.6	915.0	915.3	915.6	916.0	916.3	916.7	917.0	917.3
27.1	917.7	918.0	918.4	918.7	919.0	919.4	919.7	920.0	920.4	920.7
27.2	921.1	921.4	921.7	922.1	922.4	922.8	923.1	923.4	923.8	924.1
27.3	924.4	924.8	925.1	925.5	925.8	926.1	926.5	926.8	927.2	927.5
27.4	927.8	928.2	928.5	928.8	929.2	929.5	929.9	930.2	930.5	930.9

Table 8A (continued).—*Pressure reduced from inches of mercury to millibars.*

Inches of mercury.	0	1	2	3	4	5	6	7	8	9
27.5	931.2	931.6	931.9	932.2	932.6	932.9	933.2	933.6	933.9	934.3
27.6	934.6	934.9	935.3	935.6	936.0	936.3	936.6	937.0	937.3	937.7
27.7	938.0	938.3	938.7	939.0	939.3	939.7	940.0	940.4	940.7	941.0
27.8	941.4	941.7	942.1	942.4	942.7	943.1	943.4	943.7	944.1	944.4
27.9	944.8	945.1	945.4	945.8	946.1	946.5	946.8	947.1	947.5	947.8
28 0	948.1	948.5	948.8	949.2	949.5	949.8	950.2	950.5	950.9	951.2
28.1	951.5	951.9	952.2	952.6	952.9	953.2	953.6	953.9	954.2	954.6
28.2	954.9	955.3	955.6	955.9	956.3	956.6	957.0	957.3	957.6	958.0
28.3	958.3	958.6	959.0	959.3	959.7	960.0	960.3	960.7	961.0	961.4
28.4	961.7	962.0	962.4	962.7	963.0	963.4	963.7	964.1	964.4	964.7
28.5	965.1	965.4	965.8	966.1	966.4	966.8	967.1	967.5	967.8	968.1
28.6	968.5	968.8	969.1	969.5	969.8	970.2	970.5	970.8	971.2	971.5
28.7	971.9	972.2	972.5	972.9	973.2	973.5	973.9	974.2	974.6	974.9
28.8	975.2	975.6	975.9	976.3	976.6	976.9	977.3	977.6	977.9	978.3
28.9	978.6	979.0	979.3	979.6	980.0	980.3	980.7	981.0	981.3	981.7
29.0	982.0	982.3	982.7	983.0	983.4	983.7	984.0	984.4	984.7	985.1
29.1	985.4	985.7	986.1	986.4	986.8	987.1	987.4	987.8	988.1	988.4
29.2	988.8	989.1	989.5	989.8	990.1	990.5	990.8	991.2	991.5	991.8
29.3	992.2	992.5	992.8	993.2	993.5	993.9	994.2	994.5	994.9	995.2
29.4	995.6	995.9	996.2	996.6	996.9	997.2	997.6	997.9	998.3	998.6
29.5	998.9	999.3	999.6	1000.0	1000.3	1000.6	1001.0	1001.3	1001.7	1002.0
29.6	1002.3	1002.7	1003.0	1003.3	1003.7	1004.0	1004.4	1004.7	1005.0	1005.4
29.7	1005.7	1006.1	1006.4	1006.7	1007.1	1007.4	1007.7	1008.1	1008.4	1008.8
29.8	1009.1	1009.4	1009.8	1010.1	1010.5	1010.8	1011.1	1011.5	1011.8	1012.1
29.9	1012.5	1012.8	1013.2	1013.5	1013.8	1014.2	1014.5	1014.9	1015.2	1015.5
30.0	1015.9	1016.2	1016.6	1016.9	1017.2	1017.6	1017.9	1018.2	1018.6	1018.9
30.1	1019.3	1019.6	1019.9	1020.3	1020.6	1021.0	1021.3	1021.6	1022.0	1022.3
30.2	1022.6	1023.0	1023.3	1023.7	1024.0	1024.3	1024.7	1025.0	1025.4	1025.7
30.3	1026.0	1026.4	1026.7	1027.0	1027.4	1027.7	1028.1	1028.4	1028.7	1029.1
30.4	1029.4	1029.8	1030.1	1030.4	1030.8	1031.1	1031.5	1031.8	1032.1	1032.5
30.5	1032.8	1033.1	1033.5	1033.8	1034.2	1034.5	1034.8	1035.2	1035.5	1035.9
30.6	1036.2	1036.5	1036.9	1037.2	1037.5	1037.9	1038.2	1038.6	1038.9	1039.2
30.7	1039.6	1039.9	1040.3	1040.6	1040.9	1041.3	1041.6	1041.9	1042.3	1042.6
30.8	1043.0	1043.3	1043.6	1044.0	1044.3	1044.7	1045.0	1045.3	1045.7	1046.0
30.9	1046.4	1046.7	1047.0	1047.4	1047.7	1048.0	1048.4	1048.7	1049.1	1049.4
31.0	1049.7	1050.1	1050.4	1050.8	1051.1	1051.4	1051.8	1052.1	1052.4	1052.8
31.1	1053.1	1053.5	1053.8	1054.1	1054.5	1054.8	1055.2	1055.5	1055.8	1056 2
31.2	1056.5	1056.8	1057.2	1057.5	1057.9	1058.2	1058.5	1058.9	1059.2	1059.6
31.3	1059.9	1060.2	1060.6	1060.9	1061.2	1061.6	1061.9	1062.3	1062.6	1062.9
31.4	1063.3	1063.6	1064 0	1064.3	1064.6	1065.0	1065.3	1065.7	1066.0	1066.3
31.5	1066.7	1067.0	1067 3	1067.7	1068.0	1068.4	1068.7	1069.0	1069.4	1069.7
31.6	1070.1	1070.4	1070.7	1071.1	1071.4	1071.7	1072.1	1072.4	1072.8	1073.1
31.7	1073.4	1073.8	1074.1	1074.5	1074.8	1075.1	1075.5	1075.8	1076.1	1076.5
31.8	1076.8	1077.2	1077.5	1077.8	1078.2	1078.5	1078.9	1079.2	1079.5	1079.9
31.9	1080.2	1080.6	1080.9	1081.2	1081.6	1081.9	1082.2	1082.6	1082.9	1083.3

Example:
Pressure.................. 28.52 inches of mercury.
Table 8A gives.......... 965.8 m-bars.

Table 9 A.—*Air-temperatures reduced from Fahrenheit to centigrade.*

Degrees Fahrenheit.	0	1	2	3	4	5	6	7	8	9
−140	−95.6	−96.1	−96.7	−97.2	−97.8	−98.3	−98.9	−99.4	−100.0	−100.6
−130	−90.0	−90.6	−91.1	−91.7	−92.2	−92.8	−93.3	−93.9	− 94.4	− 95.0
−120	−84.4	−85.0	−85.6	−86.1	−86.7	−87.2	−87.8	−88.3	− 88.9	− 89.4
−110	−78.9	−79.4	−80.0	−80.6	−81.1	−81.7	−82.2	−82.8	− 83.3	− 83.9
−100	−73.3	−73.9	−74.4	−75.0	−75.6	−76.1	−76.7	−77.2	− 77.8	− 78.3
− 90	−67.8	−68.3	−68.9	−69.4	−70.0	−70.6	−71.1	−71.7	− 72.2	− 72.8
− 80	−62.2	−62.8	−63.3	−63.9	−64.4	−65.0	−65.6	−66.1	− 66.7	− 67.2
− 70	−56.7	−57.2	−57.8	−58.3	−58.9	−59.4	−60.0	−60.6	− 61.1	− 61.7
− 60	−51.1	−51.7	−52.2	−52.8	−53.3	−53.9	−54.4	−55.0	− 55.6	− 56.1
− 50	−45.6	−46.1	−46.7	−47.2	−47.8	−48.3	−48.9	−49.4	− 50.0	− 50.6
− 40	−40.0	−40.6	−41.1	−41.7	−42.2	−42.8	−43.3	−43.9	− 44.4	− 45.0
− 30	−34.4	−35.0	−35.6	−36.1	−36.7	−37.2	−37.8	−38.3	− 38.9	− 39.4
− 20	−28.9	−29.4	−30.0	−30.6	−31.1	−31.7	−32.2	−32.8	− 33.3	− 33.9
− 10	−23.3	−23.9	−24.4	−25.0	−25.6	−26.1	−26.7	−27.2	− 27.8	− 28.3
− 0	−17.8	−18.3	−18.9	−19.4	−20.0	−20.6	−21.1	−21.7	− 22.2	− 22.8
0	−17.8	−17.2	−16.7	−16.1	−15.6	−15.0	−14.4	−13.9	− 13.3	− 12.8
10	−12.2	−11.7	−11.1	−10.6	−10.0	− 9.4	− 8.9	− 8.3	− 7.8	− 7.2
20	− 6.7	− 6.1	− 5.6	− 5.0	− 4.4	− 3.9	− 3.3	− 2.8	− 2.2	− 1.7
30	− 1.1	− 0.6	0	0.6	1.1	1.7	2.2	2.8	3.3	3.9
40	4.4	5.0	5.6	6.1	6.7	7.2	7.8	8.3	8.9	9.4
50	10.0	10.6	11.1	11.7	12.2	12.8	13.3	13.9	14.4	15.0
60	15.6	16.1	16.7	17.2	17.8	18.3	18.9	19.4	20.0	20.6
70	21.1	21.7	22.2	22.8	23.3	23.9	24.4	25.0	25.6	26.1
80	26.7	27.2	27.8	28.3	28.9	29.4	30.0	30.6	31.1	31.7
90	32.2	32.8	33.3	33.9	34.4	35.0	35.6	36.1	36.7	37.2
100	37.8	38.3	38.9	39.4	40.0	40.6	41.1	41.7	42.2	42.8
110	43.3	43.9	44.4	45.0	45.6	46.1	46.7	47.2	47.8	48.3
120	48.9	49.4	50.0	50.6	51.1	51.7	52.2	52.8	53.3	53.9
130	54.4	55.0	55.6	56.1	56.7	57.2	57.8	58.3	58.9	59.4
140	60.0	60.6	61.1	61.7	62.2	62.8	63.3	63.9	64.4	65.0

PROPORTIONALITY TABLE.

Fahrenheit.	Centigrade.	Fahrenheit.	Centigrade.	Fahrenheit.	Centigrade.
0.0	0.0	0.4	0.2	0.8	0.4
0.1	0.1	0.5	0.3	0.9	0.5
0.2	0.1	0.6	0.3		
0.3	0.2	0.7	0.4		

Example:

Temperature +42.6° F. Table 9 A gives......	+42.0° F.	=	+5.6° C.
Proportionality table gives for......................	+ 0.6° F.	=	+0.3° C.
	+42.6° F.	=	+5.9° C.

Table 10 A.—*Sea-temperatures reduced from Fahrenheit to centigrade.*

Degrees Fahrenheit.	0	1	2	3	4	5	6	7	8	9
20									−2.22	−1.67
30	−1.11	−0.56	0.00	0.56	1.11	1.67	2.22	2.78	3.33	3.89
40	4.44	5.00	5.56	6.11	6.67	7.22	7.78	8.33	8.89	9.44
50	10.00	10.56	11.11	11.67	12.22	12.78	13.33	13.89	14.44	15.00
60	15.56	16.11	16.67	17.22	17.78	18.33	18.89	19.44	20.00	20.56
70	21.11	21.67	22.22	22.78	23.33	23.89	24.44	25.00	25.56	26.11
80	26.67	27.22	27.78	28.33	28.89	29.44	30.00	30.56	31.11	31.67
90	32.22	32.78	33.33	33.89	34.44	35.00	35.56	36.11	36.67	37.22

PROPORTIONALITY TABLE.

Fahrenheit.	Centigrade.	Fahrenheit.	Centigrade.	Fahrenheit.	Centigrade.
0.00	0.00	0.40	0.22	0.80	0.44
0.10	0.06	0.50	0.28	0.90	0.50
0.20	0.11	0.60	0.33		
0.30	0.17	0.70	0.39		

Example:

Temperature +54.31° F. Table 10 A gives...	+54.00° F.	=	12.22° C.
Proportionality table gives for.....................	+ 0.31° F.	=	0.17° C.
	54.31° F.	=	12.39° C.

Table 11 A. — *Virtual temperature of saturated air in degrees centigrade, the pressure being given in millimeters of mercury.*

Pressure (mm. mercury).	Temperature (°C.). -50	-40	-30	-20	-15	-10	-5	-2	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
150	0.0	0.1	0.2	0.5	0.8	1.3	2.0	2.6	3.1	3.4	3.7	4.0	4.3															
200	0.0	0.0	0.1	0.4	0.6	1.0	1.5	2.0	2.4	2.5	2.8	3.0	3.2	3.4	3.7	4.0	4.3	4.6										
250	0.0	0.0	0.1	0.3	0.5	0.8	1.2	1.6	1.9	2.0	2.2	2.4	2.6	2.7	3.0	3.2	3.4	3.7										
300	0.0	0.0	0.1	0.2	0.4	0.6	1.0	1.3	1.6	1.7	1.8	2.0	2.1	2.3	2.5	2.6	2.8	3.1	3.3	3.5	3.8	4.1	4.3					
350	0.0	0.0	0.1	0.2	0.3	0.6	0.9	1.1	1.3	1.5	1.6	1.7	1.8	2.0	2.1	2.3	2.4	2.6	2.8	3.0	3.2	3.5	3.7					
400	0.0	0.0	0.1	0.2	0.3	0.5	0.8	1.0	1.2	1.3	1.4	1.5	1.6	1.7	1.8	2.0	2.1	2.3	2.5	2.6	2.8	3.0	3.2	3.5	3.7	4.0	4.3	4.5
450	0.0	0.0	0.1	0.2	0.3	0.4	0.7	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.9	2.0	2.2	2.3	2.5	2.7	2.9	3.1	3.3	3.5	3.8	4.0
500	0.0	0.0	0.1	0.1	0.2	0.4	0.6	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	2.0	2.1	2.3	2.4	2.6	2.8	3.0	3.2	3.4	3.6
550	0.0	0.0	0.0	0.1	0.2	0.4	0.6	0.7	0.9	0.9	1.0	1.1	1.2	1.2	1.3	1.4	1.5	1.7	1.8	1.9	2.0	2.2	2.4	2.5	2.7	2.9	3.1	3.3
600	0.0	0.0	0.0	0.1	0.2	0.3	0.5	0.7	0.8	0.8	0.9	1.0	1.1	1.1	1.2	1.3	1.4	1.5	1.6	1.8	1.9	2.0	2.2	2.3	2.5	2.6	2.8	3.0
650	0.0	0.0	0.0	0.1	0.2	0.3	0.5	0.6	0.7	0.8	0.8	0.9	1.0	1.1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.9	2.0	2.1	2.3	2.4	2.6	2.8
700	0.0	0.0	0.0	0.1	0.2	0.3	0.4	0.6	0.7	0.7	0.8	0.8	0.9	1.0	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	2.0	2.1	2.3	2.4	2.6
750	0.0	0.0	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.7	0.8	0.8	0.9	1.0	1.1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	2.0	2.1	2.3	2.4
800	0.0	0.0	0.0	0.1	0.2	0.2	0.4	0.5	0.6	0.6	0.7	0.7	0.8	0.9	0.9	1.0	1.1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	2.0	2.1	2.3

Pressure (mm. mercury).	Temperature (°C.). 20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44
450	4.3	4.6	4.9	5.2	5.6																				
500	3.9	4.1	4.4	4.7	5.0	5.4	5.7	6.1	6.5	6.9															
550	3.5	3.8	4.0	4.3	4.6	4.9	5.2	5.5	5.9	6.3	6.7	7.1	7.5	8.0	8.5										
600	3.2	3.4	3.7	3.9	4.2	4.5	4.7	5.0	5.4	5.7	6.1	6.5	6.9	7.3	7.8										
650	3.0	3.2	3.4	3.6	3.8	4.1	4.4	4.7	5.0	5.3	5.6	6.0	6.4	6.7	7.2	7.6	8.1	8.6	9.1	9.7	10.2	10.9	11.5	12.2	12.9
700	2.8	2.9	3.1	3.3	3.6	3.8	4.1	4.3	4.6	4.9	5.2	5.5	5.9	6.3	6.7	7.1	7.5	8.0	8.5	9.0	9.5	10.1	10.7	11.3	11.9
750	2.6	2.7	2.9	3.1	3.3	3.6	3.8	4.0	4.3	4.6	4.8	5.2	5.5	5.8	6.2	6.6	7.0	7.4	7.9	8.3	8.8	9.4	9.9	10.5	11.1
800	2.4	2.6	2.7	2.9	3.1	3.3	3.5	3.8	4.0	4.3	4.5	4.8	5.1	5.5	5.8	6.2	6.5	6.9	7.4	7.8	8.3	8.8	9.3	9.8	10.4

Example.

Pressure 631 mm. of mercury, temperature + 2.9° C. Table 11 A gives 0.9.
74 per cent of 0.9 gives.............................. 0.7
Virtual temperature .... 3.6° C. for air of 2.9° C. and 74 per cent relative humidity at the pressure of 631 mm.

Table 12 A.—*Virtual temperature of saturated air in degrees Fahrenheit, the pressure being given in inches of mercury.*

Pressure (inches of Hg).	Temperature (° F.).																										
	−50	−30	−10	0	10	20	25	30	32	34	36	38	40	42	44	46	48	50	52	54	56	58	60	62	64	66	68
2	0.2	0.6	1.9	3.3	5.7	9.5	12.2																				
4	0.1	0.3	0.9	1.7	2.8	4.7	6.0																				
6	0.1	0.2	0.6	1.1	1.9	3.1	4.0	5.1	5.6	6.1	6.6	7.2															
8	0.0	0.1	0.5	0.8	1.4	2.3	3.0	3.8	4.2	4.6	5.0	5.4	5.9	6.4	6.9	7.5	8.1										
10	0.0	0.1	0.4	0.7	1.1	1.9	2.4	3.1	3.3	3.7	4.0	4.3	4.7	5.1	5.5	6.0	6.5	7.0	7.6	8.2	8.9	9.6					
12	0.0	0.1	0.3	0.6	0.9	1.6	2.0	2.5	2.8	3.0	3.3	3.6	3.9	4.2	4.6	5.0	5.4	5.8	6.3	6.8	7.4	8.0	8.6	9.3	10.0	10.7	11.5
14	0.0	0.1	0.3	0.5	0.8	1.3	1.7	2.2	2.4	2.6	2.8	3.1	3.3	3.6	3.9	4.3	4.6	5.0	5.4	5.8	6.3	6.8	7.3	7.9	8.5	9.2	9.9
16	0 0	0.1	0.2	0.4	0.7	1.2	1.5	1.9	2.1	2.3	2.5	2.7	2.9	3.2	3.4	3.7	4.0	4.4	4.7	5.1	5.5	5.9	6.4	6.9	7.4	8.0	8.6
18	0.0	0.1	0.2	0.4	0.6	1.0	1.3	1.7	1.9	2.0	2.2	2.4	2.6	2.8	3.0	3.3	3.6	3.9	4.2	4.5	4.9	5.3	5.7	6.1	6.6	7.1	7.6
20	0.0	0.1	0.2	0.3	0.6	0.9	1.2	1.5	1.7	1.8	2.0	2.1	2.3	2.5	2.7	3.0	3.2	3.5	3.8	4.1	4.4	4.7	5.1	5.5	5.9	6.4	6.9
22	0.0	0.1	0.2	0.3	0.5	0.9	1.1	1.4	1.5	1.6	1.8	1.9	2.1	2.3	2.5	2.7	2.9	3.2	3.4	3.7	4.0	4.3	4.6	5.0	5.4	5.8	6.2
24	0.0	0.0	0.1	0.3	0.5	0.8	1.0	1.3	1.4	1.5	1.6	1.8	1.9	2.1	2.3	2.5	2.7	2.9	3.1	3.4	3.6	3.9	4.2	4.6	4.9	5.3	5.7
26	0.0	0.0	0.1	0.3	0.4	0.7	0.9	1.2	1.3	1.4	1.5	1.6	1.8	1.9	2.1	2.3	2.5	2.7	2.9	3.1	3.4	3.6	3.9	4.2	4.6	4.9	5.3
28	0.0	0.0	0.1	0.2	0.4	0.7	0.9	1.1	1.2	1.3	1.4	1.5	1.7	1.8	2.0	2.1	2.3	2.5	2.7	2.9	3.1	3.4	3.6	3.9	4.2	4.5	4.9
30	0.0	0.0	0.1	0.2	0.4	0.6	0.8	1.0	1.1	1.2	1.3	1.4	1.5	1.7	1.8	2.0	2.1	2.3	2.5	2.7	2.9	3.2	3.4	3.7	3.9	4.2	4.5
32	0.0	0.0	0.1	0.2	0.4	0.6	0.7	0.9	1.0	1.1	1.2	1.3	1.4	1.6	1.7	1.9	2.0	2.2	2.3	2.5	2.7	3.0	3.2	3.4	3.7	4.0	4.3

Pressure (inches of Hg).	Temperature (° F.).																								
	70	72	74	76	78	80	82	84	86	88	90	92	94	96	98	100	102	104	106	108	110	112	114	116	118
16	9.3	10.0	10.7	11.5	12.4																				
18	8.2	8.8	9.5	10.2	10.9																				
20	7.4	7.9	8.5	9.2	9.9	10.6	11.3	12.1	13.0	13.9															
22	6.7	7.2	7.7	8.3	8.9	9.6	10.3	11.0	11.8	12.6	13.5	14.5	15.5	16.6	17.7										
24	6.2	6.6	7.1	7.6	8.2	8.8	9.4	10.1	10.8	11.6	12.4	13.2	14.2	15.1	16.2	17.3	18.5	19.7	22.0	22.4	24.0	25.5	27.2	28.9	30.8
26	5.7	6.1	6.5	7.0	7.5	8.1	8.7	9.3	10.0	10.7	11.3	12.2	13.1	14.0	14.9	15.9	17.0	18.2	19.4	20.7	22.0	23.5	25.0	26.6	28.3
28	5.3	5.6	6.0	6.5	7.0	7.5	8.0	8.6	9.2	9.9	10.6	11.3	12.1	12.9	13.8	14.8	15.8	16.8	17.9	19.1	20.4	21.7	23.1	24.6	26.2
30	4.9	5.3	5.6	6.1	6.5	7.0	7.5	8.0	8.6	9.2	9.9	10.5	11.3	12.1	12.9	13.8	14.7	15.7	16.7	17.8	19.0	20.2	21.5	22.9	24.4
32	4.6	4.9	5.3	5.7	6.1	6.5	7.0	7.5	8.1	8.6	9.2	9.9	10.6	11.3	12.1	12.9	13.7	14.7	15.6	16.7	17.8	18.9	20.1	21.4	22.8

Example:

Pressure 26.79 inches of mercury, temperature......... 37.5° F. Table 12 A gives 1.6.
81 per cent of 1.6 gives... 1.3° F.
Virtual temperature .... 38.8° F. for air of 37.5° F. and 81 per cent relative humidity at the pressure of 26.79 inches.

Table 13 A.—*Virtual temperature of saturated air in degrees Fahrenheit, the height being given in dynamic meters.*

Height (dynamic meters).	Temperature (°F.).																									
	−50	−30	−10	0	10	20	25	30	32	34	36	38	40	42	44	46	48	50	52	54	56	58	60	62	64	66	68
10000	0.0	0.2	0 5	0.9																							
9500	0.0	0.1	0.5	0.8																							
9000	0.0	0.1	0.4	0.8																							
8500	0.0	0.1	0.4	0.7	1.2																						
8000	0.0	0.1	0.4	0.6	1.1																						
7500	0.0	0.1	0.3	0.6	1.0	1.7																					
7000	0.0	0.1	0.3	0.6	1.0	1.6																					
6500	0.0	0.1	0.3	0.5	0.9	1.5	1.9																				
6000	0.0	0.1	0.3	0.5	0.8	1.4	1.8	2.2																			
5500	0.0	0.1	0.3	0.5	0.8	1.3	1.6	2.1	2.3																		
5000	0.0	0.1	0.2	0.4	0.7	1.2	1.5	2.0	2.1	2.4	2.5																
4500	0.0	0.1	0.2	0.4	0.7	1 1	1.4	1.8	2.0	2.2	2.4	2.6	2.8	3.0													
4000	0.0	0.1	0.2	0.4	0.6	1.1	1.3	1.7	1.9	2.1	2.2	2.4	2.6	2.8	3.1	3.3	3.6										
3500	0.0	0.1	0.2	0.3	0.6	1.0	1.3	1.6	1.8	1.9	2.1	2.3	2.5	2.7	2.9	3.1	3.4	3.7	4.0	4.3	4.6	5.0					
3000	0.0	0.1	0.2	0.3	0.6	0.9	1.2	1.5	1.6	1.8	1.9	2.1	2.3	2.5	2.7	2.9	3.2	3.4	3.7	4.0	4.3	4.7	5.0	5.4	5.9	6.3	6.7
2500	0.0	0.1	0.2	0.3	0.5	0.9	1.1	1.4	1.5	1.7	1.8	2.0	2.1	2.3	2.5	2.7	3.0	3.2	3.5	3.8	4.1	4.4	4.7	5.1	5.5	5.9	6.3
2000	0.0	0.1	0.2	0.3	0.5	0.8	1.0	1.3	1.4	1.6	1.7	1.9	2.0	2.2	2.4	2.6	2.8	3.0	3.3	3.5	3.8	4 1	4.4	4.8	5.1	5.5	5.9
1500	0.0	0.0	0.2	0.3	0.5	0.8	1.0	1.2	1.4	1.5	1.6	1.7	1.9	2.1	2.2	2.4	2.6	2.8	3.1	3.3	3.6	3.9	4.1	4.5	4.8	5.2	5.5
1000	0.0	0.0	0 1	0.3	0.4	0.7	0.9	1.2	1.3	1.4	1.5	1.6	1.8	1.9	2.1	2.3	2.5	2.7	2.9	3.1	3.4	3.6	3.9	4.2	4.5	4.9	5.2
500	0.0	0.0	0.1	0.2	0.4	0.7	0.9	1.1	1.2	1.3	1.4	1.5	1.7	1.8	2.0	2.1	2.3	2.5	2.7	2.9	3.1	3.4	3.7	4.0	4.3	4.6	4.9
0	0.0	0.0	0.1	0.2	0.4	0.6	0.8	1.0	1.1	1.2	1.3	1.4	1.6	1.7	1.8	2.0	2.2	2.3	2.5	2.7	3.0	3.2	3.4	3.7	4.0	4.3	4.6

Height (dynamic meters).	Temperature (°F.).																								
	70	72	74	76	78	80	82	84	86	88	90	92	94	96	98	100	102	104	106	108	110	112	114	116	118
2500	6.8	7.3	7.9	8.5	9.1																				
2000	6.4	6.9	7.4	7.9	8.5	9.1	9.8	10.5	11.3	12.1															
1500	6 0	6 4	6.9	7.4	8.0	8.6	9.2	9.9	10.6	11.3	12.1	12.9	13.8	14.8	15.8										
1000	5.6	6.1	6.5	7.0	7.5	8.0	8.6	9.3	9.9	10.6	11.4	12.2	13.0	13.9	15.0	15.9	16.9	18.1	19.3	20.6					
500	5.3	5.7	6.1	6.6	7.0	7.6	8.1	8.7	9.3	10.0	10.7	11.4	12.2	13.0	13.9	14.9	15.9	17.0	18.1	19.3	20.6	21.9	23.3	24.8	26.4
0	5.0	5.3	5.7	6.2	6.6	7.1	7.6	8.2	8.7	9.4	10.0	10.7	11.4	12.2	13.1	14.0	14.9	15.9	17.0	18.1	19.3	20.5	21.9	23.3	24.8

Example:

Height, 3191 dynamic meters, temperature......... 40.9° F. Table 13 A gives 2.5.
88 per cent of 2.5 gives.................................... 2.2° F.

Virtual temperature........................ ...... 43.1° F. for air of 41.9° F. and 88 per cent relative humidity at the height of 3191 dynamic meters.

Table 14 A.—*Distances in dynamic meters from the earth's surface to the nearest standard isobaric surfaces, the average virtual temperature of the sheet being 0° C. and the pressure at the station being given in millimeters of mercury.*

Pressure at the station (mm. Hg).	Standard surfaces (m-bars).			
	700	600	500	Δ
450	−1209	−1	1428	17
451	−1192	16	1445	18
452	−1174	34	1463	17
453	−1157	51	1480	17
454	−1140	68	1497	17
455	−1123	85	1514	18
456	−1105	103	1532	17
457	−1088	120	1549	17
458	−1071	137	1566	17
459	−1054	154	1583	17
460	−1037	171	1600	17
461	−1020	188	1617	17
462	−1003	205	1634	17
463	− 986	222	1651	17
464	− 969	239	1668	17
465	− 952	256	1685	17
466	− 935	273	1702	16
467	− 919	289	1718	17
468	− 902	306	1735	17
469	− 885	323	1752	17
470	− 868	340	1769	16
471	− 852	356	1785	17
472	− 835	373	1802	17
473	− 818	390	1819	16
474	− 802	406	1835	17
475	− 785	423	1852	16
476	− 769	439	1868	17
477	− 752	456	1885	16
478	− 738	472	1901	16
479	− 720	488	1917	17
480	− 703	505	1934	16
481	− 687	521	1950	16
482	− 671	537	1966	16
483	− 655	553	1982	17
484	− 638	570	1999	16
485	− 622	586	2015	16
486	− 607	602	2031	16
487	− 590	618	2047	16
488	− 574	634	2063	16
489	− 558	650	2079	16
490	− 542	666	2095	16
491	− 526	682	2111	16
492	− 510	698	2127	16
493	− 494	714	2143	16
494	− 478	730	2159	16
495	− 462	746	2175	16
496	− 446	762	2191	15
497	− 431	777	2206	16
498	− 415	793	2222	16
499	− 399	809	2238	16
500	− 383	825	2254	15
501	− 368	840	2269	16
502	− 352	856	2285	15
503	− 337	871	2300	16
504	− 321	887	2316	16
505	− 305	903	2332	15
506	− 290	918	2347	15
507	− 275	933	2362	16
508	− 259	949	2378	15
509	− 244	964	2393	16

Pressure at the station (mm. Hg).	Standard surfaces (m-bars).				
	800	700	600	500	Δ
510		−228	980	2409	15
511		−213	995	2424	15
512		−198	1010	2439	16
513		−182	1026	2455	15
514		−167	1041	2470	15
515		−152	1056	2485	15
516		−137	1071	2500	16
517		−121	1087	2516	15
518		−106	1102	2531	15
519		− 91	1117	2546	15
520		− 76	1132	2561	15
521		− 61	1147	2576	15
522		− 46	1162	2591	15
523		− 31	1177	2606	15
524		− 16	1192	2621	15
525	−1047	− 1	1207	2636	15
526	−1032	14	1222	2651	15
527	−1017	29	1237	2666	15
528	−1002	44	1252	2681	14
529	− 988	58	1266	2695	15
530	− 973	73	1281	2710	15
531	− 958	88	1296	2725	15
532	− 943	103	1311	2740	14
533	− 929	117	1325	2754	15
534	− 914	132	1340	2769	15
535	− 899	147	1355	2784	15
536	− 884	162	1370		14
537	− 870	176	1384		15
538	− 855	191	1399		14
539	− 841	205	1413		15
540	− 826	220	1428		14
541	− 812	224	1442		15
542	− 797	249	1457		14
543	− 783	263	1471		15
544	− 768	278	1486		14
545	− 754	292	1500		15
546	− 739	307	1515		14
547	− 725	321	1529		14
548	− 711	335	1543		14
549	− 697	349	1557		15
550	− 682	364	1572		14
551	− 668	378	1586		14
552	− 654	392	1600		14
553	− 640	406	1614		14
554	− 626	420	1628		15
555	− 611	435	1643		14
556	− 597	449	1657		14
557	− 583	463	1671		14
558	− 569	477	1685		15
559	− 554	492	1700		13
560	− 541	505	1713		14
561	− 527	519	1727		14
562	− 513	533	1741		14
563	− 499	547	1755		14
564	− 485	561	1769		14
565	− 471	575	1783		13
566	− 458	588	1796		14
567	− 444	602	1810		14
568	− 430	616	1824		14
569	− 416	630	1838		14

Pressure at the station (mm. Hg).	Standard surfaces (m-bars).				
	900	800	700	600	Δ
570		−402	644	1852	13
571		−389	657	1865	14
572		−375	671	1879	14
573		−361	685	1893	13
574		−348	698	1906	14
575		−334	712	1920	14
576		−320	726	1934	13
577		−307	739	1947	14
578		−293	753	1961	13
579		−280	766	1974	14
580		−266	780	1988	13
581		−253	793	2001	14
582		−239	807	2015	13
583		−226	820	2028	14
584		−212	834	2042	13
585		−199	847	2055	14
586		−185	861	2069	13
587		−172	874	2082	13
588		−159	887	2095	14
589		−145	901	2109	13
590		−132	914	2122	13
591		−119	927	2135	13
592		−106	940	2148	14
593		− 92	954	2162	13
594		− 79	967	2175	13
595		− 66	980	2188	13
596		− 53	993	2201	13
597		− 40	1006	2214	13
598		− 27	1019	2227	13
599		− 14	1032	2240	13
600	−924	− 1	1045	2253	14
601	−910	13	1059	2267	13
602	−897	26	1072	2280	13
603	−884	39	1085	2293	13
604	−871	52	1098	2306	13
605	−858	65	1111	2319	13
606	−845	78	1124	2332	12
607	−833	90	1136	2344	13
608	−820	103	1149	2357	13
609	−807	116	1162	2370	13
610	−794	129	1175	2383	13
611	−781	142	1188	2396	13
612	−768	155	1201	2409	13
613	−755	168	1214	2422	12
614	−743	180	1226	2434	13
615	−730	193	1239	2447	13
616	−717	206	1252	2460	13
617	−704	219	1265	2473	12
618	−692	231	1277	2485	13
619	−679	244	1290	2498	12
620	−667	256	1302	2510	13
621	−654	269	1315	2523	13
622	−641	282	1328	2536	12
623	−629	294	1340	2548	13
624	−616	307	1353	2561	12
625	−604	319	1365	2573	13
626	−591	332	1378	2586	12
627	−579	344	1390	2598	13
628	−566	357	1403	2611	12
629	−554	369	1415	2623	13

Table 14 A (continued).—*Distances in dynamic meters from the earth's surface to the nearest standard isobaric surfaces, the average virtual temperature of the sheet being 0° C. and the pressure at the station being given in millimeters of mercury.*

Pressure at the station (mm. Hg).	Standard surfaces (m-bars).				
	1000	900	800	700	Δ
630		−541	382	1428	12
631		−529	394	1440	13
632		−516	407	1453	12
633		−504	419	1465	12
634		−492	431	1477	13
635		−479	444	1490	12
636		−467	456	1502	12
637		−455	468	1514	13
638		−442	481	1527	12
639		−430	493	1539	12
640		−418	505	1551	12
641		−406	517	1563	13
642		−393	530	1576	12
643		−381	542	1588	12
644		−369	554	1600	12
645		−357	566	1612	12
646		−345	578	1624	13
647		−332	591	1637	12
648		−320	603	1649	12
649		−308	615	1661	12
650		−296	627	1673	12
651		−284	639	1685	12
652		−272	651	1697	12
653		−260	663	1709	12
654		−248	675	1721	12
655		−236	687	1733	12
656		−224	699	1745	12
657		−212	711	1757	12
658		−200	723	1769	12
659		−188	735	1781	11
660		−177	746	1792	12
661		−165	758	1804	12
662		−153	770	1816	12
663		−141	782	1828	12
664		−129	794	1840	12
665		−117	806	1852	11
666		−106	817	1863	12
667		− 94	829	1875	12
668		− 82	841	1887	11
669		− 71	852	1898	12
670		− 59	864	1910	12
671		− 47	876	1922	12
672		− 35	888	1934	11
673		− 24	899	1945	12
674		− 12	911	1957	11
675	−827	− 1	922	1968	12
676	−815	11	934	1980	12
677	−803	23	946	1992	11
678	−792	34	957	2003	12
679	−780	46	969	2015	11
680	−769	57	980	2026	12
681	−757	69	992	2038	11
682	−746	80	1003	2049	12
683	−734	92	1015	2061	11
684	−723	103	1026	2072	12
685	−711	115	1038	2084	11
686	−700	126	1049	2095	12
687	−688	138	1061	2107	11
688	−677	149	1072	2118	11
689	−666	160	1083	2129	12

Pressure at the station (mm. Hg).	Standard surfaces (m-bars).				
	1000	900	800	700	Δ
690	−654	172	1095	2141	11
691	−643	183	1106	2152	11
692	−632	194	1117	2163	12
693	−620	206	1129	2175	11
694	−609	217	1140	2186	11
695	−598	228	1151	2197	12
696	−586	240	1163	2209	11
697	−575	251	1174	2220	11
698	−564	262	1185	2231	11
699	−553	273	1196	2242	12
700	−541	285	1208	2254	11
701	−530	296	1219	2265	11
702	−519	307	1230	2276	11
703	−508	318	1241	2287	11
704	−497	329	1252	2298	11
705	−486	340	1263	2309	11
706	−475	351	1274	2320	12
707	−463	363	1286	2332	11
708	−452	374	1297	2343	11
709	−441	385	1308	2354	11
710	−430	396	1319	2365	11
711	−419	407	1330	2376	11
712	−408	418	1341	2387	11
713	−397	429	1352	2398	11
714	−386	440	1363	2409	11
715	−375	451	1374	2420	11
716	−364	462	1385	2431	11
717	−353	473	1396	2442	10
718	−343	483	1406	2452	11
719	−332	494	1417	2463	11
720	−321	505	1428	2474	11
721	−310	516	1439		11
722	−299	527	1450		10
723	−289	537	1460		11
724	−278	548	1471		11
725	−267	559	1482		11
726	−256	570	1493		11
727	−245	581	1504		10
728	−235	591	1514		11
729	−224	602	1525		11
730	−213	613	1536		11
731	−202	624	1547		10
732	−192	634	1557		11
733	−181	645	1568		11
734	−170	656	1579		11
735	−159	667	1590		10
736	−149	677	1600		11
737	−138	688	1611		10
738	−128	698	1621		11
739	−117	709	1632		11
740	−106	720	1643		10
741	− 96	730	1653		11
742	− 85	741	1664		10
743	− 75	751	1674		11
744	− 64	762	1685		10
745	− 54	772	1695		11
746	− 43	783	1706		10
747	− 33	793	1716		11
748	− 22	804	1727		10
749	− 12	814	1737		11

Pressure at the station (mm. Hg).	Standard surfaces (m-bars).			
	1000	900	800	Δ
750	−1	825	1748	10
751	9	835	1758	11
752	20	846	1769	10
753	30	856	1779	11
754	41	867	1790	10
755	51	877	1800	10
756	61	887	1810	11
757	72	898	1821	10
758	82	908	1831	10
759	92	918	1841	11
760	103	929	1852	10
761	113	939	1862	10
762	123	949	1872	10
763	133	959	1882	11
764	144	970	1893	10
765	154	980	1903	10
766	164	990	1913	10
767	174	1000	1923	11
768	185	1011	1934	10
769	195	1021	1944	10
770	205	1031	1954	10
771	215	1041	1964	10
772	225	1051	1974	10
773	235	1061	1984	11
774	246	1072	1995	10
775	256	1082	2005	10
776	266	1092	2015	10
777	276	1102	2025	10
778	286	1112	2035	10
779	296	1122	2045	10
780	306	1132	2055	10
781	316	1142	2065	10
782	326	1152	2075	10
783	336	1162	2085	10
784	346	1172	2095	10
785	356	1182	2105	10
786	366	1192	2115	10
787	376	1202	2125	10
788	386	1212	2135	10
789	396	1222	2145	10
790	406	1232	2155	10
791	416	1242	2165	10
792	426	1252	2175	10
793	436	1262	2185	10
794	446	1272	2195	9
795	455	1281	2204	10
796	465	1291	2214	10
797	475	1301	2224	10
798	485	1311	2234	10
799	495	1321	2244	10

Example:

	Dynamic meters.
Height of station above sea-level	39
Table 14 A gives for the 1000 m-bar surface and the pressure at the station of 762.0 mm	123
Virtual-temperature diagram giving for the sheet between the station and the 1000 m-bar surface the average virtual temperature + 25°; table 12 M gives for this temperature and the height 123 the correction	+ 11
Height of standard surface 1000 m-bars above sea-level	173

Table 15 A.—*Distances in dynamic meters from the earth's surface to the nearest standard isobaric surfaces, the average virtual temperature of the sheet being 0° F., and the pressure at the station being given in inches of mercury.*

Pressure at the station (inch Hg).	Standard surfaces (m-bars).				
	800	700	600	500	Δ
17.0		−1433	−304	1032	43
17.1		−1390	−261	1075	43
17.2		−1347	−218	1118	43
17.3		−1304	−175	1161	42
17.4		−1262	−133	1203	42
17.5		−1220	− 91	1245	41
17.6		−1179	− 50	1286	42
17.7		−1137	− 8	1328	41
17.8		−1096	33	1369	41
17.9		−1055	74	1410	41
18.0		−1014	115	1451	41
18.1		− 973	156	1492	40
18.2		− 933	196	1532	40
18.3		− 893	236	1572	40
18.4		− 853	276	1612	40
18.5		− 813	316	1652	39
18.6		− 774	355	1691	40
18.7		− 734	395	1731	39
18.8		− 695	434	1770	39
18.9		− 656	473	1809	38
19.0		− 618	511	1847	39
19.1		− 579	550	1886	38
19.2		− 541	588	1924	38
19.3		− 503	626	1962	38
19.4		− 465	664	2000	38
19.5		− 427	702	2038	37
19.6		− 390	739	2075	37
19.7		− 353	776	2112	37
19.8		− 316	813	2149	37
19.9		− 279	850	2186	37
20.0	−1220	− 242	887	2223	37
20.1	−1183	− 205	924	2260	36
20.2	−1147	− 169	960	2296	36
20.3	−1111	− 133	996	2332	36
20.4	−1075	− 97	1032	2368	36
20.5	−1039	− 61	1068	2404	35
20.6	−1004	− 26	1103	2439	36
20.7	− 968	10	1139	2475	35
20.8	− 933	45	1174	2510	35
20.9	− 898	80	1209	2545	35
21.0	− 863	115	1244	2580	35
21.1	− 828	150	1279	2615	35
21.2	− 793	185	1314	2650	34
21.3	− 759	219	1348	2684	35
21.4	− 724	254	1383	2719	34
21.5	− 690	288	1417	2753	34
21.6	− 656	322	1451	2787	34
21.7	− 622	356	1485	2821	33
21.8	− 589	389	1518	2854	34
21.9	− 555	423	1552	2888	33

Pressure at the station (inch Hg).	Standard surfaces (m-bars).					
	1000	900	800	700	600	Δ
22.0			−522	456	1585	34
22.1			−488	490	1619	33
22.2			−455	523	1652	33
22.3			−422	556	1685	33
22.4			−389	589	1718	32
22.5			−357	621	1750	33
22.6			−324	654	1783	32
22.7			−292	686	1815	32
22.8			−260	718	1847	32
22.9			−228	750	1879	32
23.0			−196	782	1911	32
23.1			−164	814	1943	32
23.2			−132	846	1975	31
23.3			−101	877	2006	31
23.4			− 70	908	2037	32
23.5		−901	− 38	940	2069	31
23.6		−870	− 7	971	2100	31
23.7		−839	24	1002	2131	31
23.8		−808	55	1033	2162	30
23.9		−778	85	1063	2192	31
24.0		−747	116	1094	2223	30
24.1		−717	146	1124	2253	31
24.2		−686	177	1155	2284	30
24.3		−656	207	1185	2314	30
24.4		−626	237	1215	2344	30
24.5		−596	267	1245	2374	30
24.6		−566	297	1275	2404	30
24.7		−536	327	1305	2434	29
24.8		−507	356	1334	2463	29
24.9		−477	386	1364	2493	29
25.0		−449	415	1393		29
25.1		−419	444	1422		29
25.2		−390	473	1451		29
25.3		−361	502	1480		29
25.4		−332	531	1509		29
25.5		−303	560	1538		28
25.6		−275	588	1566		29
25.7		−246	617	1595		28
25.8		−218	645	1623		29
25.9		−189	674	1652		28
26.0	−933	−161	702	1680		28
26.1	−905	−133	730	1708		28
26.2	−877	−105	758	1736		28
26.3	−849	− 77	786	1764		28
26.4	−821	− 49	814	1792		28
26.5	−794	− 21	842	1820		27
26.6	−766	6	869	1847		28
26.7	−738	34	897	1875		27
26.8	−711	61	924	1902		27
26.9	−683	88	951	1929		27

Pressure at the station (inch Hg).	Standard surfaces (m-bars).				
	1000	900	800	700	Δ
27.0	−656	115	978	1956	28
27.1	−629	143	1006	1984	27
27.2	−602	170	1033	2011	27
27.3	−575	197	1060	2038	26
27.4	−549	223	1086	2064	27
27.5	−522	250	1113	2091	27
27.6	−495	277	1140	2118	26
27.7	−469	303	1166	2144	26
27.8	−442	329	1192	2170	27
27.9	−416	356	1219	2197	26
28.0	−390	382	1245	2223	26
28.1	−364	408	1271	2249	26
28.2	−338	434	1297	2275	26
28.3	−312	460	1323	2301	26
28.4	−286	486	1349	2327	26
28.5	−260	511	1374		26
28.6	−234	537	1400		26
28.7	−209	563	1426		25
28.8	−183	588	1451		26
28.9	−158	614	1477		25
29.0	−133	639	1502		25
29.1	−107	664	1527		25
29.2	− 82	689	1552		25
29.3	− 57	714	1577		25
29.4	− 32	739	1602		25
29.5	− 7	764	1627		25
29.6	17	789	1652		25
29.7	42	814	1677		24
29.8	67	838	1701		25
29.9	91	863	1726		24
30.0	116	887	1750		25
30.1	140	912	1775		24
30.2	165	936	1799		24
30.3	189	960	1823		24
30.4	213	984	1847		24
30.5	237	1008	1871		24
30.6	261	1032	1895		24
30.7	285	1056	1919		24
30.8	309	1080	1943		24
30.9	332	1104	1967		23
31.0	356	1127	1990		24
31.1	380	1151	2014		23
31.2	403	1174	2037		24
31.3	427	1198	2061		23
31.4	450	1221	2084		24
31.5	473	1245	2108		23
31.6	497	1268	2131		23
31.7	520	1291	2154		23
31.8	543	1314	2177		23
31.9	566	1337	2200		23

Table 15 A (continued).—*Distances in dynamic meters from the earth's surface to the nearest standard isobaric surfaces, the average virtual temperature of the sheet being 0° F. and the pressure at the station being given in inches of mercury.*

Pressure at the station (inch Hg).	Standard surfaces (m-bars). 1000	900	800	Δ
28.80	−183	588	1451	3
28.81	−180	591	1454	2
28.82	−178	593	1456	3
28.83	−175	596	1459	2
28.84	−173	598	1461	3
28.85	−170	601	1464	2
28.86	−168	603	1466	3
28.87	−165	606	1469	2
28.88	−163	608	1471	3
28.89	−160	611	1474	2
28.90	−158	613	1476	3
28.91	−155	616	1479	3
28.92	−152	619	1482	2
28.93	−150	621	1484	2
28.94	−148	623	1487	2
28.95	−145	626	1489	3
28.96	−143	628	1492	2
28.97	−140	631	1494	3
28.98	−138	633	1497	2
28.99	−135	636	1499	3
29.00	−132	639	1502	2
29.01	−130	641	1504	3
29.02	−127	644	1507	2
29.03	−125	646	1509	3
29.04	−122	649	1512	2
29.05	−120	651	1514	3
29.06	−116	655	1517	2
29.07	−115	656	1519	3
29.08	−112	659	1522	2
29.09	−110	661	1524	3
29.10	−107	664	1527	3
29.11	−104	667	1530	2
29.12	−102	669	1532	3
29.13	− 99	672	1535	2
29.14	− 97	674	1537	3
29.15	− 94	677	1540	2
29.16	− 92	679	1542	3
29.17	− 89	682	1545	2
29.18	− 87	684	1547	3
29.19	− 84	687	1550	2
29.20	− 82	689	1552	3
29.21	− 79	692	1555	2
29.22	− 77	694	1557	3
29.23	− 74	697	1560	2
29.24	− 72	699	1562	3
29.25	− 69	702	1565	2
29.26	− 67	704	1567	3
29.27	− 64	707	1570	3
29.28	− 61	710	1573	2
29.29	− 59	712	1575	2

Pressure at the station (inch Hg).	Standard surfaces (m-bars). 1000	900	800	Δ
29.30	−57	714	1577	3
29.31	−54	717	1580	2
29.32	−52	719	1582	3
29.33	−49	722	1585	2
29.34	−47	724	1587	3
29.35	−44	727	1590	2
29.36	−42	729	1592	3
29.37	−39	732	1595	2
29.38	−37	734	1597	3
29.39	−34	737	1600	2
29.40	−32	739	1602	3
29.41	−29	742	1605	2
29.42	−27	744	1607	3
29.43	−24	747	1610	2
29.44	−22	749	1612	3
29.45	−19	752	1615	2
29.46	−17	754	1617	3
29.47	−14	757	1620	2
29.48	−12	759	1622	3
29.49	− 9	762	1625	2
29.50	− 7	764	1627	2
29.51	− 5	766	1629	3
29.52	− 2	769	1632	2
29.53	0	771	1634	3
29.54	3	774	1637	2
29.55	5	776	1639	3
29.56	8	779	1642	2
29.57	10	781	1644	3
29.58	13	784	1647	2
29.59	15	786	1649	3
29.60	18	789	1652	2
29.61	20	791	1654	3
29.62	23	794	1657	2
29.63	25	796	1659	3
29.64	28	799	1662	2
29.65	30	801	1664	3
29.66	33	804	1667	2
29.67	35	806	1669	3
29.68	38	809	1672	2
29.69	40	811	1674	3
29.70	43	814	1677	2
29.71	45	816	1679	2
29.72	47	818	1681	3
29.73	50	821	1684	2
29.74	52	823	1686	3
29.75	55	826	1689	2
29.76	57	828	1691	3
29.77	60	831	1694	2
29.78	62	833	1696	3
29.79	65	836	1699	2

Pressure at the station (inch Hg).	Standard surfaces (m-bars). 1000	900	800	Δ
29.80	67	838	1701	3
29.81	70	841	1704	2
29.82	72	843	1706	3
29.83	75	846	1709	2
29.84	77	848	1711	2
29.85	79	850	1713	3
29.86	82	853	1716	2
29.87	84	855	1718	3
29.88	87	858	1721	2
29.89	89	860	1723	2
29.90	91	862	1725	3
29.91	94	865	1728	3
29.92	97	968	1731	2
29.93	99	870	1733	2
29.94	101	872	1735	3
29.95	104	875	1738	2
29.96	106	877	1740	3
29.97	109	880	1743	2
29.98	111	882	1745	3
29.99	114	885	1748	2
30.00	116	887	1750	3
30.01	119	890	1753	2
30.02	121	892	1755	2
30.03	123	894	1757	3
30.04	126	897	1760	2
30.05	128	899	1762	3
30.06	131	902	1765	2
30.07	133	904	1767	3
30.08	136	907	1770	2
30.09	138	909	1772	3
30.10	141	912	1775	2
30.11	143	914	1777	2
30.12	145	916	1779	3
30.13	148	919	1782	2
30.14	150	921	1784	3
30.15	153	924	1787	2
30.16	155	926	1789	3
30.17	158	929	1792	2
30.18	160	931	1794	2
30.19	162	933	1796	3
30.20	165	936	1799	2
30.21	167	938	1801	3
30.22	170	941	1804	2
30.23	172	943	1806	3
30.24	175	946	1809	2
30.25	177	948	1811	2
30.26	179	950	1813	3
30.27	182	953	1816	2
30.28	184	955	1818	3
30.29	187	958	1821	2

Example:

	Dynamic meters.
Height of station above sea-level	39
Table 15 A gives for the 1000 m-bar surface and the pressure at the station of 30.00 inches of mercury	116
Virtual-temperature diagram giving for the sheet between the station and the 1000 m-bar surface the average virtual temperature 77° F.; table 16 A gives for this temperature and 116 dynamic meters	+19
Height of standard surface 1000 m-bars above sea-level	174

Table 16 A.—*Corrections to table 15 A for temperature.*

Height (dynamic meters).	Temperature (° F.).																				
	0	10	20	30	40	50	60	70	80	90	100	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	1	1	1	1	2	2	2	2	0	0	0	0	0	0	0	0	0	0
20	0	0	1	1	2	2	3	3	3	4	4	0	0	0	0	0	0	0	0	0	0
30	0	1	1	2	3	3	4	5	5	6	7	0	0	0	0	0	0	0	0	1	1
40	0	1	2	3	3	4	5	6	7	8	9	0	0	0	0	0	0	1	1	1	1
50	0	1	2	3	4	5	7	8	9	10	11	0	0	0	0	0	1	1	1	1	1
60	0	1	3	4	5	7	8	9	10	12	13	0	0	0	0	1	1	1	1	1	1
70	0	2	3	5	6	8	9	11	12	14	15	0	0	0	0	1	1	1	1	1	1
80	0	2	3	5	7	9	10	12	14	16	17	0	0	0	1	1	1	1	1	1	2
90	0	2	4	6	8	10	12	14	16	18	20	0	0	0	1	1	1	1	1	2	2
100	0	2	4	7	9	11	13	15	17	20	22	0	0	0	1	1	1	1	2	2	2
110	0	2	5	7	10	12	14	17	19	22	24	0	0	0	1	1	1	1	2	2	2
120	0	3	5	8	10	13	16	18	21	24	26	0	0	1	1	1	1	2	2	2	2
130	0	3	6	8	11	14	17	20	23	25	28	0	0	1	1	1	1	2	2	2	3
140	0	3	6	9	12	15	18	21	24	27	30	0	0	1	1	1	2	2	2	2	3
150	0	3	7	10	13	16	20	23	26	29	33	0	0	1	1	1	2	2	2	3	3
160	0	3	7	10	14	17	21	24	28	31	35	0	0	1	1	1	2	2	2	3	3
170	0	4	7	11	15	19	22	26	30	33	37	0	0	1	1	1	2	2	3	3	3
180	0	4	8	12	16	20	24	27	31	35	39	0	0	1	1	2	2	2	3	3	4
190	0	4	8	12	17	21	25	29	33	37	41	0	0	1	1	2	2	2	3	3	4
200	0	4	9	13	17	22	26	30	35	39	44	0	0	1	1	2	2	3	3	3	4
210	0	5	9	14	18	23	27	32	37	41	46	0	0	1	1	2	2	3	3	4	4
220	0	5	10	14	19	24	29	34	38	43	48	0	0	1	1	2	2	3	3	4	4
230	0	5	10	15	20	25	30	35	40	45	50	0	1	1	2	2	3	3	4	4	5
240	0	5	10	16	21	26	31	37	42	47	52	0	1	1	2	2	3	3	4	4	5
250	0	5	11	16	22	27	33	38	44	49	54	0	1	1	2	2	3	3	4	4	5
260	0	6	11	17	23	28	34	40	45	51	57	0	1	1	2	2	3	3	4	5	5
270	0	6	12	18	24	29	35	41	47	53	59	0	1	1	2	2	3	4	4	5	5
280	0	6	12	18	24	30	37	43	49	55	61	0	1	1	2	2	3	4	4	5	5
290	0	6	13	19	25	32	38	44	51	57	63	0	1	1	2	3	3	4	4	5	6
300	0	7	13	20	26	33	39	46	52	59	65	0	1	1	2	3	3	4	5	5	6
310	0	7	13	20	27	34	40	47	54	61	67	0	1	1	2	3	3	4	5	5	6
320	0	7	14	21	28	35	42	49	56	63	70	0	1	1	2	3	3	4	5	6	6
330	0	7	14	22	29	36	43	50	57	65	72	0	1	1	2	3	4	4	5	6	6
340	0	7	15	22	30	37	44	52	59	67	74	0	1	1	2	3	4	4	5	6	7
350	0	8	15	23	30	38	46	53	61	69	76	0	1	2	2	3	4	5	5	6	7
360	0	8	16	24	31	39	47	55	63	71	78	0	1	2	2	3	4	5	5	6	7
370	0	8	16	24	32	40	48	56	64	72	81	0	1	2	2	3	4	5	6	6	7
380	0	8	17	25	33	41	50	58	66	74	83	0	1	2	2	3	4	5	6	7	7
390	0	8	17	25	34	42	51	59	68	76	85	0	1	2	3	3	4	5	6	7	8
400	0	9	17	26	35	44	52	61	70	78	87	0	1	2	3	3	4	5	6	7	8
410	0	9	18	27	36	45	54	62	71	80	89	0	1	2	3	4	4	5	6	7	8
420	0	9	18	27	37	46	55	64	73	82	91	0	1	2	3	4	5	6	6	7	8
430	0	9	19	28	37	47	56	66	75	84	94	0	1	2	3	4	5	6	7	7	8
440	0	10	19	29	38	48	57	67	77	86	96	0	1	2	3	4	5	6	7	8	9
450	0	10	20	29	39	49	59	69	78	88	98	0	1	2	3	4	5	6	7	8	9
460	0	10	20	30	40	50	60	70	80	90	100	0	1	2	3	4	5	6	7	8	9
470	0	10	20	31	41	51	61	72	82	92	102	0	1	2	3	4	5	6	7	8	9
480	0	10	21	31	42	52	63	73	84	94	104	0	1	2	3	4	5	6	7	8	9
490	0	11	21	32	43	53	64	75	85	96	107	0	1	2	3	4	5	6	7	9	10
500	0	11	22	33	44	54	65	76	87	98	109	0	1	2	3	4	5	7	8	9	10

Example:

	Dynamic meters.
Given the temperature of 62.3° F. and the height of...........................	330
Table 16 A gives for 330 meters and 60° F. the correction....................	43
Table 16 A gives for 330 meters and 2.3° F. the correction..................	1
330 meters at 0° F. reduced to 62.3° F. gives the height of.........	374

Table 16 A (continued).—*Corrections to table 15 A for temperature.*

Height (dynamic meters).	Temperature (° F.).																				
	0	10	20	30	40	50	60	70	80	90	100	0	1	2	3	4	5	6	7	8	9
500	0	11	22	33	44	54	65	76	87	98	109	0	1	2	3	4	5	7	8	9	10
510	0	11	22	33	44	56	67	78	89	100	111	0	1	2	3	4	6	7	8	9	10
520	0	11	23	34	45	57	68	79	91	102	113	0	1	2	3	5	6	7	8	9	10
530	0	12	23	35	46	58	69	81	92	104	115	0	1	2	3	5	6	7	8	9	10
540	0	12	24	35	47	59	71	82	94	106	118	0	1	2	4	5	6	7	8	9	11
550	0	12	24	36	48	60	72	84	96	108	120	0	1	2	4	5	6	7	8	10	11
560	0	12	24	37	49	61	73	85	98	110	122	0	1	2	4	5	6	7	9	10	11
570	0	12	25	37	50	62	74	87	99	112	124	0	1	2	4	5	6	7	9	10	11
580	0	13	25	38	51	63	76	88	101	114	126	0	1	3	4	5	6	8	9	10	11
590	0	13	26	39	51	64	77	90	103	116	128	0	1	3	4	5	6	8	9	10	12
600	0	13	26	39	52	65	78	91	104	118	131	0	1	3	4	5	7	8	9	10	12
610	0	13	27	40	53	66	80	93	106	120	133	0	1	3	4	5	7	8	9	11	12
620	0	13	27	40	54	67	81	94	108	121	135	0	1	3	4	5	7	8	9	11	12
630	0	14	27	41	55	69	82	96	110	123	137	0	1	3	4	5	7	8	10	11	12
640	0	14	28	42	56	70	84	98	111	125	139	0	1	3	4	6	7	8	10	11	13
650	0	14	28	42	57	71	85	99	113	127	141	0	1	3	4	6	7	8	10	11	13
660	0	14	29	43	57	72	86	101	115	129	144	0	1	3	4	6	7	9	10	11	13
670	0	15	29	44	58	73	88	102	117	131	146	0	1	3	4	6	7	9	10	12	13
680	0	15	30	44	59	74	89	104	118	133	148	0	1	3	4	6	7	9	10	12	13
690	0	15	30	45	60	75	90	105	120	135	150	0	2	3	5	6	8	9	11	12	14
700	0	15	30	46	61	76	91	107	122	137	152	0	2	3	5	6	8	9	11	12	14
710	0	15	31	46	62	77	93	108	124	139	155	0	2	3	5	6	8	9	11	12	14
720	0	16	31	47	63	78	94	110	125	141	157	0	2	3	5	6	8	9	11	13	14
730	0	16	32	48	64	79	95	111	127	143	159	0	2	3	5	6	8	10	11	13	14
740	0	16	32	48	64	81	97	113	129	145	161	0	2	3	5	6	8	10	11	13	14
750	0	16	33	49	65	82	98	114	131	147	163	0	2	3	5	7	8	10	11	13	15
760	0	17	33	50	66	83	99	116	132	149	165	0	2	3	5	7	8	10	12	13	15
770	0	17	34	50	67	84	101	117	134	151	168	0	2	3	5	7	8	10	12	13	15
780	0	17	34	51	68	85	102	119	136	153	170	0	2	3	5	7	8	10	12	14	15
790	0	17	34	52	69	86	103	120	138	155	172	0	2	3	5	7	9	10	12	14	15
800	0	17	35	52	70	87	104	122	139	157	174	0	2	3	5	7	9	10	12	14	16
810	0	18	35	53	71	88	106	123	141	159	176	0	2	4	5	7	9	11	12	14	16
820	0	18	36	54	71	89	107	125	143	161	178	0	2	4	5	7	9	11	12	14	16
830	0	18	36	54	72	90	108	126	145	163	181	0	2	4	5	7	9	11	13	14	16
840	0	18	37	55	73	91	110	128	146	165	183	0	2	4	5	7	9	11	13	15	16
850	0	19	37	56	74	93	111	130	148	167	185	0	2	4	6	7	9	11	13	15	17
860	0	19	37	56	75	94	112	131	150	168	187	0	2	4	6	7	9	11	13	15	17
870	0	19	38	57	76	95	114	133	152	170	189	0	2	4	6	8	9	11	13	15	17
880	0	19	38	57	77	96	115	134	153	172	192	0	2	4	6	8	10	11	13	15	17
890	0	19	39	58	77	97	116	136	155	174	194	0	2	4	6	8	10	12	14	15	17
900	0	20	39	59	78	98	118	137	157	176	196	0	2	4	6	8	10	12	14	16	18
910	0	20	40	59	79	99	119	139	158	178	198	0	2	4	6	8	10	12	14	16	18
920	0	20	40	60	80	100	120	140	160	180	200	0	2	4	6	8	10	12	14	16	18
930	0	20	40	61	81	101	121	142	162	182	202	0	2	4	6	8	10	12	14	16	18
940	0	20	41	61	82	102	123	143	164	184	205	0	2	4	6	8	10	12	14	16	18
950	0	21	41	62	83	103	124	145	165	186	207	0	2	4	6	8	10	12	14	17	19
960	0	21	42	63	84	104	125	146	167	188	209	0	2	4	6	8	10	13	15	17	19
970	0	21	42	63	84	106	127	148	169	190	211	0	2	4	6	8	11	13	15	17	19
980	0	21	43	64	85	107	128	149	171	192	213	0	2	4	6	9	11	13	15	17	19
990	0	22	43	65	86	108	129	151	172	194	215	0	2	4	6	9	11	13	15	17	19
1000	0	22	44	65	87	109	131	152	174	196	218	0	2	4	7	9	11	13	15	17	20

Example:

	Dynamic meters.
Given the temperature 44° F. and the height of	1462
Table 16 A gives for 1000 meters and 40° F.	87
Table 16 A gives for 1000 meters and 4° F.	9
Table 16 A gives for 462 meters and 40° F.	40
Table 16 A gives for 462 meters and 4° F.	4
1462 meters at 0° F. reduced to 44° F. gives	1602

Table 17 A.—*Temperature correction to be added to the virtual temperature at the earth's surface in order to give the most probable average virtual temperatures in the sheet between the earth's surface and the nearest standard isobaric surfaces (based upon statistics).*

Height of standard surfaces above station (dynamic meters).	Temperature correction (° F.).											
	Winter.			Spring.			Summer.			Autumn.		
	High.	Mean.	Low.	High.	Mean.	Low.	High.	Mean.	Low.	High.	Mean.	Low.
2400	4.6	1.6	−2.9	−5.8	−10.1	−10.1	−7.3	−11.3	−11.2	−2.0	−8.4	−11.5
2300	4.8	1.9	−2.7	−5.4	− 9.6	− 9.7	−6.7	−10.9	−10.7	−1.7	−8.1	−11.0
2200	4.9	2.1	−2.4	−5.0	− 9.2	− 9.2	−6.6	−10.5	−10.1	−1.4	−7.6	−10.5
2100	5.0	2.3	−2.2	−4.7	− 8.8	− 8.8	−6.2	−10.1	− 9.7	−1.1	−7.3	−10.0
2000	5.1	2.5	−2.0	−4.3	− 8.3	− 8.4	−5.8	− 9.7	− 9.2	−0.9	−6.9	− 9.6
1900	5.1	2.7	−1.8	−4.0	− 7.9	− 7.9	−5.5	− 9.3	− 8.6	−0.6	−6.6	− 9.1
1800	5.1	2.8	−1.6	−3.6	− 7.5	− 7.5	−5.1	− 8.9	− 8.1	−0.4	−6.2	− 8.6
1700	5.1	2.9	−1.4	−3.2	− 7.1	− 7.1	−4.7	− 8.5	− 7.6	−0.1	−5.9	− 8.2
1600	5.0	3.0	−1.2	−2.9	− 6.6	− 6.7	−4.4	− 8.1	− 7.1	+0.1	−5.5	− 7.7
1500	4.9	3.1	−1.1	−2.5	− 6.2	− 6.3	−4.0	− 7.7	− 6.7	+0.3	−5.2	− 7.2
1400	4.7	3.1	−1.0	−2.2	− 5.8	− 5.9	−3.6	− 7.3	− 6.2	+0.5	−4.8	− 6.8
1300	4.5	3.1	−0.8	−1.9	− 5.4	− 5.5	−3.3	− 6.9	− 5.7	+0.7	−4.5	− 6.3
1200	4.2	3.0	−0.7	−1.6	− 5.0	− 5.1	−2.9	− 6.5	− 5.2	+0.8	−4.1	− 5.8
1100	4.0	2.9	−0.6	−1.3	− 4.6	− 4.7	−2.5	− 6.1	− 4.8	+0.9	−3.8	− 5.4
1000	3.8	2.7	−0.5	−1.1	− 4.2	− 4.3	−2.2	− 5.6	− 4.3	+1.0	−3.4	− 4.9
900	3.6	2.5	−0.5	−0.9	− 3.8	− 3.9	−1.9	− 5.2	− 3.9	+1.0	−3.1	− 4.5
800	3.3	2.3	−0.4	−0.7	− 3.4	− 3.4	−1.6	− 4.7	− 3.4	+1.0	−2.7	− 4.0
700	2.9	2.1	−0.4	−0.5	− 3.0	− 3.0	−1.2	− 4.1	− 3.0	+0.9	−2.5	− 3.5
600	2.5	1.8	−0.3	−0.4	− 2.5	− 2.6	−1.0	− 3.6	− 2.5	+0.8	−2.1	− 3.0
500	2.2	1.5	−0.2	−0.3	− 2.2	− 2.2	−0.8	− 3.0	− 2.1	+0.7	−1.9	− 2.5
400	1.7	1.2	−0.1	−0.3	− 1.7	− 1.7	−0.6	− 2.5	− 1.6	+0.6	−1.4	− 2.1
300	1.3	0.9	0	−0.2	− 1.3	− 1.3	−0.4	− 2.1	− 1.2	+0.5	−1.1	− 1.6
200	0.9	0.6	0	−0.1	− 0.9	− 0.9	−0.3	− 1.4	− 0.7	+0.3	−0.7	− 1.1
100	0.5	0.3	0	−0.1	− 0.4	− 0.4	−0.2	− 0.7	− 0.4	+0.2	−0.4	− 0.5
0	0	0	0	0	0	0	0	0	0	0	0	0

Extrapolation below the earth's surface (common for all pressures and seasons).

Dynamic meters.	Temperature correction (° F.).	Dynamic meters.	Temperature correction (° F.).	Dynamic meters.	Temperature correction (° F.).
0	0	−500	2.3	−1000	4.5
−100	0.5	−600	2.7	−1100	5.0
−200	0.9	−700	3.2	−1200	5.4
−300	1.4	−800	3.6		
−400	1.8	−900	4.1		

Example: Low pressure, spring; at station pressure 28.87 inches of mercury; virtual temperature + 50.1° F.

1	2	3	4	5	6
800	1469	−6.2	+43.9	141	1610
900	606	−2.6	+47.5	63	669
1000	−165	+0.7	+50.8	−18	−183

Column 1. Standard surfaces.
2. Approximate height of these surfaces, found from table 15 A for the pressure of 28.87 inches of mercury at the station.
3. Temperature corrections according to table 17 A for low pressure spring and for the heights of column 2.
4. Most probable average virtual temperature of the sheets between the earth and the standard surfaces of column 1, found by addition of the corrections of column 3 to the virtual temperature at the station + 50.1° F.
5. Corrections to the height of column 2, found from table 16 A for the heights of column 2 and the average virtual temperature of column 4.
6. Height of the standard surfaces above the station found by addition of the approximate heights of column 1 and the corrections of column 5.

Table 18 A.—*Change of isobaric charts, given in millimeters of mercury for sea-level, into charts of dynamic topography of the 1000 m-bars isobaric surface.*

Height (dyn. meters).	Temperature (° C.) in sea-level.										
	−50	−40	−30	−20	−10	0	10	20	30	40	50
−500	693	696	698	700	702	703	705	707	708	709	711
−450	699	701	703	705	706	708	709	711	712	713	714
−400	704	706	708	710	711	713	714	715	716	717	718
−350	710	712	713	715	716	717	718	720	721	721	722
−300	716	717	718	720	721	722	723	724	725	725	726
−250	721	722	724	725	726	726	727	728	729	730	730
−200	727	728	729	730	730	731	732	733	733	734	734
−150	733	734	734	735	735	736	736	737	737	738	738
−100	738	739	739	740	740	741	741	741	742	742	742
− 50	744	745	745	745	745	745	746	746	746	746	746
0	750	750	750	750	750	750	750	750	750	750	750
50	756	756	755	755	755	755	755	755	754	754	754
100	762	761	761	760	760	760	759	759	759	758	758
150	768	767	766	766	765	765	764	764	763	763	762
200	774	773	772	771	770	769	769	768	767	767	766
250	780	779	777	776	775	774	774	773	772	771	770
300	786	784	783	782	780	779	778	777	776	775	775
350	792	790	789	787	786	784	783	782	781	780	779
400	798	796	794	793	791	789	788	786	785	784	783
450	804	802	800	798	796	794	793	791	790	788	787
500	811	808	806	803	801	799	797	796	794	793	791

Given the isobaric chart in millimeters of mercury and the isothermic chart, centigrade, both for sea-level, the latter reduced to sea-level under the supposition of a fall of temperature of 0.5° C. for every hundred dynamic meters of height. On account of the smallness of the reductions no distinction between true and virtual temperature is required.

(1) Required the level curve of height 0 on the 1000 m-bars surface. The table shows this curve to be identical with the isobaric curve of 750 mm. pressure.

(2) Required the level curve of height 250 dynamic meters on the 1000 m-bars surface. The table shows that it passes closely by the points of intersection of the isothermic curve —50° and the isobaric 780 mm.; the isothermic —40° and the isobaric 779 mm.; the isothermic —30° and the isobaric 777 mm., and so on.

(3) Required the level curve of height —150 dynamic meters on the 1000 m-bars surface. The table shows that it passes closely by the points of intersection of the isothermic curve —50° with the isobaric 733 mm.; of the isothermic curves —40° and —30° with the isobaric 734 mm.; of the isothermic curves —20° and —10° with the isobaric 735 mm., and so on.

The main result is a close accordance of the level lines for the interval of 50 dynamic meters with the isobaric lines for the interval of 5 mm. of mercury.

Table 19 A.—*Change of isobaric charts, given in inches of mercury for sea-level, into charts of dynamic topography of the 1000 m-bars isobaric surface.*

Height (dyn. meters).	Temperature (° F.) at sea-level.															
	−50	−40	−30	−20	−10	0	10	20	30	40	50	60	70	80	90	100
−500	27.34	27.39	27.44	27.49	27.53	27.57	27.61	27.65	27.60	27.73	27.76	27.79	27.83	27.85	27.89	27.91
−450	27.55	27.60	27.64	27.68	27.73	27.76	27.80	27.84	27.87	27.90	27.93	27.96	27.99	28.02	28.05	28.07
−400	27.77	27.81	27.85	27.88	27.92	27.96	27.99	28.02	28.05	28.08	28.11	28.13	28.16	28.18	28.21	28.23
−350	27.98	28.02	28.05	28.09	28.12	28.15	28.18	28.21	28.23	28.26	28.28	28.31	28.33	28.35	28.37	28.39
−300	28.20	28.23	28.26	28.29	28.32	28.34	28.37	28.39	28.42	28.44	28.46	28.48	28.50	28.52	28.53	28.55
−250	28.42	28.44	28.47	28.49	28.52	28.54	28.56	28.58	28.60	28.62	28.63	28.65	28.67	28.68	28.70	28.71
−200	28.64	28.66	28.68	28.70	28.72	28.74	28.75	28.77	28.78	28.80	28.81	28.82	28.84	28.85	28.86	28.88
−150	28.86	28.87	28.89	28.90	28.92	28.93	28.94	28.96	28.97	28.98	28.99	29.00	29.01	29.02	29.03	29.04
−100	29.08	29.09	29.10	29.11	29.12	29.13	29.14	29.15	29.16	29.16	29.17	29.18	29.18	29.19	29.20	29.20
− 50	29.31	29.31	29.32	29.32	29.33	29.33	29.33	29.34	29.34	29.35	29.35	29.35	29.36	29.36	29.36	29.37
0	29.53	29.53	29.53	29.53	29.53	29.53	29.53	29.53	29.53	29.53	29.53	29.53	29.53	29.53	29.53	29.53
50	29.76	29.75	29.75	29.74	29.74	29.73	29.73	29.72	29.72	29.72	29.71	29.71	29.71	29.70	29.70	29.70
100	29.99	29.98	29.97	29.96	29.95	29.94	29.93	29.92	29.91	29.90	29.90	29.89	29.88	29.88	29.87	29.86
150	30.22	30.20	30.18	30.17	30.16	30.14	30.13	30.11	30.10	30.09	30.08	30.07	30.06	30.05	30.04	30.03
200	30.45	30.43	30.41	30.39	30.37	30.35	30.33	30.31	30.30	30.28	30.27	30.25	30.24	30.22	30.21	30.20
250	30.68	30.65	30.63	30.60	30.58	30.55	30.53	30.51	30.49	30.47	30.45	30.43	30.42	30.40	30.38	30.37
300	30.92	30.88	30.85	30.82	30.79	30.76	30.74	30.71	30.69	30.66	30.64	30.62	30.60	30.58	30.56	30.54
350	31.15	31.11	31.08	31.04	31.00	30.97	30.94	30.91	30.88	30.85	30.83	30.80	30.77	30.75	30.73	30.71
400	31.39	31.34	31.30	31.26	31.22	31.18	31.15	31.11	31.08	31.05	31.02	30.99	30.96	30.93	30.90	30.88
450	31.63	31.58	31.53	31.48	31.44	31.39	31.35	31.31	31.28	31.24	31.21	31.17	31.14	31.11	31.08	31.05
500	31.87	31.81	31.76	31.70	31.65	31.61	31.56	31.52	31.48	31.44	31.40	31.36	31.32	31.29	31.26	31.23

Given the isobaric chart in inches of mercury and the isothermic chart, Fahrenheit, both for sea-level, the latter reduced to sea-level under the supposition of a fall of temperature of 1° F. for every 100 dynamic meters of height. On account of the smallness of the reductions no distinction between true and virtual temperature is needed.

(1) Required the level curve of height 0 on the 1000 m-bars surface. The table shows this curve to be identical with the isobaric curve of 29.53 inches pressure.

(2) Required the level curve of height 250 dynamic meters on the 1000 m-bars surface, The table shows it to pass closely by the points of intersection of the isothermic curve —50° F. with the isobaric 30.68 inches; of the isothermic —40° F. with the isobaric 30.65 inches; of the isothermic —30° F. with the isobaric 30.63 inches, and so on.

(3) Required the level curve of height —50 dynamic meters on the 1000 m-bars surface. The table shows it to pass closely by the points of intersection of the isothermic curves —50 and —40° F. with the isobaric 29.31 inches; of the isothermic curves —30 and —20° F. with the isobaric 29.32 inches; of the isothermic curves —10, 0, and +10° F. with the isobaric 29.33 inches, and so on.